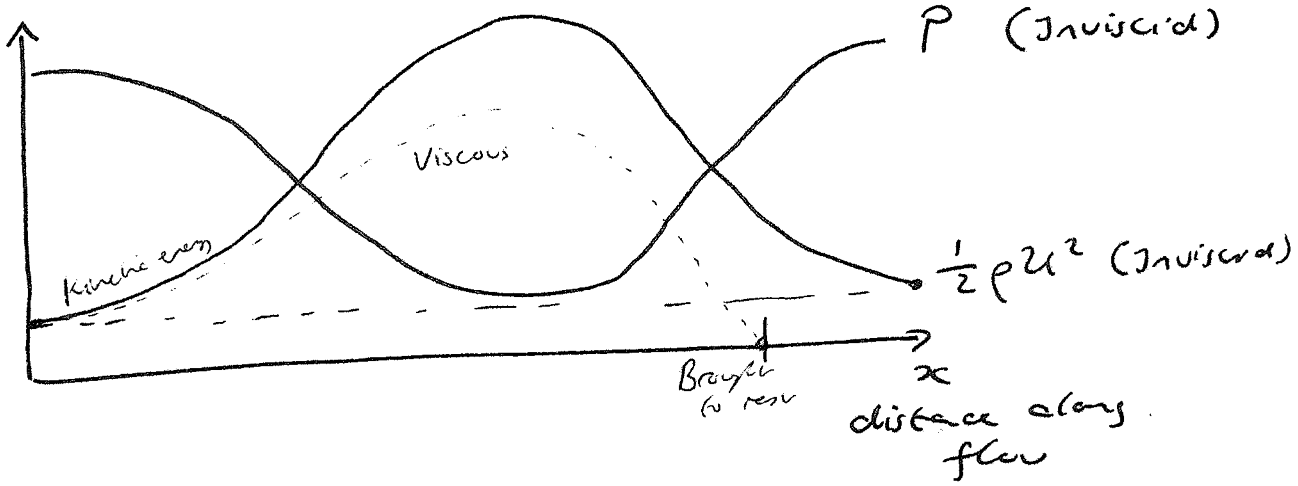
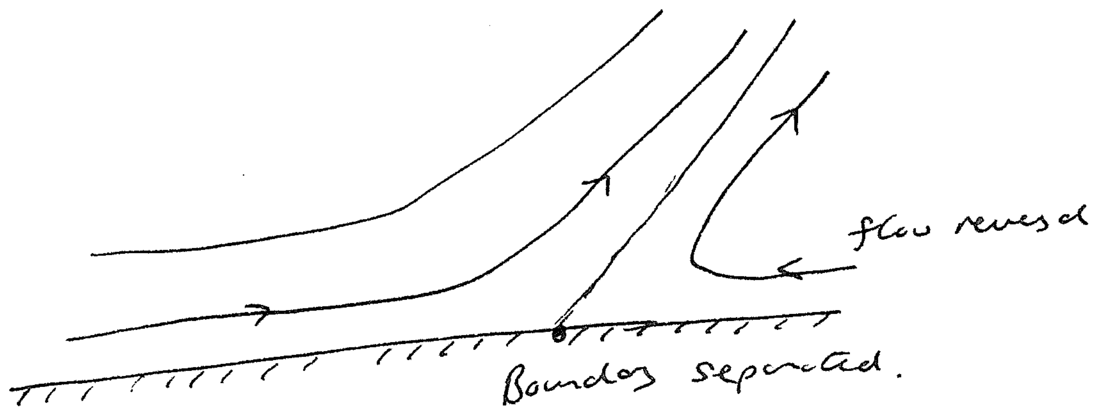
Lecture 12

Boundary layer separation

If an adverse pressure gradient exists, i.e. pressure increases in the direction of flow, viscosity in the boundary layer can lead to separation



In the absence of viscosity, the fluid gains just enough kinetic energy to get over the pressure 'hill'. Viscosity removes kinetic energy, potentially bringing the fluid to a halt. To conserve mass the flow must leave the surface



From the momentum equation

$$\rho\left[\frac{\partial \underline{u}}{\partial t} + \underline{u}\cdot\nabla\underline{u}\right] = -\nabla p + \mu\nabla^2\underline{u}$$

We took the component along the surface x in steady state, assumed a thin boundary layer to get Prandtl's boundary layer equations

$$u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u_x}{\partial y^2}$$

$U(x)$ = flow along the surface in the free stream (outside boundary layer), determined by Bernoulli's equation.

