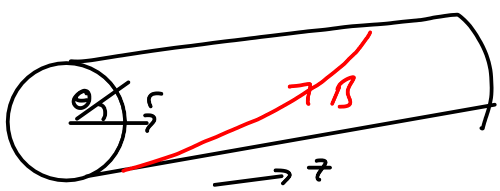
Flux Coordinates

Contents

- Clebsch form of B field
- Magnetic flux in a torus
- Poloidal angle
- Safety factor, local field pitch



$$\ell = \frac{B}{B_z} z$$

$$\underset{\sim}{e}_\ell = \frac{\underset{\sim}{B}}{B} \quad \text{unit vector}$$

Const. along B $\begin{cases} r \\ \eta = \theta - \dfrac{B_\theta}{B r} \ell \end{cases}$

$$B \cdot \nabla = B \frac{\partial}{\partial \ell} \quad \text{parallel derivatives}$$

$$\underset{\sim}{B} \cdot \nabla r = 0$$

$$B \cdot \nabla \eta = 0$$

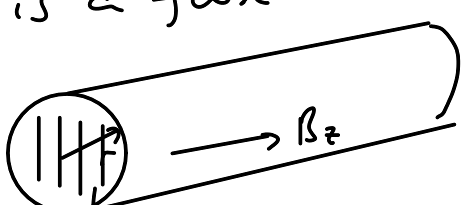
$$\underset{\sim}{B} \propto \nabla r \times \nabla \eta = \frac{\underset{\sim}{B}}{r B_z}$$

$$\underbrace{r B_z \nabla r}_{\nabla \psi} \times \nabla \eta = \underset{\sim}{B}$$

$$\psi = B_z \frac{r^2}{2} \quad \text{flux per radian}$$

$$\underset{\sim}{B} = \nabla \psi \times \nabla \eta \quad \underline{\text{Clebsch}}$$

ψ is a flux



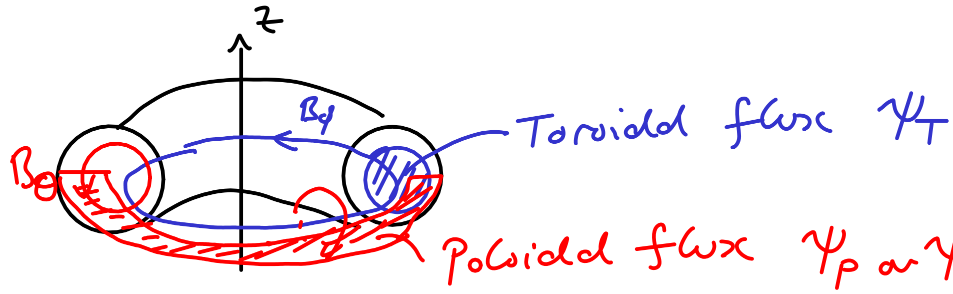
Area × B / 2π

$$\underset{\sim}{A} = \eta \nabla \psi \quad \text{or} \quad \psi \nabla \eta$$

vector potential

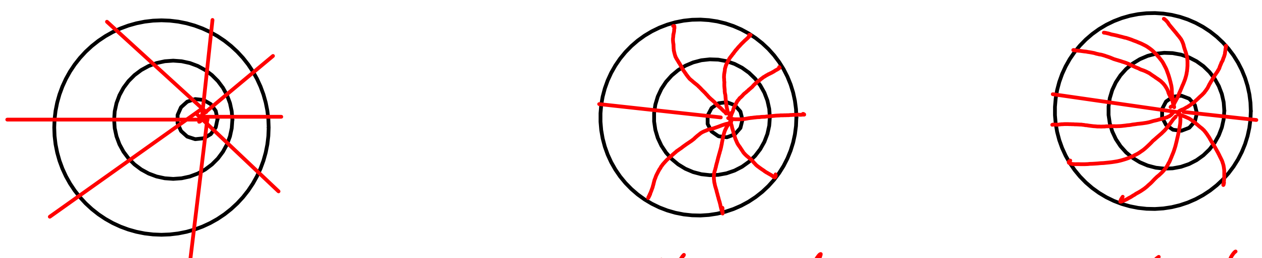
Flux surfaces in a torus

Field lines lie on surfaces of constant ψ
(and constant ψ)



