Shafranov shift

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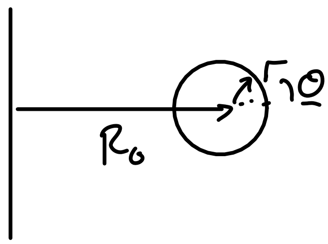


$$\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\psi - \frac{1}{R}\left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\psi$$

$$= -\mu_0 R^2 p' - ff'$$

$$(R_0 + r\cos\theta)^2 = R_0^2 + r2\cos\theta + r^2\cos^2\theta$$

$$\psi(r,\theta) = \psi_0(r) + \psi_1(r,\theta)$$

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \psi_0}{\partial r} = -\mu_0 R^2 p'(\psi_0) - ff'(\psi_0)$$

Corrections $\psi_1(r,\theta) = \bar{\psi}_1(r)\cos\theta$

$$p'(\psi) \simeq p'(\psi_0) + p''(\psi_0)\psi_1$$

$$ff'(\psi) \simeq ff'(\psi_0) + (ff')'\psi_1$$

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \bar{\psi}_1}{\partial r} - \frac{1}{r^2}\bar{\psi}_1 - \underbrace{\frac{1}{R_0}\frac{\partial \psi_0}{\partial r}}_{B_\theta} = -\mu_0 R_0^2 p'' \bar{\psi}_1 - 2\mu_0 r R_0 p' - (ff')'\bar{\psi}_1$$

$$p'' = \frac{1}{R_0 B_\theta}\frac{d}{dr}p'$$

$$(ff')' = \frac{1}{R_0 B_\theta}\frac{d}{dr}(ff')$$

$$\frac{d}{d\psi_0} = \frac{dr}{d\psi_0}\frac{d}{dr} = \frac{1}{R_0 B_\theta}\frac{d}{dr}$$

$\times\, rB_\theta$

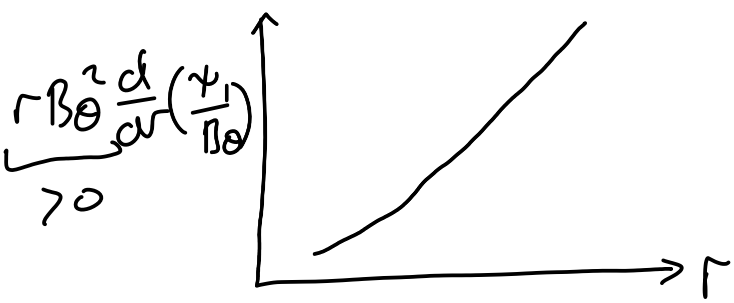
$$B_\theta \frac{d}{dr}\left(r\frac{d\bar{\psi}_1}{dr}\right) - \cancel{\frac{B_\theta}{r}\bar{\psi}_1} - rB_\theta^2 = \boxed{\frac{r}{R_0}\frac{d}{dr}\underline{\left[-\mu_0 R_0^2 p' - ff'\right]}\bar{\psi}_1} - 2\mu_0 r^2 \frac{dp}{dr}$$

$$\frac{1}{r}\frac{d}{dr}r\underbrace{\frac{d\psi_0}{dr}}_{R_0 B_\theta}$$

$$r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rB_\theta)\right] = \underbrace{\frac{d}{dr}\left(r\frac{dB_\theta}{dr}\right)} - \cancel{\frac{B_\theta}{r}}$$

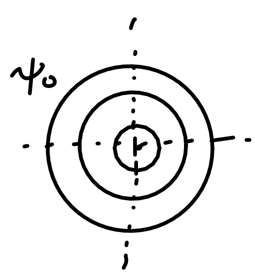
$$\underbrace{B_\theta \frac{d}{dr}\left(r\frac{d\bar{\psi}_1}{dr}\right) - \frac{d}{dr}\left(r\frac{dB_\theta}{dr}\right)\bar{\psi}_1}_{\frac{d}{dr}\left[rB_\theta^2 \frac{d}{dr}\left(\frac{\bar{\psi}_1}{B_\theta}\right)\right]} - rB_\theta^2 = -2\mu_0 r^2\frac{dp}{dr}$$

$$\boxed{\frac{d}{dr}\left[rB_\theta^2 \frac{d}{dr}\left(\frac{\bar{\psi}_1}{B_\theta}\right)\right] = rB_\theta^2 - 2\mu_0 r^2\frac{dp}{dr}} > 0$$

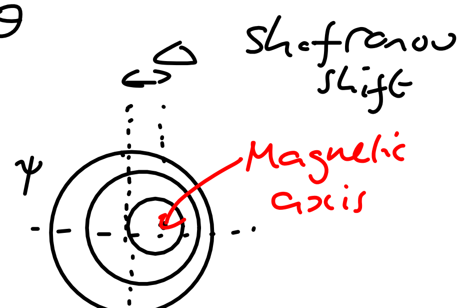


Shifted circles

$$\psi(r,\theta) = \psi_0(r) + \bar{\psi}_1\cos\theta$$



$\Rightarrow$



$\psi(R_0+\Delta) = \psi_0(R_0)$

$$\psi(R) \simeq \psi_0(R-\Delta)$$

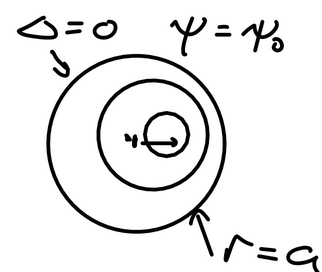
$$\simeq \psi_0(r) - \underbrace{\frac{\partial \psi_0}{\partial R}}\,\Delta(r)$$

$$\frac{d\psi_0}{dr}\frac{dr}{dR} = \frac{d\psi_0}{dr}\cos\theta$$

$$\Rightarrow \psi(r,\theta) \simeq \psi_0 - \underbrace{\frac{d\psi_0}{dr}(r)}_{R_0 B_\theta}\cos\theta\,\Delta(r)$$

$$= \psi_0 + \bar{\psi}_1 \cos\theta$$

$$\Rightarrow \boxed{\bar{\psi}_1(r) = -R_0 B_\theta \Delta(r)}$$



$\Rightarrow$