# Toroidal devices

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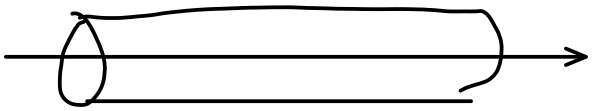
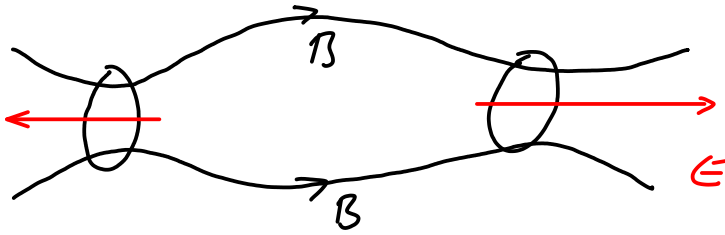
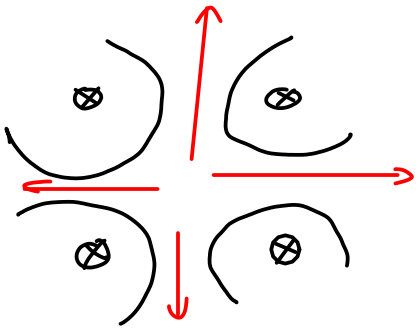



Θ pinch

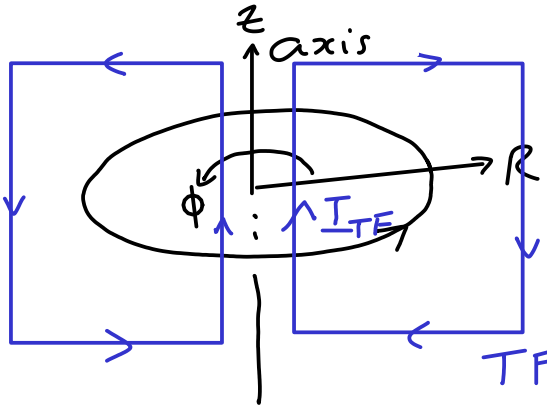



Mirror

End losses

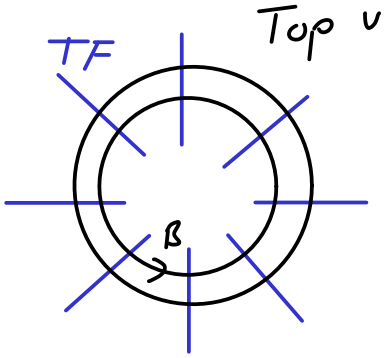


Cusp




$\phi$ toroidal angle

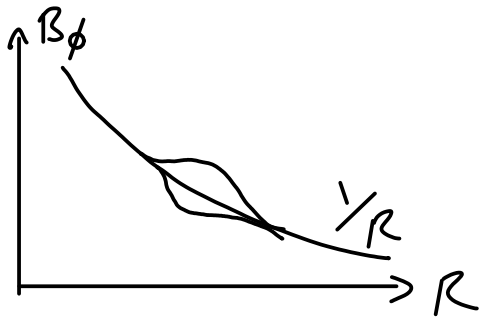
TF coils




Top view

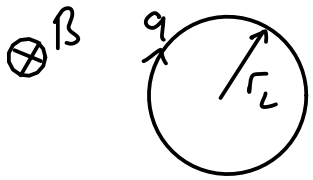
$$\oint \underset{\sim}{B} \cdot dl = \mu_0 I \leftarrow N I_{TF}$$

