Stellarators

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6th February 2015

Previously...

- Toroidal devices need a rotational transform so that the ∇B drift doesn't lead to charge separation
- In tokamaks, this is provided by a toroidal current, some of which needs to be driven using transformer action, radio waves or neutral beams
- Particle trapping (neoclassical effects) lead to the bootstrap current, which can provide much of the toroidal current
- The distributon of toroidal current has important effects on plasma performance and stability
- In this lecture we'll look at other ways to produce a rotational transform without a toroidal current

Stellarators

It can seem counter-intuitive that you could create a toroidal machine with rotational transform without having a plasma current

- As an example, consider the first such design
- Proposed in 1951 by Lyman Spitzer, Jr[1] [2]
- Called a **stellarator** since it was inspired by the sun

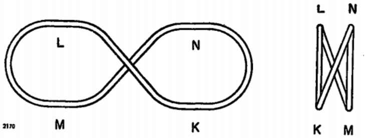
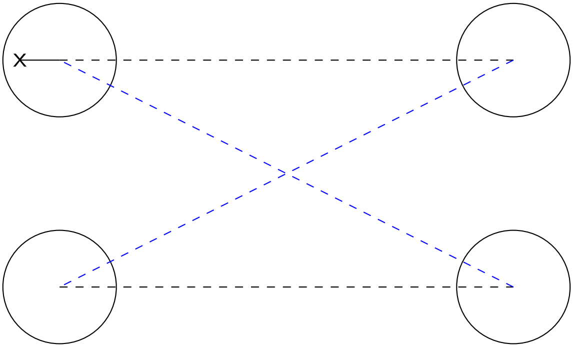


Figure 2. Top and end views of a figure-eight stellarator

[1] Project Matterhorn. Declassified, renamed PPPL in 1961

[2] L.Spitzer, Jr "The Stellarator Concept" IAEA conference 1958. Available from `http://www-naweb.iaea.org/napc/physics/2ndgenconf`

If we follow a field-line around this figure of eight



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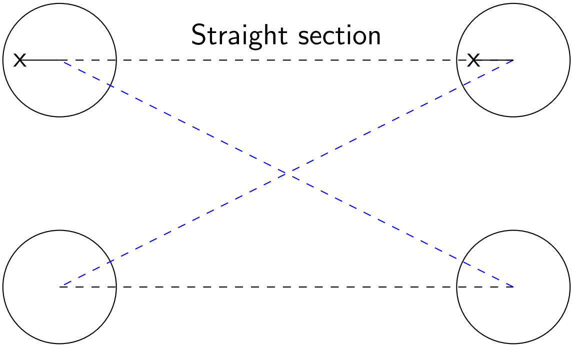

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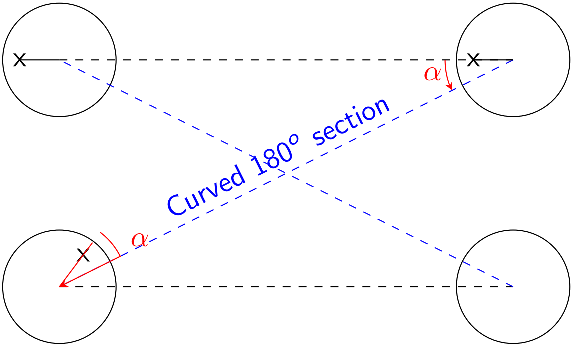

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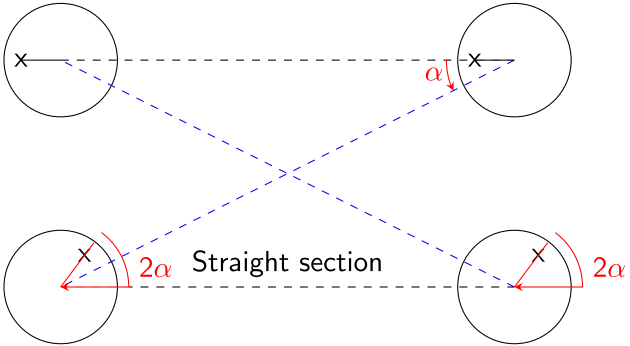

