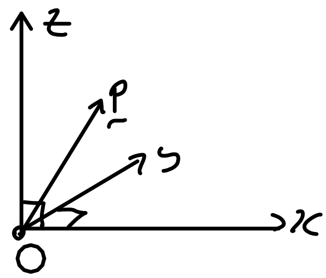
Coordinate systems

Contents

- Cylindrical coordinates
- Transforming vectors and derivatives
- Tangent and reciprocal basis vectors

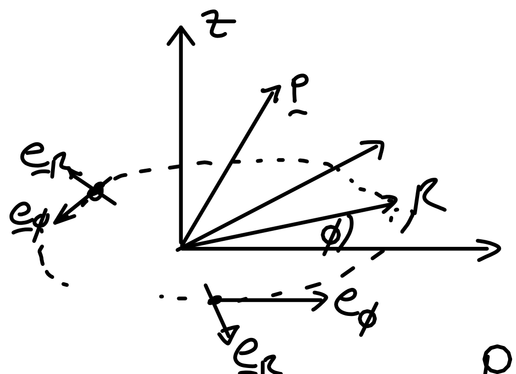


$$\underset{\sim}{p} = p_x \hat{\underset{\sim}{x}} + p_y \hat{\underset{\sim}{y}} + p_z \hat{\underset{\sim}{z}}$$

$$\underset{\sim}{e}_x = \frac{\partial \underset{\sim}{p}}{\partial x} \quad \underset{\sim}{e}_y = \frac{\partial \underset{\sim}{p}}{\partial y} \quad \underset{\sim}{e}_z = \frac{\partial \underset{\sim}{p}}{\partial z}$$

Tangent

Cylindrical



$$x = R\cos\phi \qquad R = \sqrt{x^2+y^2}$$

$$y = R\sin\phi \qquad \tan\phi = y/x$$

Tangent vectors

$$\underset{\sim}{p} = R\cos\phi\, \hat{\underset{\sim}{x}} + R\sin\phi\, \hat{\underset{\sim}{y}} + z\, \hat{\underset{\sim}{z}}$$

Tangent basis vectors

$$\underset{\sim}{e}_R = \frac{\partial \underset{\sim}{p}}{\partial R} = \cos\phi\, \hat{\underset{\sim}{x}} + \sin\phi\, \hat{\underset{\sim}{y}}$$

$$\underset{\sim}{e}_\phi = \frac{\partial \underset{\sim}{p}}{\partial \phi} = -R\sin\phi + R\cos\phi\, \hat{y}$$

$$\underset{\sim}{e}_R \cdot \underset{\sim}{e}_R = 1 \qquad \underset{\sim}{e}_\phi \cdot \underset{\sim}{e}_\phi = R^2 = h_\phi^2 \quad \text{scale factor}$$

$$\underset{\sim}{p} = p^R \underset{\sim}{e}_R + p^\phi \underset{\sim}{e}_\phi + p^z \underset{\sim}{e}_z$$

Derivatives $\longrightarrow$ Reciprocal basis

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\nabla = \nabla x\frac{\partial}{\partial x} + \nabla y\frac{\partial}{\partial y} + \nabla z\frac{\partial}{\partial z}$$

Cylindrical $\quad \nabla = \nabla R\frac{\partial}{\partial R} + \nabla\phi\frac{\partial}{\partial \phi} + \nabla z\frac{\partial}{\partial \phi}$

Reciprocal basis

$$e^R,\ e^\phi,\ e^z$$

e_R, e_ϕ, e_z

tangent

$$e_i \cdot e^j = \delta_i^j \qquad e_R \perp e^\phi, e^z$$

$$\nabla R \cdot p = p^R \overset{1}{\cancel{\nabla R\, e_R}} = p^R$$

What are $\nabla R, \nabla\phi, \nabla z$?

$R = \sqrt{x^2+y^2} \qquad \nabla R = \frac{\partial R}{\partial x}\hat{x} + \frac{\partial R}{\partial y}\hat{y} + \cancel{\frac{\partial R}{\partial z}\hat{z}}$

$\phi = \tan^{-1}(y/x) \qquad = \cos\phi\,\hat{x} + \sin\phi\,\hat{y}$

$$\nabla\phi = -\frac{\sin\phi}{R}\hat{x} + \frac{\cos\phi}{R}\hat{y}$$

$\nabla R \cdot \nabla R = 1 \qquad \nabla\phi\cdot\nabla\phi = \frac{1}{R^2} \quad \left[= \frac{1}{h_\phi^2}\right]$

orthogonal

Two sets of basis vectors

tangent: e_R, e_ϕ, e_z

Reciprocal: $e^R, e^\phi, e^z \quad [= \nabla R, \nabla\phi, \nabla z]$

$$\mathbf{e}_i \cdot \mathbf{e}^j = \delta_i{}^j \qquad \begin{cases} = 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{u} = \sum_i u^i \mathbf{e}_i \qquad u^i = \mathbf{u} \cdot \mathbf{e}^i$$

u^i contravariant component

$$\mathbf{u} = \sum_i u_i \mathbf{e}^i \qquad u_i = \mathbf{u} \cdot \mathbf{e}_i$$

u_i covariant component