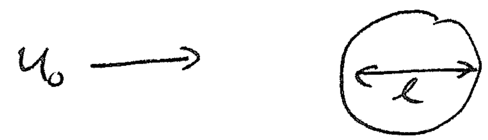
u_x, u_y - velocity in the boundary layer.

This is augmented by incompressible flow condition

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$$

Dimensionless form

If we have flow around a body of characteristic length l, at speed characteristic u_0



then we can cast the boundary layer into dimensionless form

12.2

$$x' = \frac{x}{l} \qquad y' = \frac{y}{l}\sqrt{Re} \qquad u'_x = \frac{u_x}{U_0} \qquad u'_y = \frac{u_y}{U_0}\sqrt{Re} \qquad U' = \frac{U}{U_0}$$

where $Re = \dfrac{l U_0}{\nu}$ is the Reynolds number

Substituting, we get

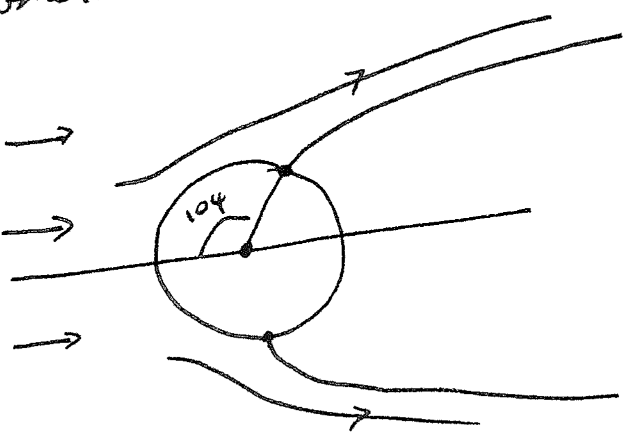
$$u'_x \frac{\partial u'_x}{\partial x'} + u'_y \frac{\partial u'_y}{\partial y'} = U' \frac{dU'}{dx'} + \frac{\partial^2 u'_x}{\partial y'^2}$$

$$\frac{\partial u'_x}{\partial x'} + \frac{\partial u'_y}{\partial y'} = 0$$

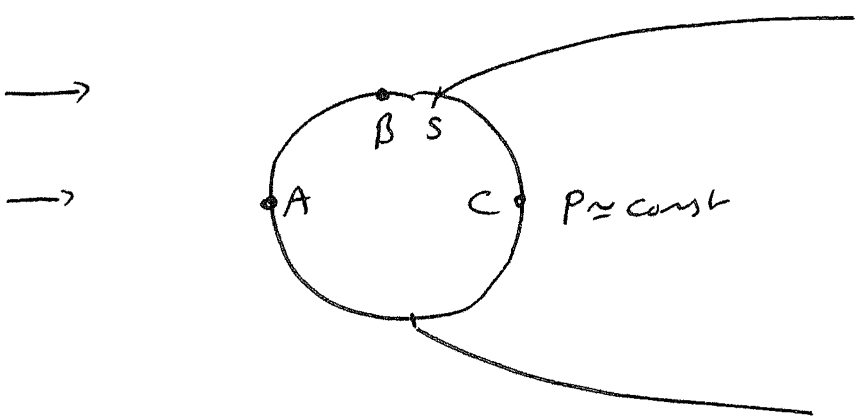
These equations do not depend on fluid properties or even Reynolds number, and so neither do the solutions.

