9.1

Lecture 9 Aerofoil theory

1. Effect of circulation on flow, stagnation points
2. Aerofoils - general shape, flow
3. Flow around a camber
4. Kutta condition
5. Thin aerofoil theory.

For a cylinder in a flow of speed u

$$\phi = u\left(r + \frac{a^2}{r}\right)\cos\theta + \frac{\Gamma\theta}{2\pi}$$

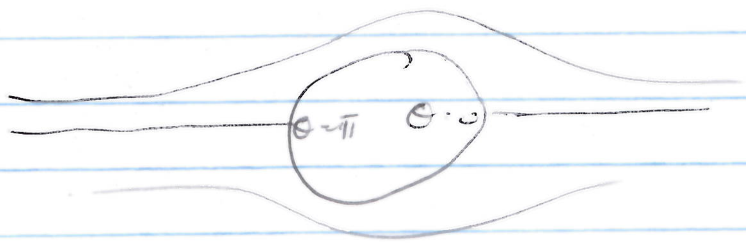
and so the velocity $\underline{V} = \nabla\phi$

$$V_r = \frac{\partial\phi}{\partial r} \qquad V_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta}$$

To find stagnation points, want $V_\theta = 0$

$$V_\theta = -2u\sin\theta + \frac{\Gamma}{2\pi a}$$

So if $\Gamma = 0$, stagnation point at $\theta = 0$ and $\theta = \pi$

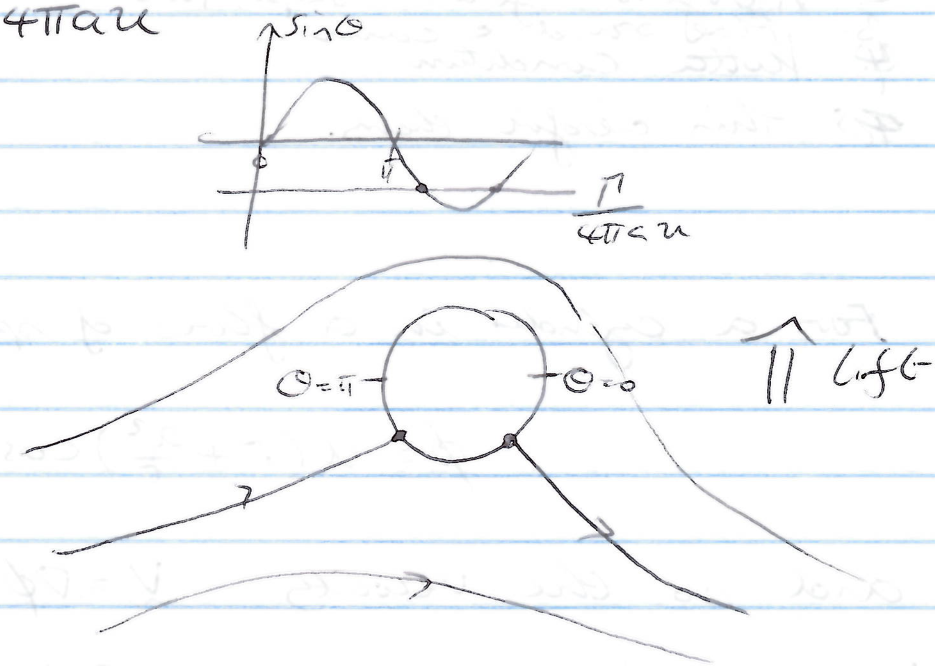


If $\Gamma > 0$

$$2u\sin\theta = \frac{\Gamma}{2\pi a}$$

(reverse $\Gamma < 0$ for positive lift)

$$\sin\theta = \frac{\Gamma}{4\pi a u}$$



The location of these stagnation points is important for flows over aerofoils as we'll see.