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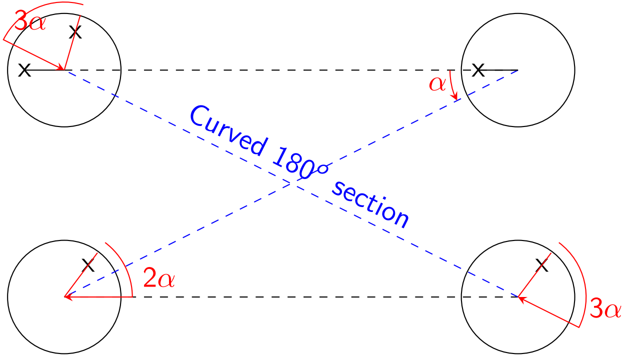

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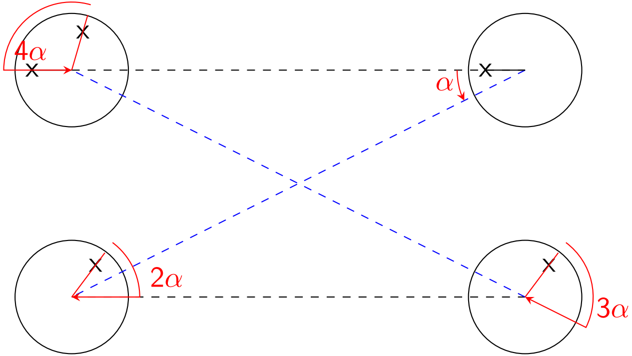


- In the figure of eight design, field-lines are shifted by an angle of 4α each time they go around the machine. This shift is the **rotational transform**
- As in a tokamak, this leads to the formation of flux surfaces

Producing rotational transform

- The first designs like the racetrack demonstrate the principle, but did not have very good confinement: they turned out to be unstable with a finite plasma pressure.
- They are complicated to build, and the fields from neighbouring coils tended to interfere with each other

Producing rotational transform

- The first designs like the racetrack demonstrate the principle, but did not have very good confinement: they turned out to be unstable with a finite plasma pressure.
- They are complicated to build, and the fields from neighbouring coils tended to interfere with each other
- There are three ways to get a rotational transform in toroidal devices:
  - A toroidal current, either driven externally or by the Bootstrap current (i.e. tokamak-like)
  - A deformation in 3D (torsion) of the magnetic axis, as in the racetrack machine
  - Non-circular deformation of the magnetic surfaces in resonance with magnetic field lines

Helical Axis Stellarators

To produce a twist in the magnetic axis, draw a helical path, then position the toroidal field coils perpendicular to this path. This produces a configuration called a Helical Axis stellarator, or **Heliac**

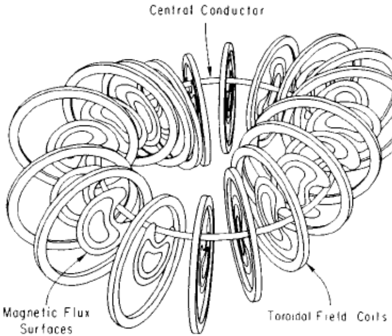
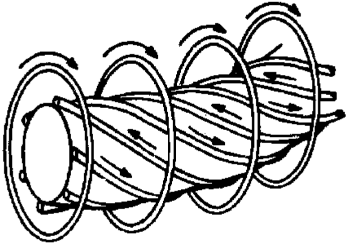


Figure: Heliac configuration produced by displacing toroidal field coils[3]

[3] A.H.Boozer, Phys. Plasmas **5**, 1647 (1998)

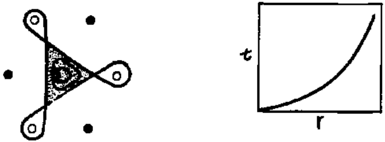
Classical Stellarators

Rather than twisting the magnetic axis, so-called **classical stellarators** deform the flux surface shape by adding helical coils with currents in alternating directions (so field cancels out on axis).



