Vertical Magnetic Field

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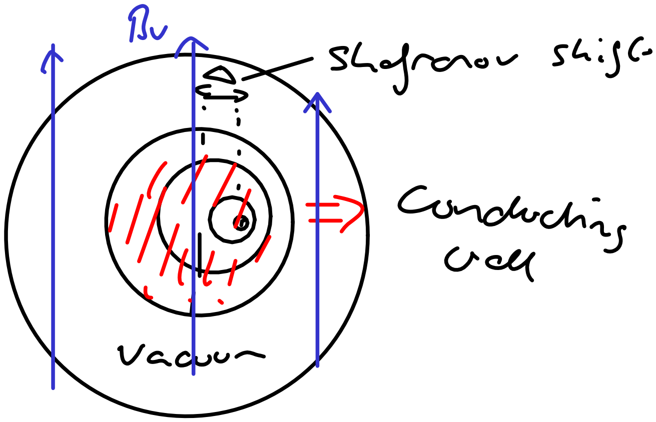


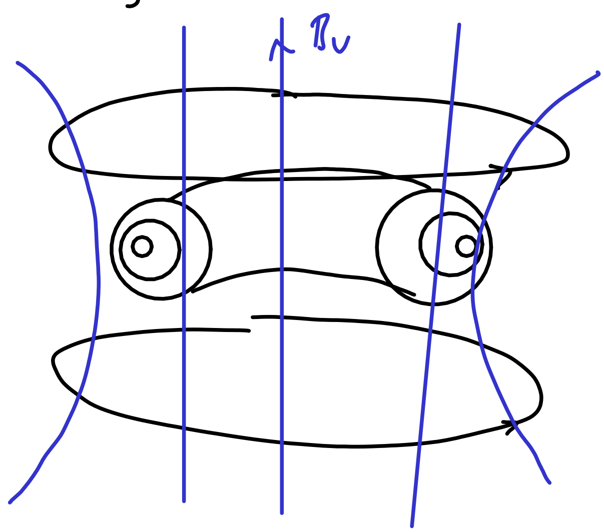
plasma moves outwards

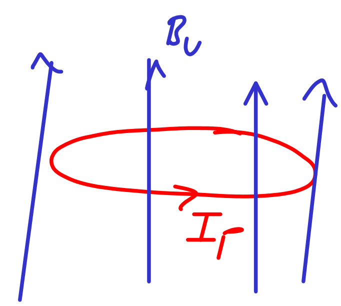
→ B increases on outside
decrease on inside

→ eventually hit wall

Control radial location

⇒ need vertical field





Outside the plasma, the field is that of a circular wire (the plasma) + a vertical field

Field due to a wire of radius R_0

$$\psi_p = \frac{-\mu_0 I}{\pi}\frac{\sqrt{RR_0}}{2k}\left[(2-k^2)K - 2E\right] + c$$

Elliptic integrals

$$k^2 = \frac{4RR_0}{(R+R_0)^2 + z^2}$$

for $a << R << R_0$

$$\psi_p = \frac{-\mu_0 I R_0}{2\pi}\left[\ln\frac{8R_0}{r} - 2 + c + \left(\ln\frac{8R_0}{r} - 1\right)\frac{r}{2R_0}\cos\theta\right]$$

Field due to plasma

+ vertical magnetic field $\psi_V = R_0 B_V \cos\theta$

Terms constant in θ are $\psi = \frac{-\mu_0 I R_0}{2\pi}\ln\left(\frac{8R_0}{r}\right)$

$$B_\theta(r) = \frac{1}{R_0}\frac{\partial\psi}{\partial r} = \underline{\underline{\frac{\mu_0 I}{2\pi r}}}$$

field due to straight wire

$\cos\theta$:

$$\psi_1(r) = \frac{-\mu_0 I R_0}{2\pi}\left(\ln\frac{8R_0}{r} - 1\right)\frac{r}{2R_0} + R_0 B_V r \quad *$$

outer solution

$$\psi(r,\theta) = \psi_0(r) + \psi_1(r)\cos\theta + \dots$$

Last video (Shafranov shift)

$$\frac{d}{dr}\left[r B_\theta^2 \frac{d}{dr}\left(\frac{\psi_1}{B_\theta}\right)\right] = r B_\theta^2 - 2\mu_0 r^2 \frac{dp}{dr}$$