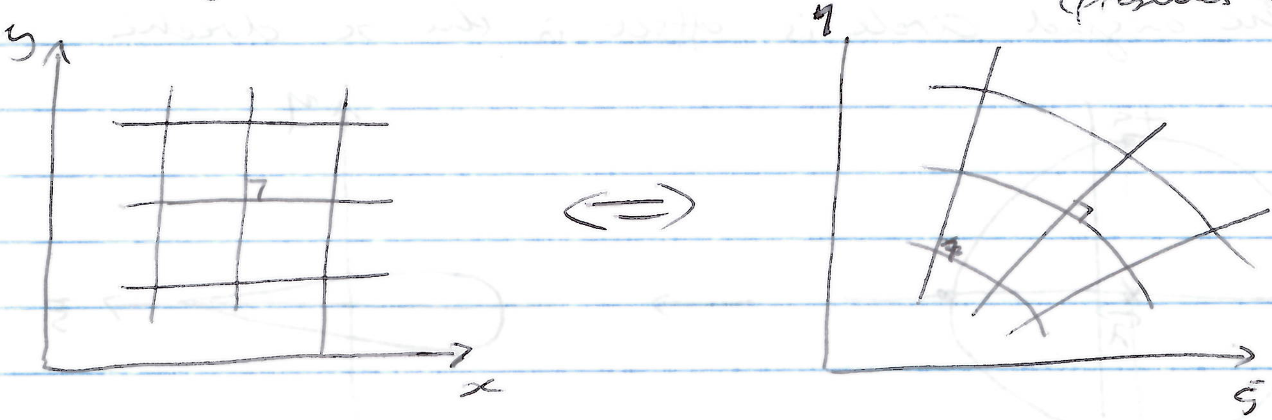
9.2

Conformal mapping

In order to apply theory from potential theory on a cylinder to more complicated shapes, conformal mapping can be used

→ Relies on complex analysis, will look at shortly but for now

(preserves angles)

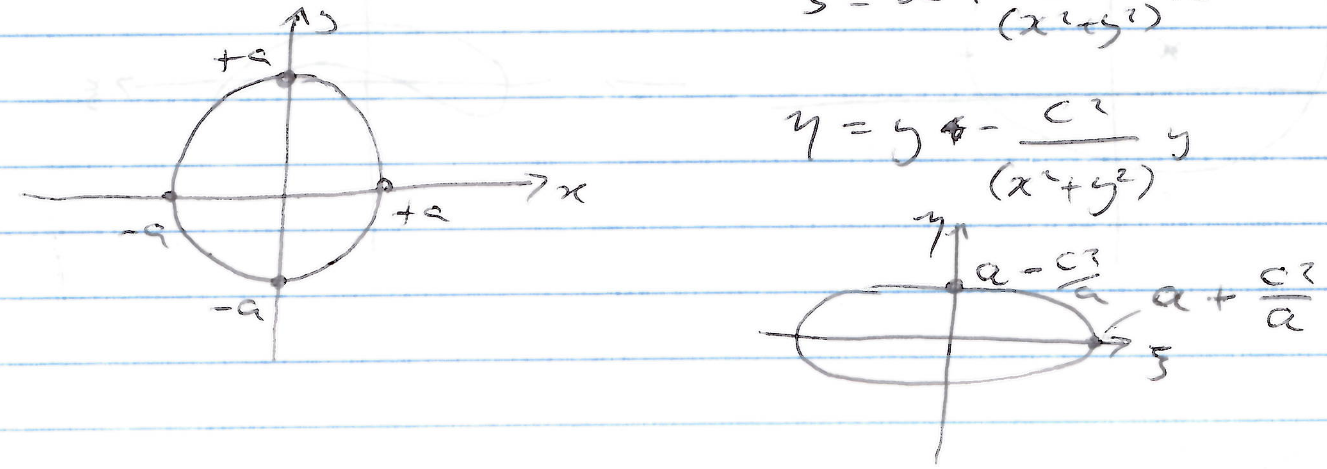


It can be shown * (later) that solutions in one space (x-y) can be mapped onto solutions in another space (ξ, η) if the map satisfies the right properties