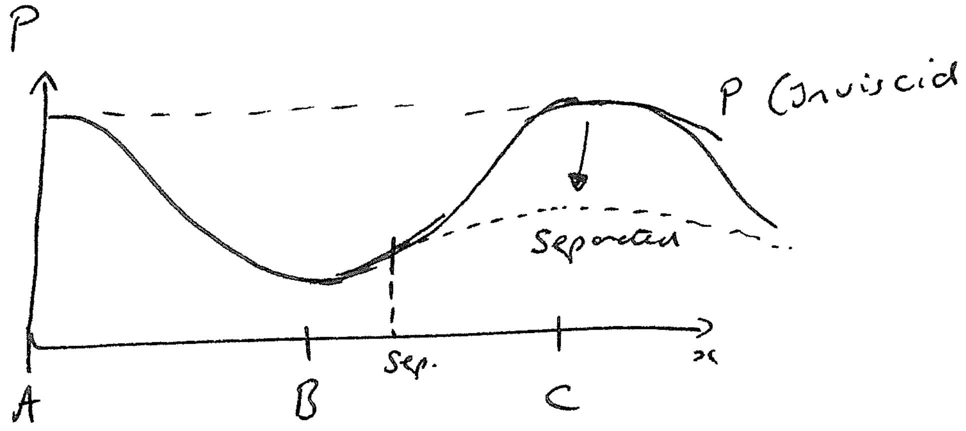
If flow in the boundary layer is laminar (not turbulent) then these boundary layer equations apply. When flow separates, the assumptions fail, so separation points appear as singularities in the boundary layer equations. Because the boundary layer equations are independent of Reynolds number, the location of this separation is independent of Reynolds number for geometrically similar flows

For a cylinder the angle is $104.5°$



Flow separation modifies the pressure at the back of the cylinder





pressure drops at the back of the cylinder

This produces a fore-aft asymmetry which causes a pressure force 'drag', and resolves d'Alambert's "paradox".

This drag is called 'Form' or 'Pressure' drag and is due to the wake formed behind unstreamlined bodies. This wake typically has a size comparable to the size of the body.

This drag varies with the pressure differen

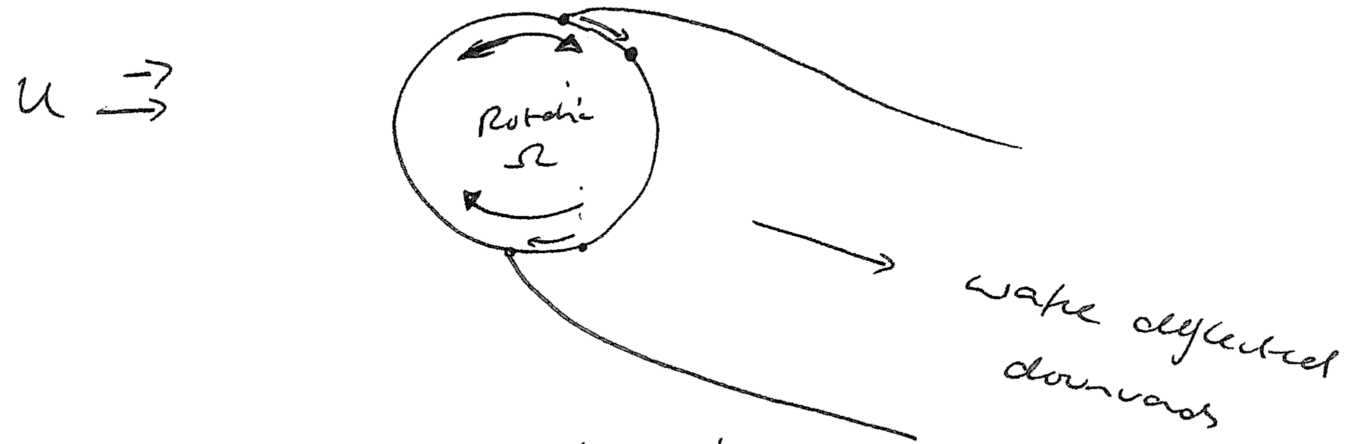
$$\Delta P \sim \rho U^2$$

and the size of the body, S : $F \sim \rho U^2 S$

12.3

Magnus effect

The friction depends on the relative motion of the fluid and surface. By rotating a body, the location of separation can be modified



