Blast Waves

Blast waves occur when a large amount of energy is deposited suddenly in a background fluid. Examples are supernovae and nuclear explosions. These have several stages:

1. Free expansion, in which the mass of the ejecta is larger than the background gas

   e.g. In a supernova $\sim 10^{44}$ J of kinetic energy is released, expelling around 1 solar mass ($\sim 10^{30}$ kg) so a velocity of $\sim 10^4$ km/s.

2. As the shock expands it sweeps up the background gas. If the upstream pressure of the background gas is negligible compared to the dynamic/"ram" pressure ρu^2, and the radiative loss of energy is negligible, then there is a scale-free solution called a Sedov-Taylor blast wave.

3. Eventually the shock/blast wave slows and on longer timescales radiation losses become important

   For typical supernova values, the blast wave solution is valid from ~ 50 years to ~ 50,000 years after the supernova.

If the pressure of the background doesn't matter nor the mass ejected, then the only parameters are

- The density of the background, ρ_1
- The amount of energy released, E

Dimensional analysis: $\rho \; [M L^{-3}]$

$E \; [M L^2 T^{-2}]$

How to make a dimensionless variable given only ρ, E, radius of the blast r, and time t?

$$x = r\, t^l \rho^m E^n$$

$$L\, T^l M^m L^{-3m} M^n L^{2n} T^{-2n}$$

L: $1 - 3m + 2n = 0$

M: $m + n = 0$

T: $l - 2n = 0$

$n = -\frac{1}{5}$

$m = \frac{1}{5}$

$l = -\frac{2}{5}$

$$x = r\, t^{-2/5} \rho^{1/5} E^{-1/5}$$

A solution is

$$r(t) = x \left(\frac{E t^2}{\rho} \right)^{1/5}$$

where x is a constant which depends on the adiabatic ratio γ and is $\simeq 1$ (1.17 if air) $\gamma = 5/3$

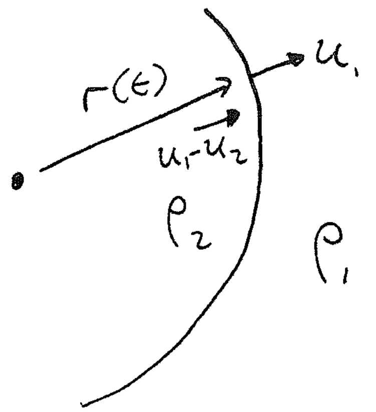
16.2

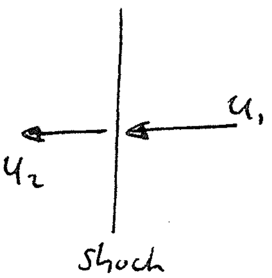
This is a "self-similar" solution, in which the solution at any time t looks "similar" just scaled. It was discovered independently in the '40s by John von Neumann (USA), L.I. Sedov (USSR) and G.I. Taylor (UK).

The speed of the blast is

$$u(t) = \frac{dr}{dt} = x\,\frac{2}{5}\left(\frac{E}{\rho\, t^3}\right)^{\frac{1}{5}}$$

We can use the Rankine-Hugoniot equations to derive properties of the expanding blast wave





In a shock
$$\frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}$$

If the background pressure is negligible then $M_1 \to \infty$

so
$$\frac{\rho_2}{\rho_1} \to \frac{\gamma+1}{\gamma-1} \simeq 4 \quad \text{if } \gamma = 5/3$$

The pressure is given by

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1}$$

where $M_1^2 = \frac{u_1^2}{c_1^2} = u_1^2 \frac{\rho_1}{\gamma P_1}$

$$\text{so} \quad P_2 = \frac{2\cancel{\gamma}}{\gamma+1} u_1^2 \frac{\rho_1}{\cancel{\gamma}} - \frac{(\gamma-1)}{\gamma+1} P_1$$

Since we're ignoring the background pressure

$$P_2 \simeq \frac{2}{\gamma+1} \rho_1 u_1^2$$

$$= \frac{2x^2}{\gamma+1}\left(\frac{2}{5}\right)^2 \rho_1 \left(\frac{E}{\rho_1 t^3}\right)^{2/5}$$

$$= \frac{8}{25} \frac{x^2}{\gamma+1} \left(\frac{E \rho_1^{3/2}}{t^3}\right)^{2/5}$$

Given the density and pressure, we can estimate the temperature behind the shock assuming ideal gas $P = \frac{\rho}{m_a} k_B T$

$$\text{so} \quad T \simeq \frac{8}{25} \frac{m_a}{k_B} \frac{x^2}{(\gamma+1)} \left(\frac{E}{\rho_1 t^3}\right)^{2/5} \underbrace{\left(\frac{\gamma-1}{\gamma+1}\right)}_{\text{since } \rho_2 \sim 4\rho_1}$$

16.3

So for a supernova releasing 10^{44} J of kinetic energy into a background with ~ 1 atom per cubic centimetre

$$\rightarrow \quad \rho_1 \simeq 2\times10^{-21}\,\mathrm{kg/m^3}$$

gives

$$r = 6.8 \left(\frac{E}{10^{44}\,\mathrm{J}}\right)^{1/5} \left(\frac{\rho}{2\times10^{-21}\,\mathrm{kg/m^3}}\right)^{-1/5} \left(\frac{t}{100\,\mathrm{years}}\right)^{2/5}$$

light years

$$u = 8.1\times10^{6} \left(\frac{E}{10^{44}\,\mathrm{J}}\right)^{1/5} \left(\frac{\rho}{2\times10^{-21}\,\mathrm{kg/m^3}}\right)^{-1/5} \left(\frac{t}{100\,\mathrm{years}}\right)^{-3/5}$$

m/s

$$P = 1\times10^{-7} \left(\frac{E}{10^{44}\,\mathrm{J}}\right)^{2/5} \left(\frac{\rho}{2\times10^{-21}\,\mathrm{kg/m^3}}\right)^{3/5} \left(\frac{t}{100\,\mathrm{years}}\right)^{-6/5}$$

Pascals

$$T = 1.5\times10^{9} \left(\frac{E}{10^{44}\,\mathrm{J}}\right)^{2/5} \left(\frac{\rho}{2\times10^{-21}\,\mathrm{kg/m^3}}\right)^{-2/5} \left(\frac{t}{100\,\mathrm{years}}\right)^{-6/5}$$

kelvin
(assuming $m_a = 1.67\times10^{-27}$ kg)

High temperatures lead to radiative cooling, so by ~10^5 years the radiation loss has become significant, and the blast-wave approximations no longer apply.

The same physics can be used in other areas where a large amount of energy is quickly deposited in a fluid background. It was developed to study atomic bomb tests

e.g. Article by G.I. Taylor (1949) analysed public photographs of the Trinity test

The blast wave solution is found to work well between ~ 0.5 and 100 ms

e.g. $t = 1.93$ ms

$r = 48.7$ m

Density of air $\rho \simeq 1.2\ \mathrm{kg/m^3}$

$$E = \frac{\rho}{t^2}\left(\frac{r}{x}\right)^5$$

using 1 ton TNT $\simeq 4.25 \times 10^9$ J

for $x = 1$

$$E = 20.8\left(\frac{\rho}{1.2\,\mathrm{kg/m^3}}\right)\left(\frac{t}{1.93\,\mathrm{ms}}\right)^{-2}\left(\frac{r}{48.7\,\mathrm{m}}\right)^{5}(x)^{-5}$$

kton

$x = 1.17$ ($\gamma = 5/3$) gives $E \simeq 9$ kton

$x \simeq 1$ for air $E \simeq 20$ kton