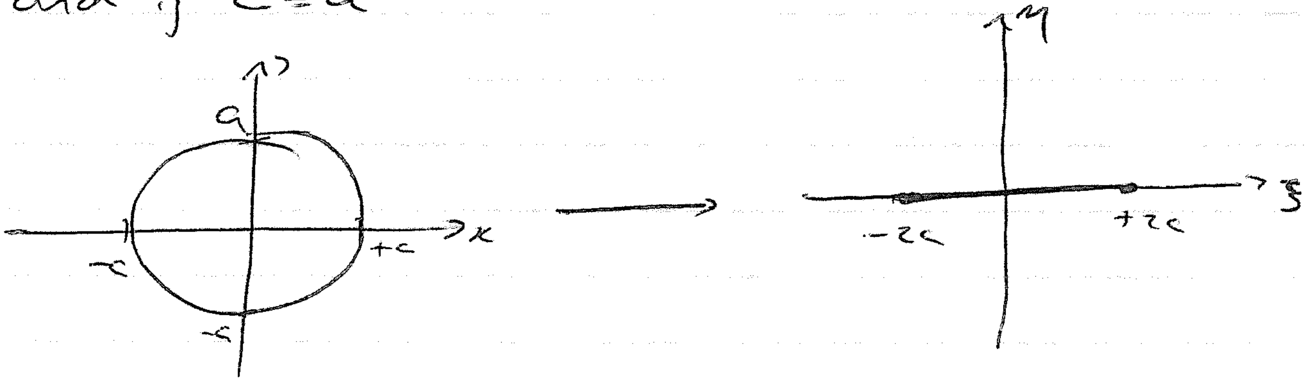
Joukowski transform

$$\xi = x + \frac{c^2}{(x^2+y^2)}x$$

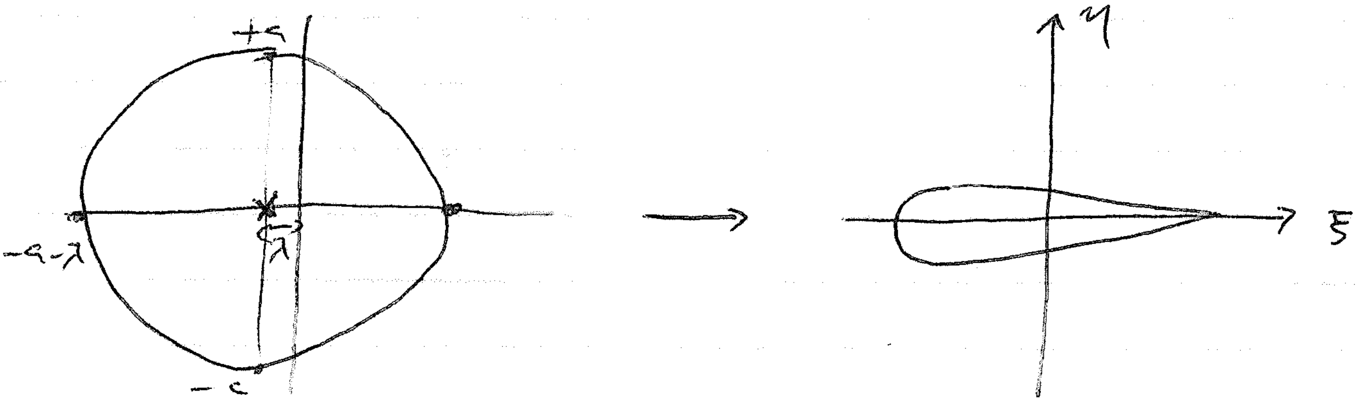
$$\eta = y - \frac{c^2}{(x^2+y^2)}y$$



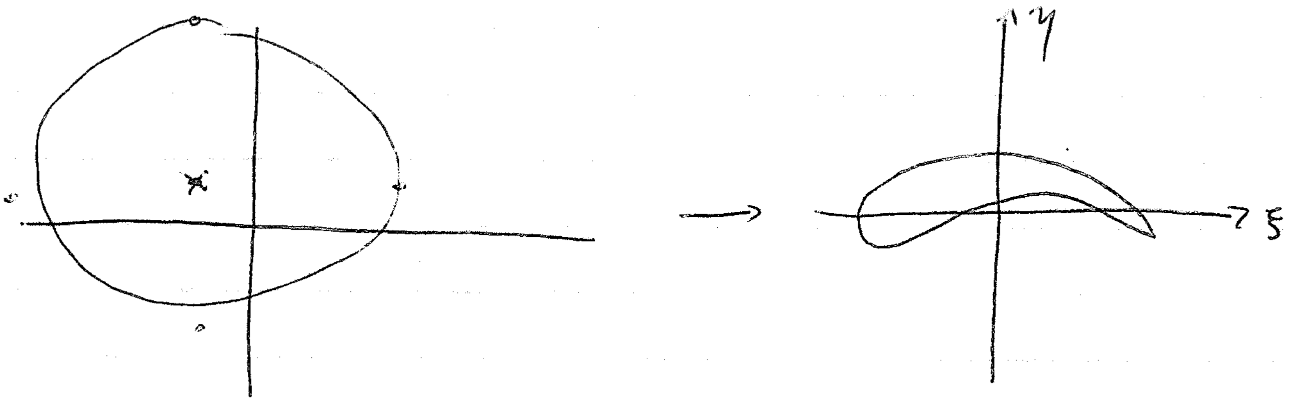
and if $c = a$



if the original circle is offset in the x direction



