Force free equilibria

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- Force free solutions JxB = 0
- Reversed Field Pinch configuration

$$\underset{\sim}{J} \times \underset{\sim}{B} = 0 \qquad \mu_0 \underset{\sim}{J} = \nabla \times B$$

$$(\nabla \times \underset{\sim}{B}) \times \underset{\sim}{B} = 0 \qquad \boxed{\nabla \times \underset{\sim}{B} = \overset{\alpha(\underset{\sim}{x})}{\alpha} \underset{\sim}{B}}$$

$$\nabla \cdot (\nabla \times \underset{\sim}{B}) = \nabla \cdot (\alpha \underset{\sim}{B})$$

$$0 = \alpha \underbrace{\cancel{\nabla \cdot \underset{\sim}{B}}}_{0} + \underbrace{\underset{\sim}{B} \cdot \nabla \alpha}$$

α constant along magnetic field

Lowest energy state subject to a constraint on the magnetic helicity

- Woltjer's theorem

$$\nabla \times (\nabla \times \underset{\sim}{B}) = \nabla \times (\alpha \underset{\sim}{B})$$

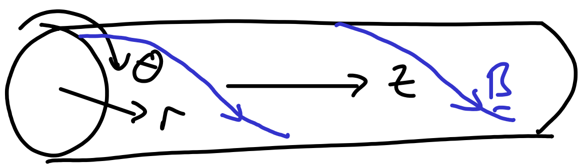
$$\nabla(\underbrace{\cancel{\nabla \cdot \underset{\sim}{B}}}_{0}) - \nabla^2 \underset{\sim}{B} = \alpha \underbrace{\nabla \times \underset{\sim}{B}}_{\alpha \underset{\sim}{B}} + \nabla \alpha \times \underset{\sim}{B}$$

$$\boxed{\nabla^2 \underline{B} + \alpha^2 \underline{B} + \nabla\alpha \times \underline{B} = 0}$$

Reversed field pinch

$\alpha = \text{const}$

(r, θ, z)



symmetric

$$\frac{\partial}{\partial\theta}, \frac{\partial}{\partial z} \to 0$$

B_θ, B_z $\qquad$ $B_r = 0$

$$\theta: \quad \frac{\partial^2 B_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial B_\theta}{\partial r} - \frac{B_\theta}{r^2} + \alpha^2 B_\theta = 0$$

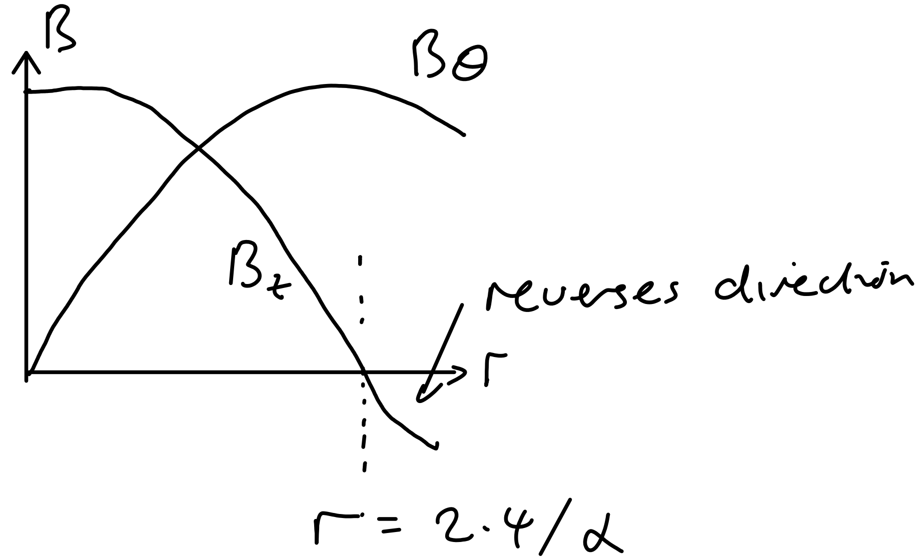
$$r^2 \frac{d^2 B_\theta}{dr^2} + r\frac{dB_\theta}{dr} - (1 - \alpha^2 r) B_\theta = 0$$

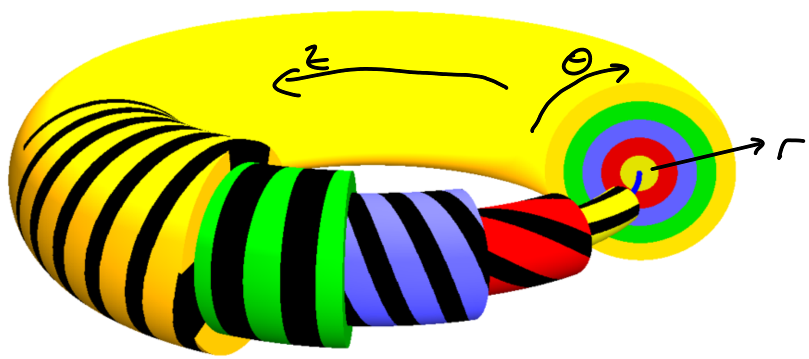
Bessel's equation $\qquad B_\theta = B_{\theta 0} J_1(\alpha r)$

$$z: \quad \frac{\partial^2 B_z}{\partial r^2} + \frac{1}{r}\frac{\partial B_z}{\partial r} + \alpha^2 B_z = 0$$

$$r^2 \frac{dB_z}{dr} + r\frac{dB_z}{dr} + \alpha^2 r^2 B_z = 0$$

$$B_z = B_{z0} J_0(\alpha r)$$


B
Bθ
Bz
reverses direction
r
r = 2.4/α


z
θ
r