Downwards deflection of the wake leads to an upwards force on the cylinder

The force on a rotating object can quite often be well described by the (inviscid) Kutta-Joukowski formula

$$F_y = -\rho U \Gamma \qquad \Gamma = \oint \underset{\sim}{u} \cdot dl$$

For a cylinder rotating at an angular velocity Ω

$$\underset{\sim}{u} = \Omega a \qquad \text{so} \qquad \Gamma = \Omega a \cdot 2\pi a = 2\pi \Omega a^2$$

The magnus effect can be seen in many ball games, where balls are deflected in the direction of spin

e.g backspin, topspin, side-spin

Also can be important in ballistics
Has been the basis of a sailing ship driven by rotating cylinders (Flettner rotors) and an aircraft

12.4

Form drag due to boundary layer separation therefore has a drag coefficient:

$$C_D = \frac{F_x}{\frac{1}{2}\rho U^2 S} \sim \text{constant, independent of Reynolds no.}$$

Drag

We have seen three forms of drag

1. Form drag from separated (laminar) flows

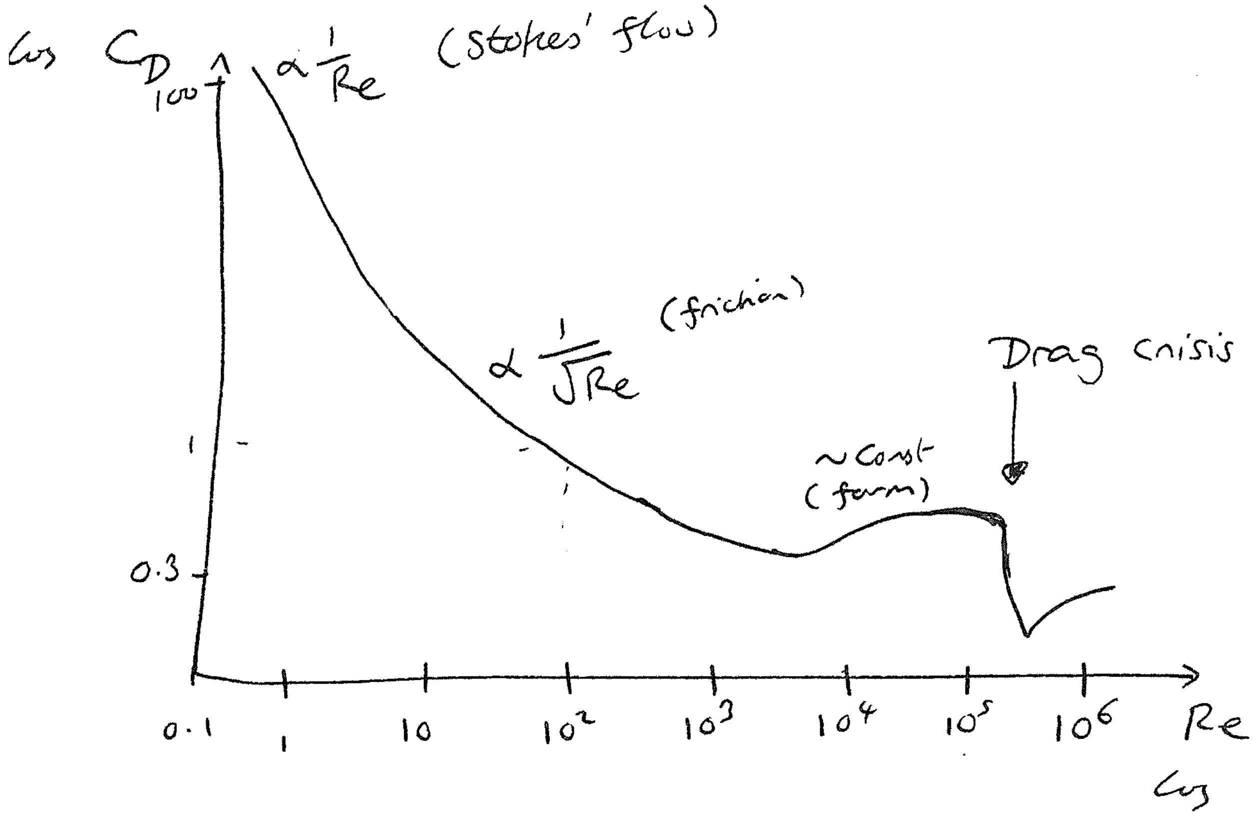
$$C_D \sim \text{constant, independent of } Re.$$