STELLARATOR (ℓ=3)

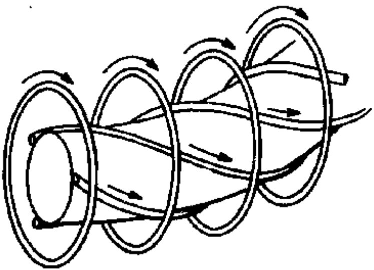
$l = 3$ stellarator with $2l = 6$ helical windings



The original C-stellarator had very poor performance, but subsequent designs improved on this: the Wendelstein-I, W-II and W-7A at Max-Planck institute.

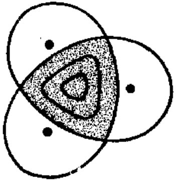
[1]R.L.Miller, R.A.Krakowski LANL report LA-8978-MS (1981)

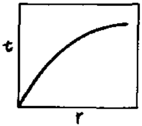
Following declassification of fusion research in 1958, stellarator research started in Japan. An alternative design to the classical stellarator was developed[4] which uses only half the number of coils, all carrying current in the same direction



HELIOTRON (ℓ=3)

Figure: R.L.Miller, R.A.Krakowski LANL report LA-8978-MS (1981)



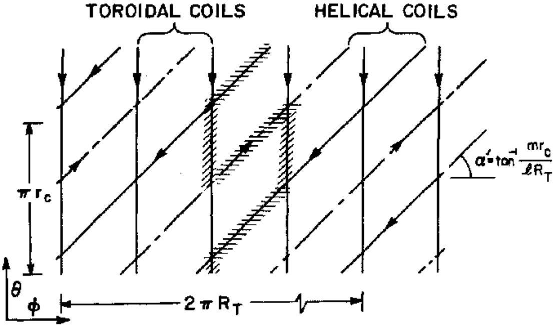


- Heliotrons are easier to build since fewer coils, and the forces between them are reduced
- The Large Helical Device (LHD) is of this design

[4]K.Uo, J. Physical Soc. Japan **16**(7):1380 (1961)

Modular coils

- The helical coils in classical stellarators, heliotrons and variants are hard to build, because they are interlocking, and inside the toroidal coils
- Plot the currents in these coils as a function of θ and ϕ



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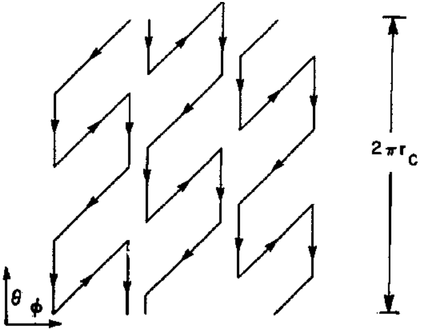
Modular coils

- The helical coils in classical stellarators, heliotrons and variants are hard to build, because they are interlocking, and inside the toroidal coils
- Plot the currents in these coils as a function of θ and ϕ
- Replace with an essentially equivalent set of modular coils



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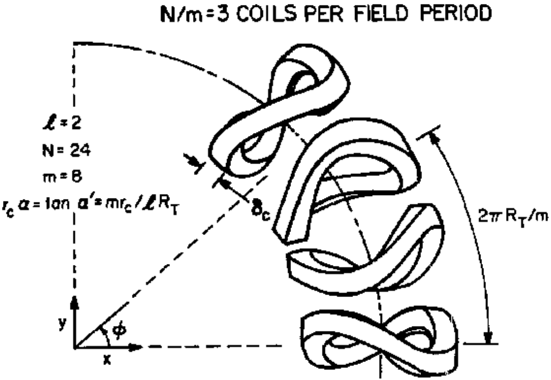
Modular coils

