Toroidal pinches and current-driven instabilities

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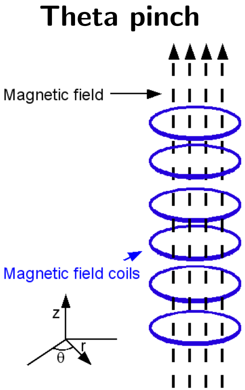
18^{th} February 2015

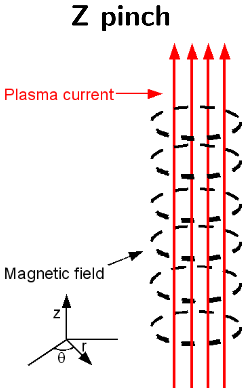
Plasma stability

- So far you have seen some magnetic confinement schemes, how plasmas are heated and fuelled, and some mechanisms for energy loss
- All of this assumes that the plasma is stable, and doesn't throw itself against the walls of the machine!

- Historically the main challenge of magnetic confinement was to find configurations which are stable at high plasma pressures needed for fusion
  – Edward Teller once said that confining plasmas with magnetic fields was like *"trying to confine jelly with rubber bands"*
- From this work the tokamak has emerged as the most promising approach. In the following few lectures we will look at why this is.

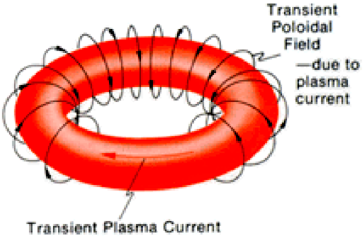
Passing a current through a plasma leads to an inwards “pinch” force which can be used to compress and heat to high temperatures.



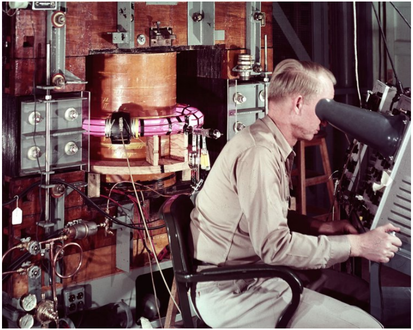


Toroidal pinches

To avoid end losses, wrap the pinch into a torus



By using a transformer coil, a large current can be driven transiently in the torus. This compresses the plasma as in a Z pinch.



The Perhapsatron, Los Alamos 1952. James L. Tuck.

Toroidal pinches: Pinch ratio Θ

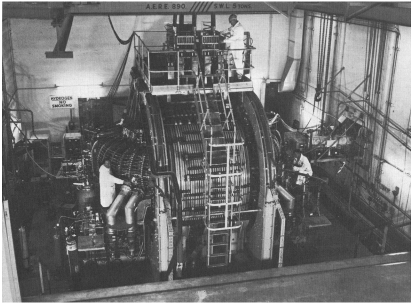
- An initial toroidal field of magnitude B_ϕ is "frozen in" to the compressing plasma, and so is amplified as the plasma compresses
- The magnetic pressure driving the compression is due to the poloidal field B_p

$$\oint B_\theta dl_\theta \simeq 2\pi a B_\theta = \mu_0 I \qquad B_\theta = \mu_0 I / 2\pi a$$

- The compression of the plasma therefore depends on the ratio of the toroidal and magnetic fields: the **pinch parameter** or **pinch ratio**

$$\Theta = \frac{B_\theta\left(r = a\right)}{B_\phi} \simeq \frac{\mu_0 I}{2\pi a B_\phi} \qquad \left[\Theta = 2I/aB_\phi \text{ in cgs units}\right]$$

- ZETA was a large toroidal pinch with $R = 1.5$m and $r = 0.5$m.
- Plasma currents of up to 200 kA driven by transformer coil, and pulse lengths of several milliseconds
- Early measurements indicated temperatures of 1 - 5 million ^{o}C, and bursts of neutrons were seen