2. Friction drag from the boundary layer. This is the dominant drag for streamlined bodies

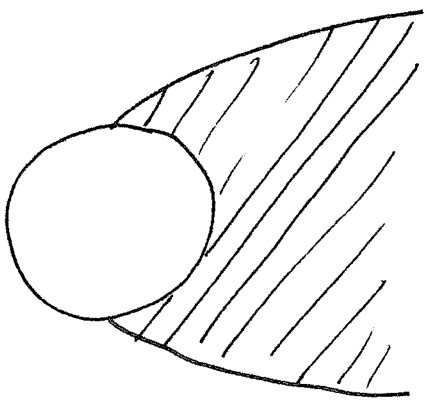
$$C_D \sim Re^{-1/2}$$

3. Induced drag. In 3D, aerofoils shed a vortex from the trailing edge. This puts energy into motion of the fluid, and so work must be done. This is usually small compared to form and friction drag.

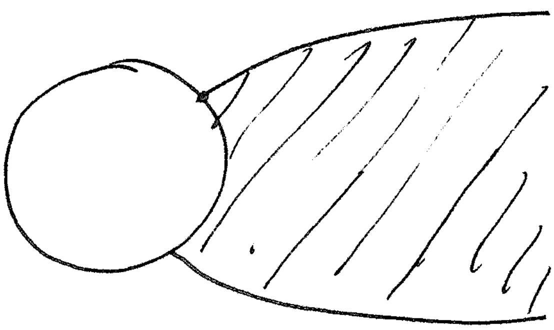
Experimentally measured drag coefficient



For a sphere between $Re \sim 10^4$ and 10^5 the laminar boundary separation theory works, and $C \sim$ const. Around $Re \sim 2 \times 10^5$ the drag drops by a factor of ~3-5. This point is called the 'drag crisis'.



Laminar

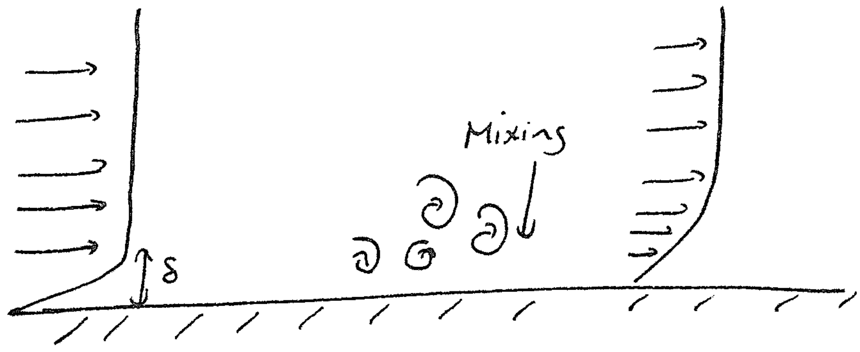


turbulent

Reduction in drag

12.5

At sufficiently high Reynolds numbers, the laminar boundary layer becomes unstable, and transitions to turbulence.



Turbulence transports momentum, speeding up the boundary layer and delaying separation

This reduces the size of the wake, hence reducing drag.