Integrate in r

$$\int_0^r \left[r B_\theta^2 \, dr - 2\mu_0 r^2 \frac{dp}{dr}\right] dr$$

plasma for $0 < r < a$, vacuum for $r > a$

$$= \underline{\int_0^a r B_\theta^2 \, dr} + \int_a^r r B_\theta^2 \, dr - \underbrace{2\mu_0 \int_0^a r^2 \frac{dp}{dr} dr}_{\text{integrate by parts}}$$

vacuum

$$B_\theta = B_\theta(a)\frac{a}{r}$$

$$= 4\mu_0 \int_0^a p(r)\, r \, dr$$

$$= a^2 B_\theta^2(a) \beta_p$$

poloidal beta

$$= a^2 B_\theta^2(a)\left(\frac{l_i}{2} + \ln\left(\frac{r}{a}\right) + \beta_p\right)$$

l_i = normalised internal plasma inductance per unit length

$$l_i = \frac{L_i / 2\pi R_0}{\mu_0 / 4\pi} \qquad \frac{1}{2} L_i I^2 = \int \frac{B_\theta^2}{2\mu_0} d\underline{x}$$

$$\text{so } l_i = \frac{2}{a^2 B_\theta^2(a)} \int_0^a B_\theta^2 \, r \, dr$$

$$\text{So } r B_\theta^2 \frac{d}{dr}\left(\frac{\psi_1}{B_\theta}\right) = a^2 B_\theta^2(a)\left(\frac{l_i}{2} + \ln\frac{r}{a} + \beta_p\right)$$

Integrate again in r

$$\psi_1 = 0 \quad \text{at } r = a$$

$$\frac{\psi_1}{B_\theta} = a^2 B_\theta^2(a)\left(\frac{l_i}{2} + \beta_p\right)\overbrace{\int_a^r \frac{1}{r B_\theta^2}\,dr}^{\frac{1}{a^2 B_\theta^2(a)}\,\frac{1}{2}(r^2-a^2)} + a^2 B_\theta^2(a)\underbrace{\int_a^r \ln\frac{r}{a}\,\frac{1}{r B_\theta^2}\,dr}_{=\frac{1}{2}r^2\ln(r/a) - \frac{1}{4}(r^2-a^2)}$$

$B_\theta = \dfrac{\mu_0 I}{2\pi r}$

in vacuum

vacuum

$B_\theta(r) = B_\theta(a)\dfrac{a}{r}$

$$\Rightarrow \psi_1 = \frac{\mu_0 I}{4\pi r}\left[\left(\beta_p + \frac{l_i}{2} - \frac{1}{2}\right)(r^2 - a^2) + r^2 \ln\frac{r}{a}\right]$$

If $r \gg a$ match against outer solution

$$\psi_1^{inner} \rightarrow \frac{\mu_0 I}{4\pi}\left(\beta_p + \frac{l_i}{2} - \frac{1}{2} + \ln(r/a)\right) r$$

Match $*$ set $\psi_1^{inner} = \psi_1^{outer}$

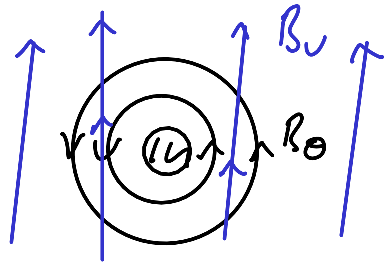
$$\psi_1^{outer} \simeq -\frac{\mu_0 I}{4\pi}\left(\ln\frac{8R_0}{r} - 1\right) r + R_0 B_v r$$

rearrange to find

$$\Rightarrow R_0 B_v r = \frac{\mu_0 I}{4\pi}\left(\beta_p + \frac{l_i}{2} - \frac{1}{2} + \underbrace{\ln\frac{r}{a} + \ln\frac{8R_0}{r}}_{\ln\frac{8R_0}{a}} - 1\right) r$$

$$\Rightarrow B_v = \frac{\mu_0 I}{4\pi R_0}\left(\beta_p + \frac{l_i - 3}{2} + \ln\frac{8R_0}{a}\right) > 0$$

Gives a field which increases B_θ on outside
decreases B_θ on inside



$\Rightarrow$ poloidal field pressure
pushes plasma inwards.