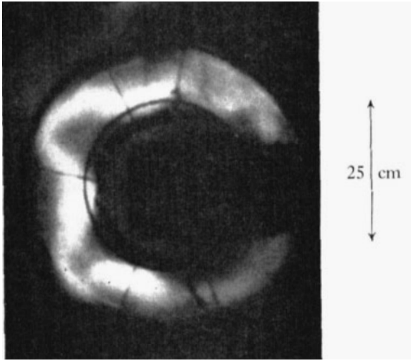
The Zero Energy Toroidal Assembly (ZETA) at Harwell, UK 1957

[Bodin and Newton Nucl. Fusion 20 (1980) 1255]

Kink instabilities

Unfortunately, the ZETA results were not what they appeared. Instabilities were creating high energy ions which were then leading to fusion reactions. Bulk of plasma much colder ($\sim 500,000^o$C)

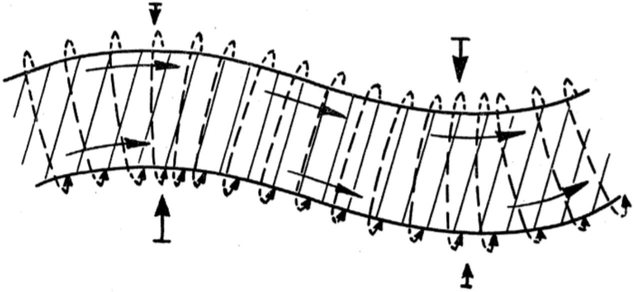
- Plasmas were observed to wriggle or "kink" in smaller devices
- It was thought that by using a conducting wall these would be stabilised



[1] "Observations of the Instability of Constricted Gaseous Discharges" by R.Carruthers, P.A.Davenport (1957)

Kink instabilities

This instability had been predicted theoretically[1] and are driven by the plasma current



- Magnetic field is compressed in some places, and expanded in others
- The magnetic field pressure $\propto B^2$ acts to enhance the motion

[1] "Some instabilities of a completely ionized plasma" by M.Kruskal and M.Schwarzschild (1954)

To understand and predict large-scale instabilities, ideal MHD is often used. Equations for mass density ρ, fluid velocity $\underline{u}$, pressure P and magnetic field $\underline{B}$

Makes several assumptions

- Timescales much longer than the (ion) cyclotron frequency
- Length scales much longer than gyro-radius
- Collisional plasma, close to Maxwellian
- No dissipation

$$\tau \gg \frac{1}{\Omega_{ci}} \qquad L \gg \rho_i \qquad L \gg \lambda \qquad \eta = 0$$

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \underline{u}\cdot\nabla\rho &= -\rho\nabla\cdot\underline{u} \\
\rho\left[\frac{\partial \underline{u}}{\partial t} + \underline{u}\cdot\nabla\underline{u}\right] &= \underline{J}\times\underline{B} - \nabla P \\
\frac{\partial P}{\partial t} + \underline{u}\cdot\nabla P &= -\gamma P\nabla\cdot\underline{u} \\
\frac{\partial \underline{B}}{\partial t} &= \nabla\times\left(\underline{u}\times\underline{B}\right) \\
\underline{J} &= \frac{1}{\mu_0}\nabla\times\underline{B}
\end{aligned}$$

Plasma stability and Ideal MHD

If Ideal MHD makes so many (dubious) assumptions, why use it?

- Ideal MHD equations include the essential physics of plasma instabilities in a (relatively) simple set of equations
- Additional (non-ideal) effects tend to allow new types of instability. Resistivity in particular allows field-lines to reconnect, however:
- Instabilities described by ideal MHD (ideal instabilities) tend to be the fastest and most violent. A plasma which is ideally unstable probably won't last long.
- Many non-ideal instabilities are variations on ideal instabilities, and lots of the jargon is from ideal MHD

Plasma stability and Ideal MHD

Main reason to use ideal MHD is: It works much better than it should do, even in hot (i.e. nearly collisionless) plasmas

- Perpendicular to the B field, movement is restricted and the effective mean-free-path is approximately the gyro-radius $\Rightarrow$ As long as perpendicular length-scales are long compared with the gyro-radius then the fluid approximation is ok
- Parallel to the field, the mean-free-path is very long. Gradients in this direction also tend to be very small.
- Kinetic modifications to MHD primarily modify parallel dynamics.
- As we shall see, parallel dynamics are not very important for determining when linear instabilities start.