Applications

1) In ball games such as football, tennis, baseball etc. a ball after the shot starts out above the drag crisis. As the ball slows down, it can drop below the critical Reynolds number triggering a sharp increase in drag

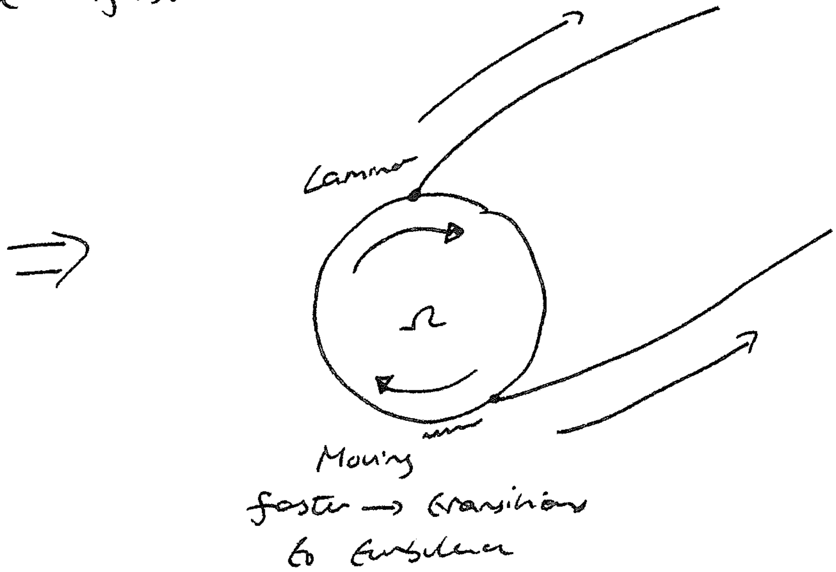
e.g. football: a typical goal or free kick $U \simeq 25$ m/s so Reynolds number for a ball of diameter 22cm is

$$Re \sim \frac{25 \times 0.22}{1.5 \times 10^{-5}\ [m^2/s]} \leftarrow \text{kinematic viscosity of air}$$

$$\sim 3.7 \times 10^5$$

[Note: boundary layer $\delta \sim l/\sqrt{Re} \sim 0.4$ mm]

(2) This transition to turbulence can ~~be stabilised~~ ~~triggered~~ occur on one side of a spinning ball first



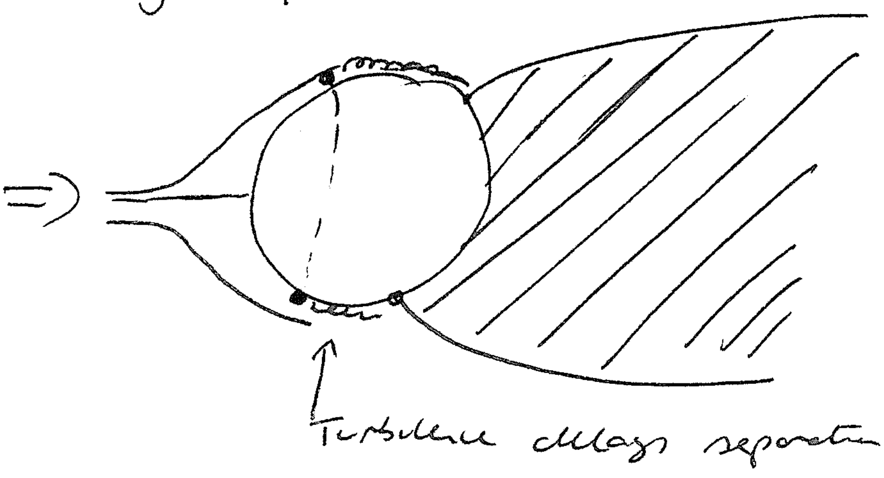
Here the faster moving boundary goes unstable whilst the slower moving boundary remains laminar

⟹ Reverse Magnus effect

This can happen on smooth balls, and can make their deflection less predictable

(3) The transition to turbulence can be deliberately triggered by making the ball non-smooth