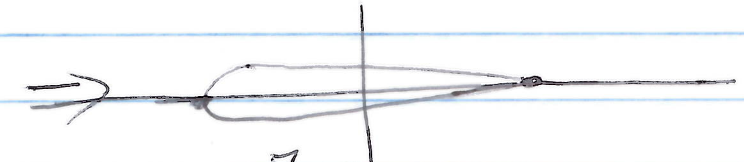
and in the y direction



9.3

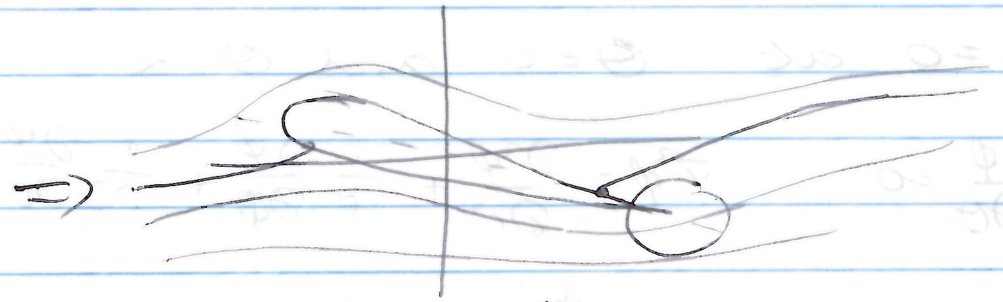
Flow past an aerofoil (qualitative)

Take this case:



$\Gamma = 0$

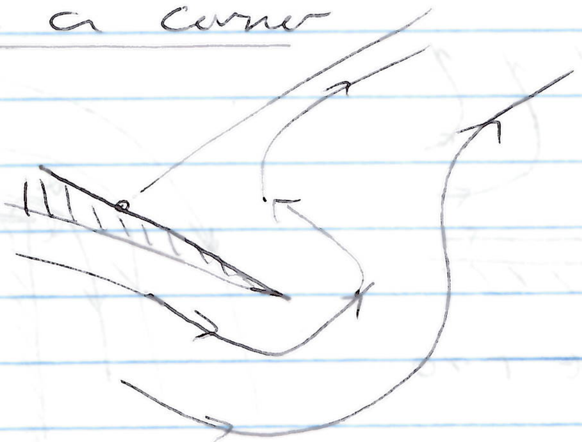
now tilt the aerofoil an angle α (or the flow an angle α)