Poloidal coordinates

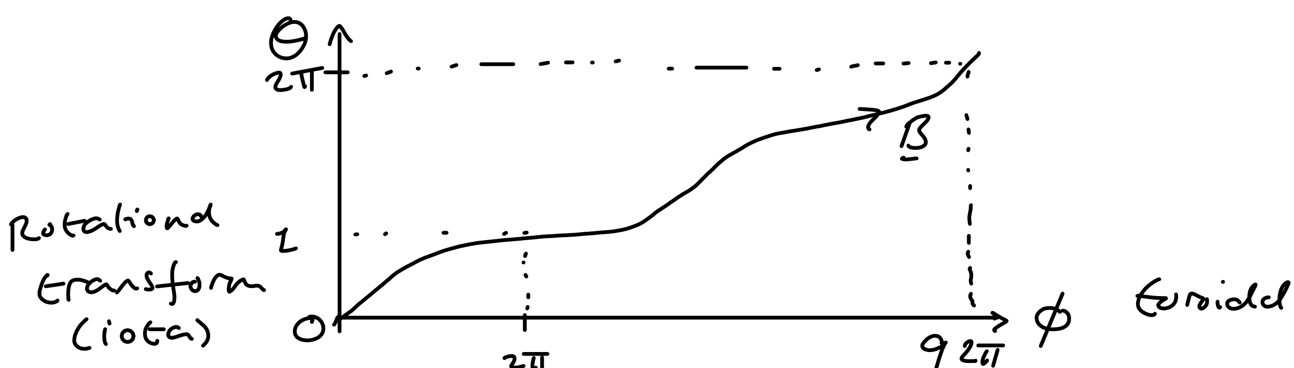
Many choices of poloidal angle θ



Geometric — orthogonal — Constant arc-length

Magnetic field

$\psi = \text{const}$ (on flux surface)



q = Safety factor

$$d\phi = ?\, d\theta$$

$$\underset{\sim}{B}\cdot\nabla\theta = B\frac{d\theta}{dl}$$ l distance along B

$$\underset{\sim}{B}\cdot\nabla\phi = B\frac{d\phi}{dl}$$

along B $$\frac{d\phi}{d\theta} = \frac{\underset{\sim}{B}\cdot\nabla\phi}{\underset{\sim}{B}\cdot\nabla\theta} = \nu(\psi,\theta)$$

local pitch

$$q = \frac{1}{2\pi}\int_{\theta=0}^{\theta=2\pi} d\phi = \frac{1}{2\pi}\int_0^{2\pi}\frac{\underset{\sim}{B}\cdot\nabla\phi}{\underset{\sim}{B}\cdot\nabla\theta}\,d\theta$$

Field-aligned coordinates

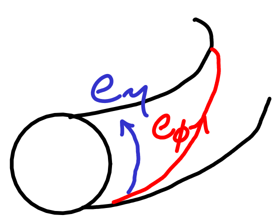
A) Re-define poloidal angle

$$\eta = \theta - \int_0^{\phi}\underbrace{\frac{\underset{\sim}{B}\cdot\nabla\theta}{\underset{\sim}{B}\cdot\nabla\phi}}_{\frac{1}{\nu}}\,d\phi$$

η const. along B

if ν const $\Rightarrow$ $\eta = \theta - \frac{1}{q}\phi$

$\underset{\sim}{e}_\phi$ aligned ω B



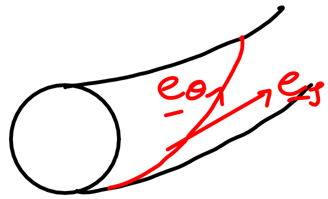
(ψ, η, ϕ)

B) Re-define toroidal angle

$$\zeta = \phi - \int_0^{\theta}\underbrace{\frac{\underset{\sim}{B}\cdot\nabla\phi}{\underset{\sim}{B}\cdot\nabla\theta}}_{\nu(\psi,\theta)}\,d\theta$$

if $v = \text{const}$

$$\zeta = \phi - q\theta$$



$\underline{e}_\zeta$ in toroidal direction