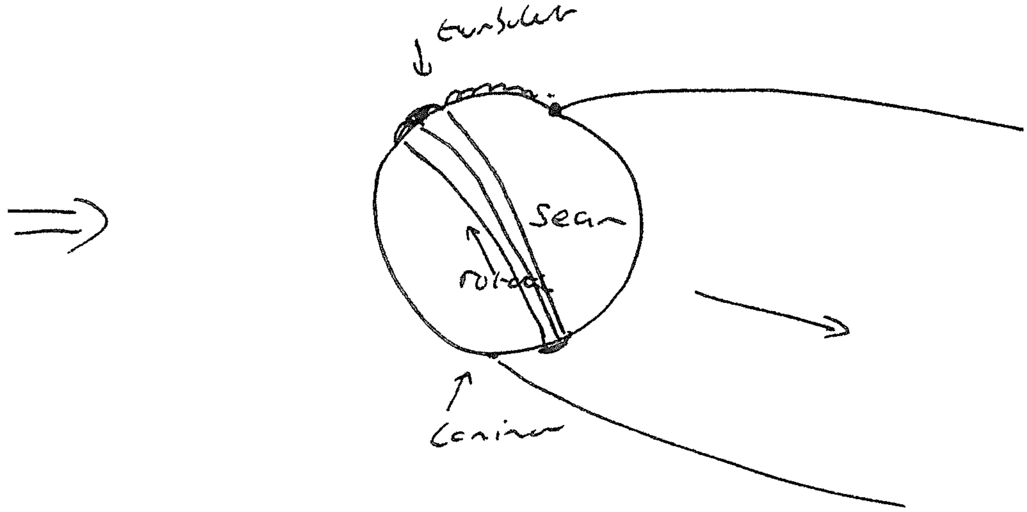
e.g. 'tripwire' on a ball:



12.6

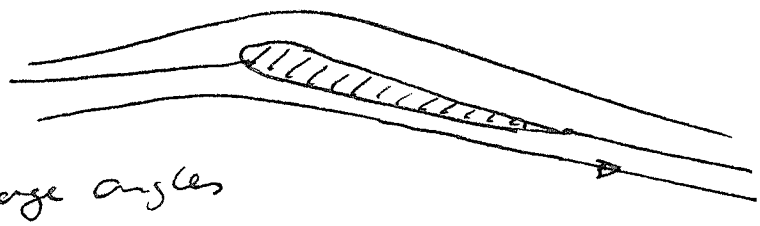
The dimples on a golf ball have a similar effect, reducing the drag crisis Reynolds number to $Re \sim 3\times10^4$ and approximately doubling their range. Similarly for the fuzz on the surface of a tennis ball.

By triggering the transition in an asymmetric way, the ball can be deflected e.g. swing bowling

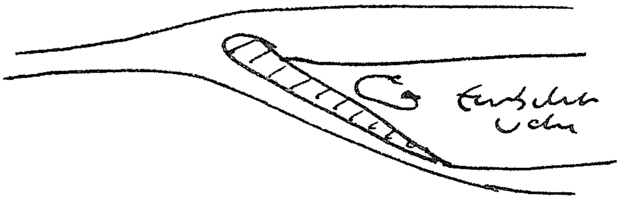


④ Aircraft wings

At small angles of attack (α) the flow is smooth (laminar) and the drag is dominated by friction drag.



At too large angles separation occurs



Drag greatly increases (form drag) and lift decreases

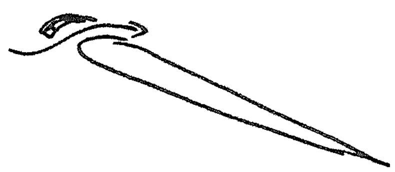
This is a stall, and can happen suddenly during high-lift situations e.g. takeoff

Several methods are used to avoid or recover from this situation

- Put nose of aircraft down, reducing angle of attack

- Deliberately induce turbulence in the boundary layer to delay separation
  e.g. Vortex Generators on wings

- Slats at the leading edge of wings



allow air from high pressure region onto upper surface

used on many airlines during takeoff and landing to increase lift and delay stall.