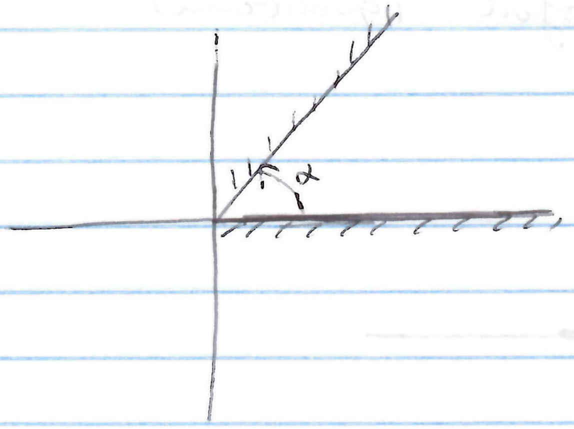


$\Gamma = 0$

Note that the stagnation point is on the upper surface if $\Gamma = 0$
A particular problem occurs at the trailing edge

Flow around a corner





two planes (walls) at $\theta = 0$ and $\theta = \alpha$

Using polar coordinates again, boundary condition of no flow through walls

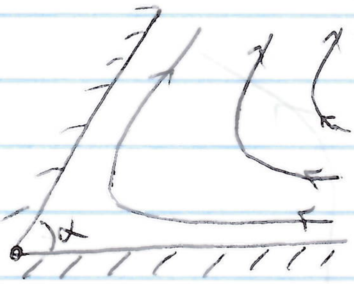
$$V_\theta = 0 \quad \text{at} \quad \theta = 0 \quad \text{and} \quad \theta = \alpha$$

$$\frac{1}{r}\frac{\partial \phi}{\partial \theta} = 0 \qquad \nabla^2 \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial \phi} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0$$

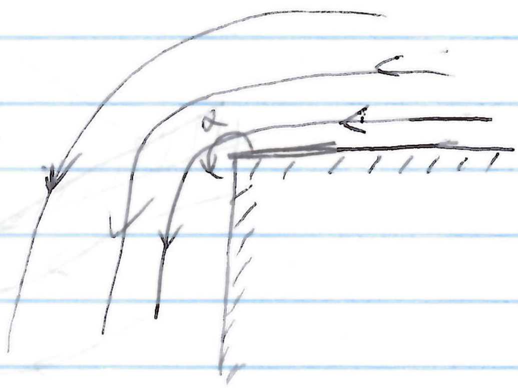
Solution $\phi = A r^n \cos(n\theta)$ $\quad n = \pi/\alpha$

$$V_r = nAr^{n-1}\cos n\theta \qquad V_\theta = -nAr^{n-1}\sin n\theta$$

if $\alpha < \pi$, $n > 1$ $\qquad$ if $\alpha > \pi$, $n < 1$



$V_r \to 0$ at $r \to 0$



$V_r \to \infty$ at $r = 0$

9.4

clearly an infinite velocity is unphysical.