Modular coils were a big breakthrough in stellarator design

- Coils can be independently built and then assembled. Design is more difficult, but result is more practical for large reactors



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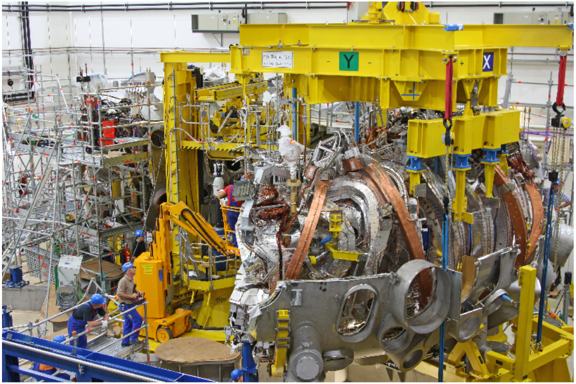

Figure: Wendelstein 7-X under construction, 2010

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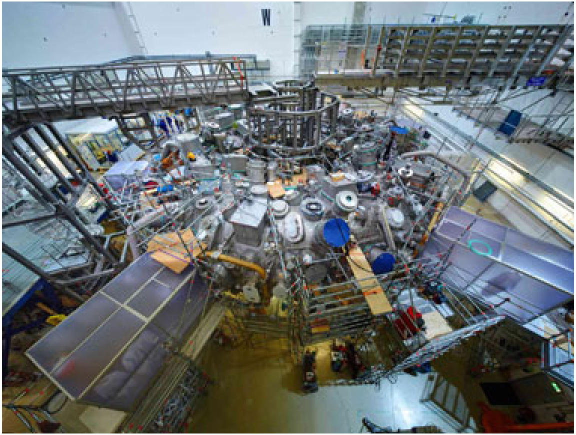


Figure: Wendelstein 7-X nearly finished, October 2013

Modular coils

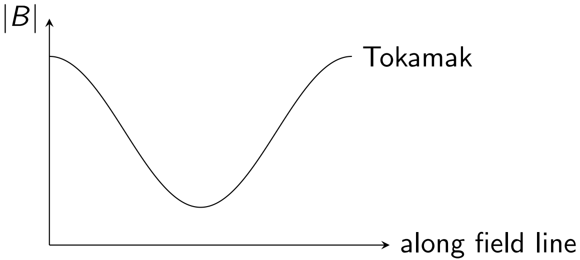
Modular coils were a big breakthrough in stellarator design

- Coils can be independently built and then assembled. Design is more difficult, but result is more practical for large reactors
- More importantly, using modular coils allows stellarators to be designed "plasma first", rather than "coil first" [5]. We can design the plasma equilibrium based on physics considerations, and then design a set of coils to produce the required field.
- Stellarators designed this way are called **advanced stellarators**, the first example of which was Wendelstein 7-AS.
- Performing and optimising these designs has only become possible with advances in computing power

[5] R.F.Schmitt PhD thesis, Columbia U. 2008

Particle orbits in stellarators

- Following a field-line around a tokamak the magnetic field strength is approximately sinusoidal



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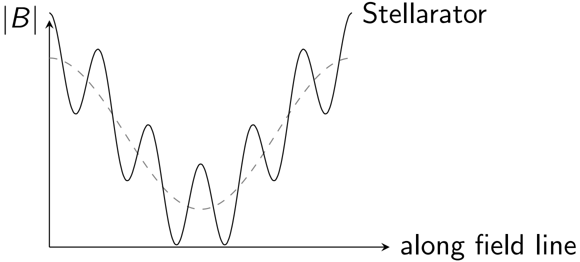

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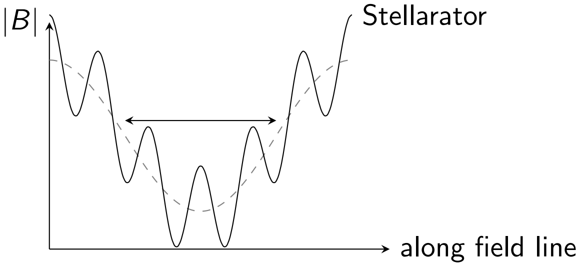