$$B_\phi 2\pi R = \mu_0 I \qquad B_\phi = \frac{\mu_0 I}{2\pi R}$$




$$B_\phi \sim \frac{1}{R}$$

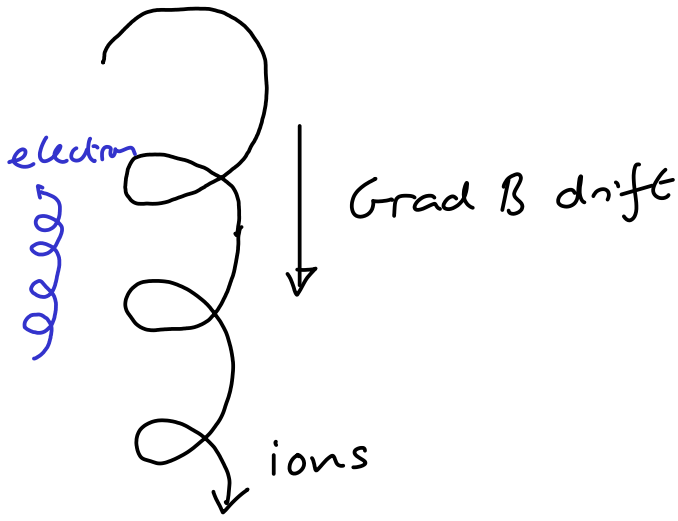
## Particle dynamics

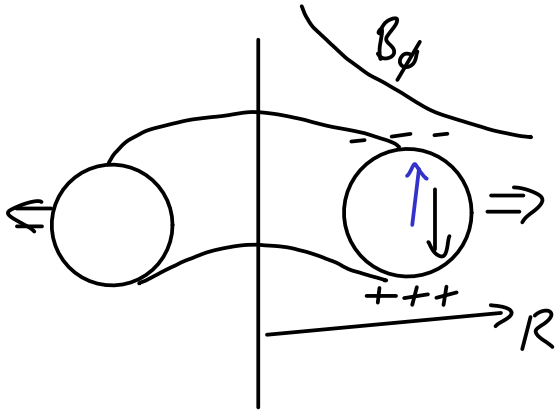



$$r_L = \frac{v_\perp}{\Omega} \qquad \Omega = \frac{qB}{m}$$

$$\Rightarrow \quad r_L \propto \frac{1}{B}$$

high B $\xleftarrow{\nabla B}$ low B




## MHD description

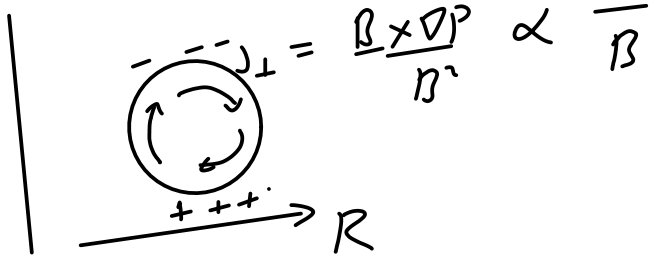
Force balance $\underset{\sim}{J} \times \underset{\sim}{B} = \nabla P$

$$\underset{\sim}{J}_\perp = \frac{\underset{\sim}{B}\times\nabla P}{B^2} \qquad \underset{\sim}{J} = \underset{\sim}{J}_\perp + J_\parallel \frac{\underset{\sim}{B}}{B}$$

$$\nabla\cdot\underset{\sim}{J}_\perp = \nabla\cdot\left(\frac{\underset{\sim}{B}\times\nabla P}{B^2}\right) = -\nabla\cdot\left(P\nabla\times\frac{\underset{\sim}{B}}{B^2}\right) = -\nabla\times\frac{\underset{\sim}{B}}{B^2}\cdot\nabla P$$

$$\nabla\times\left(P\frac{\underset{\sim}{B}}{B^2}\right) = \overbrace{\nabla P\times\frac{\underset{\sim}{B}}{B^2}} + P\nabla\times\left(\frac{\underset{\sim}{B}}{B^2}\right)$$

if $B$ non-uniform, then $\nabla\cdot\underset{\sim}{J}_\perp \neq 0$




$\nabla\cdot\underset{\sim}{J} = 0$ to avoid charge separation

$$\underset{\sim}{J} = \underset{\sim}{J}_\perp + J_\parallel \frac{\underset{\sim}{B}}{B}$$

$\uparrow$

$\nabla\cdot\underset{\sim}{J}_\perp \neq 0$

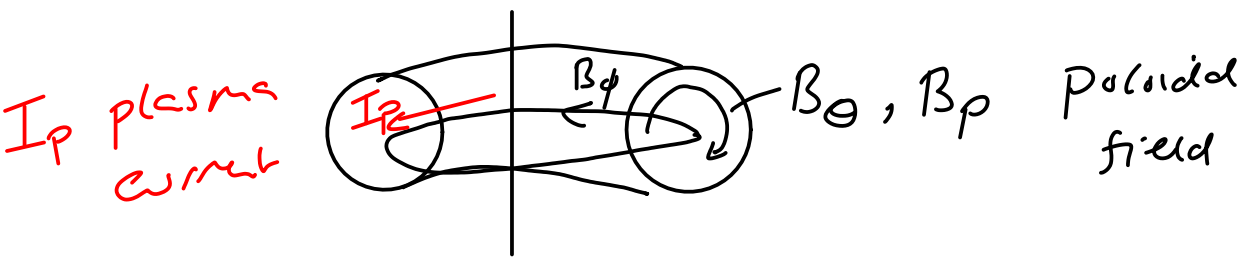
if purely toroidal

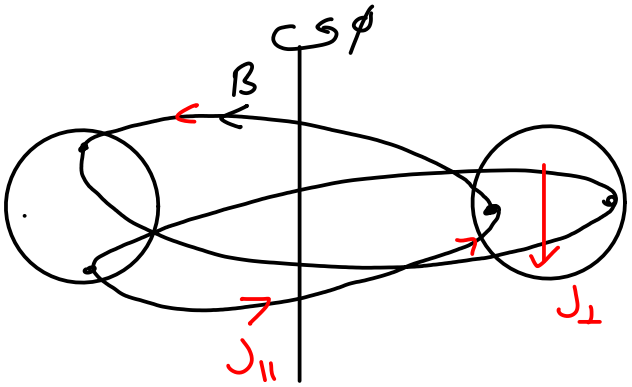
$$\underset{\sim}{B} = B_\phi\hat{\phi}$$

$$\nabla\cdot(J_\parallel\hat{\phi}) = 0$$

$$\Rightarrow \nabla\cdot\underset{\sim}{J} \neq 0$$

need a poloidal magnetic field






$$\nabla \cdot \underset{\sim}{J}_{\parallel} = -\nabla \cdot \underset{\sim}{J}_{\perp}$$

$$\Rightarrow \nabla \cdot \underset{\sim}{J} = 0$$