Non-orthogonal Coordinates

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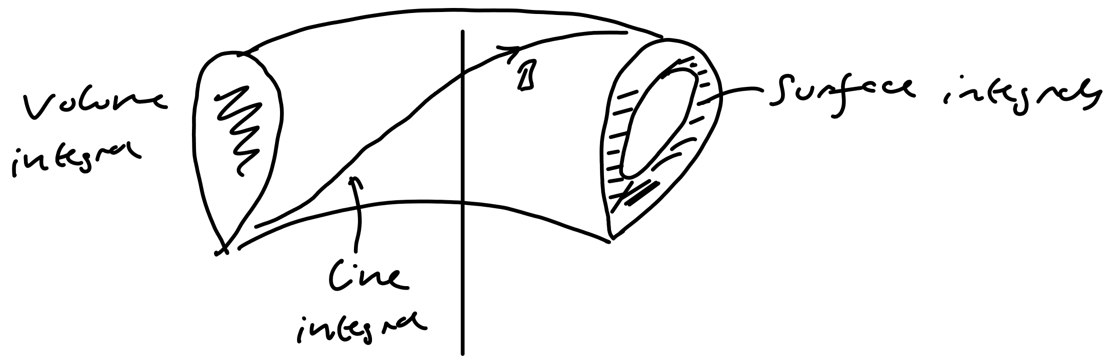
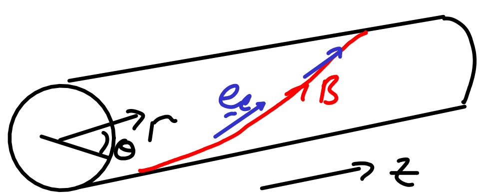


Example: Screw pinch

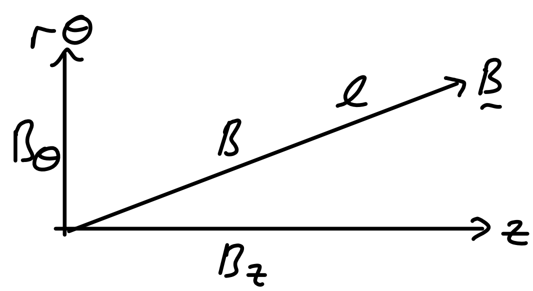


$$\underset{\sim}{B} = B_\theta \underset{\sim}{\hat{\theta}} + B_z \underset{\sim}{\hat{z}}$$

want to find coordinate system where $\underset{\sim}{e}_l \propto \underset{\sim}{B}$

$$\underset{\sim}{e}_l \propto \underset{\sim}{B} \quad \longrightarrow \quad \underset{\sim}{B} \cdot \nabla \rightarrow \frac{\partial}{\partial l}$$

Define l to be length along B



$$l = \frac{B}{B_z} z$$

(r, θ, l)

$$\underset{\sim}{e}_l = \frac{B}{B_z} \underset{\sim}{e}_z \qquad \text{not aligned to } B$$

$\underline{e}_\ell = \frac{\partial \underline{P}}{\partial \ell}$ keep r, θ constant

How does θ vary along B?

$$r\theta = \frac{B_\theta}{B}\ell \Rightarrow \theta = \frac{B_\theta}{rB}\ell$$

along B field

Define

$$\eta = \theta - \frac{B_\theta}{rB}\ell$$ constant along B

<u>Coordinates (r, η, ℓ)</u>

$$x = r\cos\theta = r\cos\left(\eta + \frac{B_\theta}{rB}\ell\right)$$

$$y = r\sin\theta = r\sin\left(\eta + \frac{B_\theta}{rB}\ell\right)$$

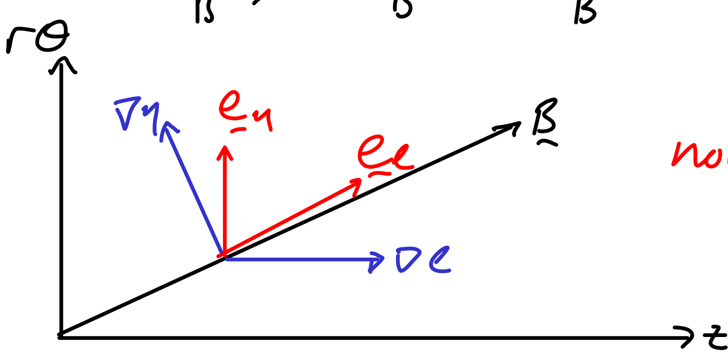
$$z = \frac{B_z}{B}\ell$$

Tangent basis

$$\underline{e}_\eta = \frac{\partial \underline{P}}{\partial \eta} = -r\sin\theta\,\hat{\underline{x}} + r\cos\theta\,\hat{\underline{y}}$$

$$\underline{e}_\ell = \frac{\partial \underline{P}}{\partial \ell} = -r\sin\theta\,\frac{B_\theta}{Br}\hat{\underline{x}} + r\cos\theta\,\frac{B_\theta}{Br}\hat{\underline{y}} + \frac{B_z}{B}\hat{\underline{z}}$$

$$= \frac{B_\theta}{B}\hat{\underline{\theta}} + \frac{B_z}{B}\hat{\underline{z}} = \frac{\underline{B}}{B}$$ unit vector in B direction



non-orthogonal

$$\underline{e}_\eta \cdot \underline{e}_\ell = r\frac{B_\theta}{B}$$

Reciprocal basis

$$\nabla = \underset{\sim}{e}^r \frac{\partial}{\partial r} + \underset{\sim}{e}^\eta \frac{\partial}{\partial \theta} + \underset{\sim}{e}^l \frac{\partial}{\partial l}$$

$$\nabla r \qquad \eta = \theta - \frac{B_\theta}{rB} l \qquad \nabla l = \frac{B}{B_t} \nabla t$$

$$\nabla \eta = \nabla \theta - \frac{B_\theta}{rB} \nabla l \quad \left[+ \nabla \left(\frac{B_\theta}{rB} \right) l \right]$$

Derivatives along B

$$\underset{\sim}{B} = B \underline{\underline{e_l}} \qquad \nabla = \nabla r \frac{\partial}{\partial r} + \nabla \eta \frac{\partial}{\partial \eta} + \underline{\underline{\nabla l}} \frac{\partial}{\partial l}$$

$$\underset{\sim}{B} \cdot \nabla = B \frac{\partial}{\partial l}$$