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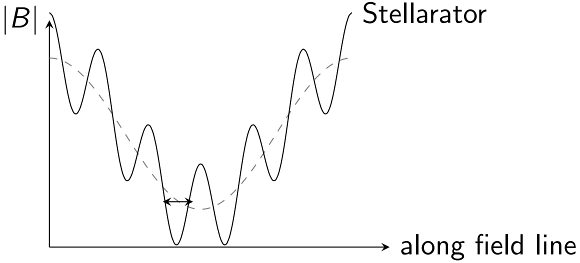

- As in a tokamak, particles can be passing or trapped due to toroidicity
- There are also particles which get trapped in local minima

Super-banana orbits

- These particles trapped in local minima are confined to regions on the upper or lower half of the flux surface $\Rightarrow$ their vertical ∇B drift doesn't cancel out, and they drift straight out of the machine. Called **super-banana** or **direct loss** orbits [6]
- This drift is different for electrons and ions, and so leads to electric fields.
- The same process can happen in tokamaks due to toroidal ripple produced by a limited number of toroidal field coils
- This direct loss lead to pretty poor performance in the 1960s and interest moved to tokamaks

[6] J.R.Cary, C.L.Hedrick, J.S.Tolliver, Phys. Fluids **31**, 1586 (1988)

Particle orbits in a tokamak revisited

So why do particle orbits stay near flux surfaces in tokamaks?

Particle orbits in a tokamak revisited

So why do particle orbits stay near flux surfaces in tokamaks?

Consider the toroidal angular momentum of a particle in a tokamak Rv_ϕ

$$m\frac{d}{dt}\left(Rv_\phi\right) = qR\left(\underline{v}\times\underline{B}\right)_\phi$$

Using the expressions for an axisymmetric poloidal field

$$\underline{B}_\theta = \nabla\psi\times\nabla\phi \qquad \underline{v}\times\left(\nabla\psi\times\nabla\phi\right) = \nabla\psi\left(\underline{v}\cdot\nabla\phi\right) - \nabla\phi\left(\underline{v}\cdot\nabla\psi\right)$$

$$\Rightarrow m\frac{d}{dt}\left(Rv_\phi\right) = -q\underline{v}\cdot\nabla\psi$$

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$$\Rightarrow m\frac{d}{dt}\left(Rv_\phi\right) = -q\underline{v}\cdot\nabla\psi$$

Since $\underline{v}\cdot\nabla\psi = \frac{\partial\psi}{\partial t}$, this becomes

$$\boxed{\frac{dp_\phi}{dt} = 0} \qquad \text{where} \quad p_\phi = mRv_\phi + q\psi$$

Canonical momentum

- $p_\phi = mRv_\phi + q\psi$ is called the **canonical momentum**, and is conserved in an axisymmetric configuration
- In lecture 5, we derived an expression for the width of a banana orbit by considering the bounce time and ∇B drift velocity
- An alternative way is to consider the change $|\delta\psi| \sim |\nabla\psi|\,\delta r_b$
- This gives an upper bound on the particle orbit width

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A more elegant way to derive the canonical momentum is through the particle Lagrangian. Symmetries lead to conserved quantities, and in this case axisymmetry in a tokamak leads to conservation of canonical momentum

$\Rightarrow$ Toroidal symmetry leads to a limit on how far particles can deviate from flux surfaces without collisions.

