Z-pinch

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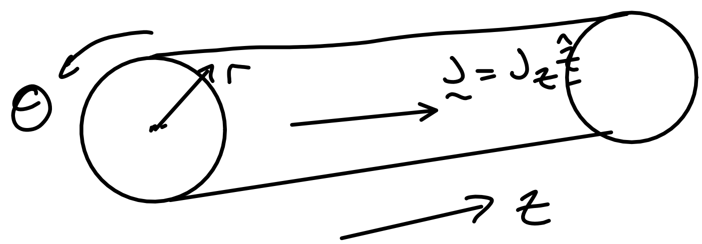


$\underset{\sim}{J} = J_z(r)\,\hat{\underset{\sim}{z}}$

Ideal MHD $\quad \underset{\sim}{J} \times \underset{\sim}{B} = \nabla P$

$\nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{J} \qquad \frac{\partial}{\partial \theta}, \frac{\partial}{\partial z} \to 0$

$$\frac{1}{r}\frac{\partial}{\partial r}(r B_\theta) = \mu_0 J_z \qquad B_\theta = \frac{1}{r}\int r \mu_0 J_z \, dr$$

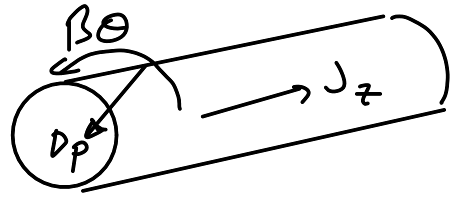
assume J_z constant

$$\Rightarrow B_\theta = \frac{\mu_0 J_z}{2} r$$

$$\underset{\sim}{J} \times \underset{\sim}{B} = \underset{\sim}{J_z} \times \underset{\sim}{B_\theta}$$

$$= J_z^2 \frac{\mu_0}{2} r \underbrace{\hat{\underset{\sim}{z}} \times \hat{\underset{\sim}{\theta}}}_{-\hat{\underset{\sim}{r}}}$$

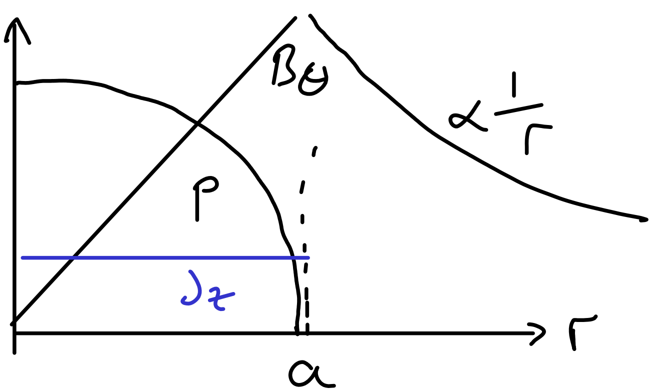
$$\underset{\sim}{J} \times \underset{\sim}{B} = \nabla P = \frac{\partial P}{\partial r}\hat{\underset{\sim}{r}} \qquad -J_z^2 \frac{\mu_0}{2} r = \frac{\partial P}{\partial r}$$

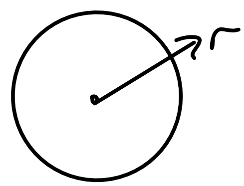


$p(r)$ at $r=a$ (edge of plasma) $p = 0$

$$p(r) = \int_a^r \left(-J_z^2 \frac{\mu_0}{2} r\right) dr$$

$$= \left[-\frac{\mu_0}{2} J_z^2 \frac{1}{2} r^2\right]_a^r = \frac{\mu_0 J_z^2}{4}\left(a^2 - r^2\right)$$





General $J_z(r)$

$$\underset{\sim}{J} \times \underline{B} = \frac{1}{r\mu_0}\frac{\partial}{\partial r}(r B_\theta)\,\underset{\sim}{\hat{z}} \times B_\theta\, \underset{\sim}{\hat{\theta}}$$

$$= -\frac{1}{r\mu_0}\underbrace{\frac{\partial}{\partial r}(r B_\theta)}\, B_\theta\, \underset{\sim}{\hat{r}}$$

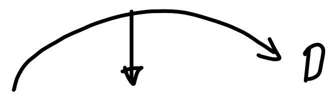
$$r\frac{\partial B_\theta}{\partial r} + B_\theta$$

$$\underset{\sim}{J} \times B = \left[-\frac{B_\theta^2}{r\mu_0} - \frac{1}{\mu_0}\frac{\partial}{\partial r}\left(\frac{B_\theta^2}{2}\right)\right]\underset{\sim}{\hat{r}}$$

$$= \frac{\partial p}{\partial r}\hat{\underset{\sim}{r}}$$

$$\frac{\partial}{\partial r}\left(p + \frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{r\mu_0} = 0$$

fluid pressure — Magnetic pressure — Magnetic tension



General force balance

$$\underset{\sim}{J} = \frac{1}{\mu_0}\nabla \times \underline{B}$$

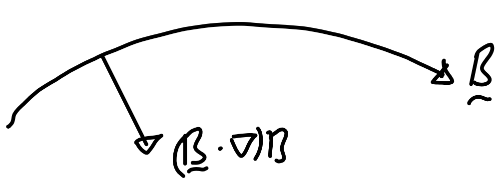
$$\underset{\sim}{J} \times \underline{B} = \frac{1}{\mu_0}(\nabla \times B) \times \underline{B} = \frac{1}{\mu_0}\left[(\underset{\sim}{B}\cdot\nabla)\underline{B} - \nabla\left(\frac{B^2}{2}\right)\right]$$

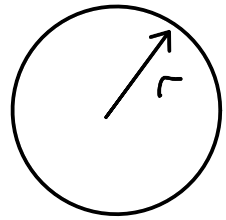
(exact, vector identity)

$$= \frac{1}{\mu_0}(\underline{B}\cdot\nabla)B - \nabla\left(\frac{B^2}{2\mu_0}\right)$$

so in general

$$\nabla\left(p + \frac{B^2}{2\mu_0}\right) - \underbrace{\frac{1}{\mu_0}(\underset{\sim}{B}\cdot\nabla)\underset{\sim}{B}}_{\text{Magnetic tension}} = 0$$





$$(\underline{B}\cdot\nabla)\underline{B} = -\frac{B^2}{r}\hat{\underline{r}}$$