From Bernoulli's equation $\frac{P}{\rho} + \frac{1}{2}u^2 = \text{const}$

this would imply a negative pressure

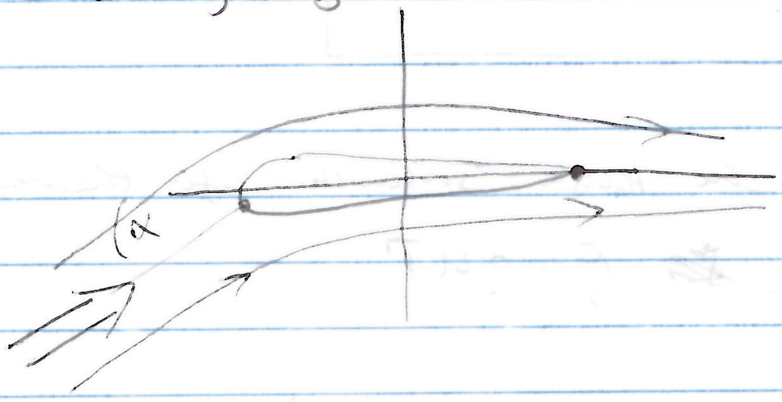
→ usually implies some kind of cavitation in fluids.

* Flow around corners should be avoided in incompressible, irrotational flow

But this is the solution for $\Gamma = 0$ around an airfoil

Kutta condition: the circulation will be such that the flow leaves the trailing edge of the wing smoothly

This implies that the stagnation point should be on the trailing edge



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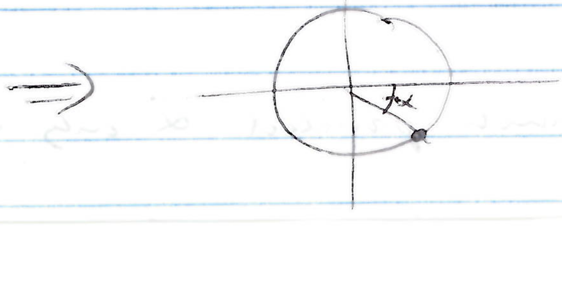
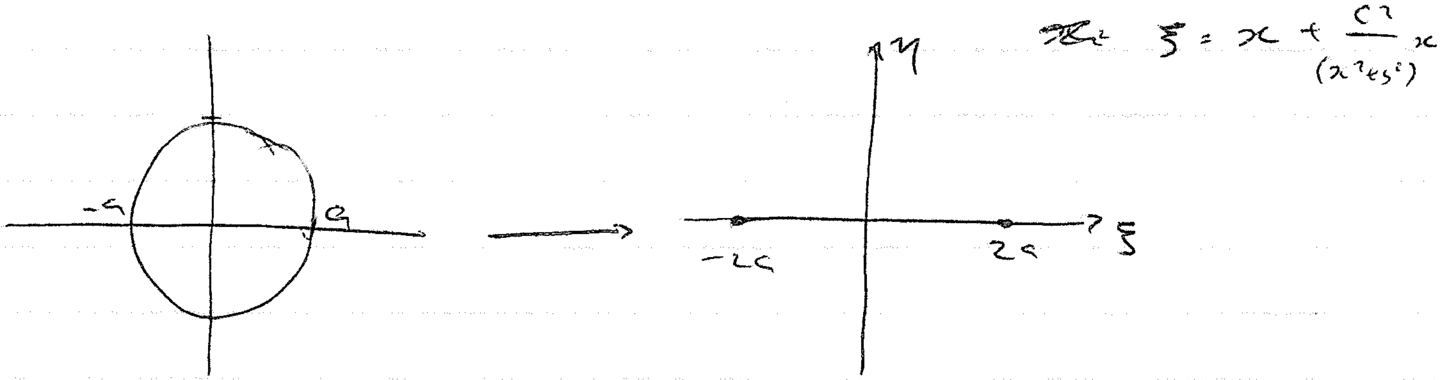


We already found the locations of the stagnation points

$$\sin\theta = \frac{\Gamma}{4\pi a u}$$

$$\Rightarrow \quad \Gamma = -4\pi a u \sin\alpha$$

What is a?



So L, the length of the wing $= 4a$

$$\boxed{\Gamma_k = -\pi u L \sin\alpha}$$

and so using the Kutta - Joukowski lift theorem

$$F_y = -\rho u \Gamma$$

$$\Rightarrow \quad F_y = \pi \rho u^2 L \sin\alpha$$

Agrees well with experiment provided α only a few (6-12) degrees.