Quasi-symmetric stellarators

- In classical stellarators, there is no toroidal symmetry. Hence canonical momentum is not conserved, and there is no bound on particle excursions from a flux surface.
- Quasi-symmetric stellarators[7] [8] aim to introduce a new symmetry angle and so a canonical momentum

[7] A.H.Boozer, Phys. Fluids **26**(2):496-499 (1983)
[8] J.Nuhrenberg and R.Zille, Phys. Lett. A **129**, 113 (1988)

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- Quasi-symmetric stellarators[7] [8] aim to introduce a new symmetry angle and so a canonical momentum
- In these designs, $|B|$ a function of only ψ and a linear combination $M\theta - N\phi$ (in Boozer coordinates)
  - When $N = 0$, the field has Quasi-Axial Symmetry (QAS)
  - When $M = 0$, the field has Quasi-Poloidal Symmetry (QPS)
  - When $M = 1$ and $N \neq 0$ then the field has Quasi-Helical Symmetry (QHS)
- In this case there is a symmetry direction, and a conserved canonical momentum. This leads to a limit on the deviation of particles from flux surfaces, and the same toroidal neoclassical theory applies

[7] A.H.Boozer, Phys. Fluids **26**(2):496-499 (1983)

[8] J.Nuhrenberg and R.Zille, Phys. Lett. A **129**, 113 (1988)

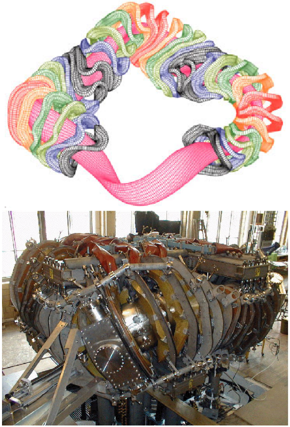

Figure: HSX
http://www.hsx.wisc.edu

- Helically symmetric stellarator, constructed at Wisconsin as first test of quasi-symmetry
- Began operation in 1999
- Major radius 1.20m, minor radius 0.15m
- Confirmed reduction of direct loss orbits

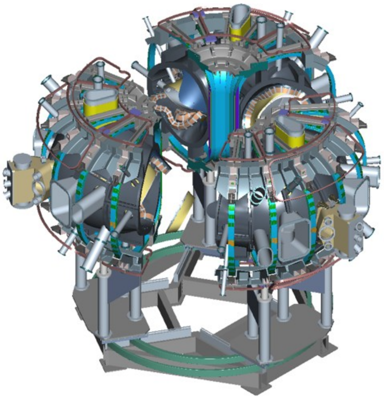

Figure: National Compact Stellarator eXperiment at PPPL http://ncsx.pppl.gov/

- Low aspect-ratio, high performance (β) machines called "compact stellarators" have been designed
- At low aspect-ratio, quasi-helicity cannot be attained[a]
- Instead, another symmetry can be used such as quasi-axisymmetry
- Major radius 1.42m, minor radius 0.33m

[a] D.A.Garren, A.H.Boozer, Phys. Fluids B **3**:2822 (

Note: The NCSX project was mothballed in 2008

Quasiomnigeneous stellarators

A second way to reduce neoclassical transport in stellarators is to make the radial particle drift average out over a bounce orbit.

- These are called omnigenous or linked mirror designs [9][10], and need not be symmetric
- The bounce-average radial drift can be minimised for a chosen population of particles (i.e. super-bananas), hence the "quasi-"

[9] L.S.Hall, B.McNamara, Phys. Fluids **18**:552 (1975)
[10] H.E.Mynick, T.K.Chu, A.H.Boozer Phys. Rev. Lett **48**(5):322 (1986)

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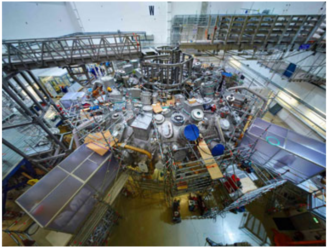
- The bounce-averaged radial drift can be minimised for a chosen population of particles (i.e. super-bananas), hence the "quasi-"
- The variation in field-strength ∇B within a flux surface is proportional to the curvature
- Since super-banana particles are trapped in regions of low B, minimise the curvature in these regions, and put the curvature needed to bend plasma into a torus in regions of high B

[9] L.S.Hall, B.McNamara, Phys. Fluids **18**:552 (1975)
[10] H.E.Mynick, T.K.Chu, A.H.Boozer Phys. Rev. Lett **48**(5):322 (1986)

A large Quasi-Omnigenous stellarator being built at IPP Greifswald



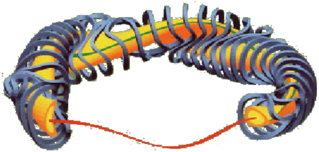


Figure: A.H.Boozer, Phys. Plasmas **5**(5):1647 (1998)

- Straight sections with triangular cross-section and low B, and curved sections with crescent cross-sections and high B.
- Major radius 5.5m, minor radius 0.52m, magnetic field 3T
- Expected to have JET-like plasma performance, and discharge length of $\sim$ 30 minutes
- Planned completion date 2014 at cost of $\sim$ \$300 million

Stellarator equilibria

Stellarator equilibria are quite different to a tokamak:

- In some ways it is easier: currents in the plasma are usually optimised to be small, so the coil currents alone determine the equilibrium
- The Grad-Shafranov equation cannot be used, so now the full 3D $\underline{J} \times \underline{B} = \nabla P$ must be solved. Two codes which do this are VMEC[11] and PIES[12]

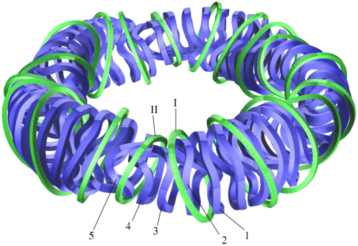


Figure: W7-X coils: M. Drevlak *et al.* Nucl. Fusion 45, 731 (2005)

[11] S.P. Hirschman, *et al.* Phys. Fluids 26, 3553 (1983)

[12] A. Reiman and H.S. Greenside, Comput. Phys. Commun. 43, 157 (1986)

Optimising Stellarators

Stellarator design is about optimising trade-offs

- First a plasma equilibrium must be constructed with desired aspect ratio, pressure and rotational transform
- Quasi-symmetric or quasi-omnigenous configurations minimise neoclassical transport

[13] See e.g. work by Pavlos Xanthopoulos using GS2

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- The plasma must avoid large-scale instabilities
- The Pfirsch-Schüter and Bootstrap currents are calculated. These are usually minimised to have maximum control over the plasma configuration
- In the last couple of years it has become possible to start optimising anomalous (turbulent) transport[13]

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- The Pfirsch-Schüter and Bootstrap currents are calculated. These are usually minimised to have maximum control over the plasma configuration
- In the last couple of years it has become possible to start optimising anomalous (turbulent) transport[13]
- A design of coils is produced which create the desired field
- These coils may not be practical, so modify the design
- Repeat until an optimised design is produced

H.E.Mynick, Phys. Plasmas **13**, 058102 (2006)

[13] See e.g. work by Pavlos Xanthopoulos using GS2

Problems with Stellarators

There were several problems with Stellarators

- Complicated coil configurations need to be very precise ($\sim$ 1mm), and are difficult to design and expensive to build. They must be capable of carrying Mega-Ampéres of current, and withstanding huge forces
- Generally, stellarator configurations cannot be varied to the extent that tokamaks can. This makes experimentation more difficult
- Achieving good particle confinement is more difficult in stellarators than tokamaks
- So far, stellarators have not reached the same performance (pressure β and density) as tokamaks
- The divertor and heat-handling regions have a more complicated geometry than tokamaks, complicating the engineering

Advantages of stellarators

- Stellarators are intrinsically steady-state, as there is no need to drive a plasma current
- Lack of plasma current removes a large class of instabilities which we'll see later in tokamaks
- Because the rotational transform and position of the plasma is set by external coils and not by currents in the plasma, stellarators do not suffer violent disruptions
- Stellarators have a much greater range of designs than tokamaks, potentially allowing greater optimisation of performance
- With the construction of Wendelstein 7-X, new concepts such as quasi-symmetry, and the interest in 3D fields in tokamaks, there is renewed interest in stellarator research.