Linearisation

To analyse the stability of an equilibrium, we can calculate the evolution of small perturbations.

- All quantities have an equilibrium value (which might vary in space) e.g. $\rho_0\left(\underline{x}\right), p_0\left(\underline{x}\right), \ldots$
- Add a small perturbation e.g

$$\rho = \rho_0\left(\underline{x}\right) + \epsilon\rho_1\left(\underline{x}, t\right)$$

- If the ϵ is "small" then ϵ^2 is extremely small, and terms can be neglected. This is **linearisation**: Keep terms which are linear in perturbations, but neglect higher order terms

$$\frac{\partial\rho}{\partial t} + \underline{u}\cdot\nabla\rho = -\rho\nabla\cdot\underline{u}$$

$$\frac{\partial\epsilon\rho_1}{\partial t} + \left(\underline{u}_0 + \epsilon\underline{u}_1\right)\cdot\nabla\left(\rho_0 + \epsilon\rho_1\right) = -\left(\rho_0 + \epsilon\rho_1\right)\nabla\cdot\left(\underline{u}_0 + \epsilon\underline{u}_1\right)$$

$$\frac{\partial\rho_1}{\partial t} + \underline{u}_0\cdot\nabla\rho_1 + \underline{u}_1\cdot\nabla\rho_0 = -\rho_0\nabla\cdot\underline{u}_1 - \rho_1\nabla\cdot\underline{u}_0 + \ldots$$

Fourier transform

It is often useful to Fourier transform in periodic directions

- In a cylinder $f(r, \theta, z, t) \rightarrow \hat{f}(r, m, k, t)\, e^{im\theta} e^{-ikz}$
- In a torus $f(r, \theta, \phi, t) \rightarrow \hat{f}(r, m, n, t)\, e^{im\theta} e^{-in\phi}$

- In a linear system, different Fourier modes (k, m, n) can often be decoupled, and solved separately
- Note that in a tokamak the equilibrium is not symmetric in θ, so in general the m modes are not independent

Linear systems can also be Fourier transformed in time:

$$f(\ldots, t) = \hat{f}(\ldots)\, e^{-i\omega t}$$

where ω is the complex frequency

Kruskal-Shafranov limit

Adding a magnetic field down the axis of a Z-pinch, or toroidally in a toroidal pinch (tokamak) can stabilise the kink

Force balance from $\mathbf{J} \times \mathbf{B}$:

$$\nabla_{\perp} \left(p + \frac{B^2}{2\mu_0} \right) - \frac{1}{\mu_0}\mathbf{B} \cdot \nabla \mathbf{B} = 0$$

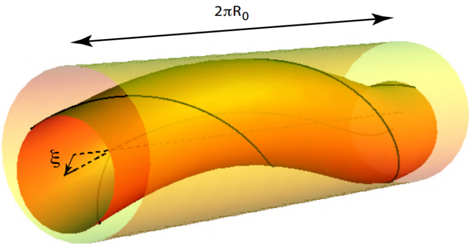
- Bending a magnetic field gives rise to a tension force $\frac{1}{\mu_0}\mathbf{B} \cdot \nabla \mathbf{B}$
- Kink distortions of the axial (toroidal) field produce a restoring force
- Short wavelength instabilities require more bending of the magnetic field
- Long wavelengths tend to be unstable to $m = 1$ kink modes

- In a torus, if the wavelength of the instability is too long to fit, then it cannot be unstable
- So for stability to the $m = 1$ mode we need

$$L_s = \frac{2\pi r B_\phi}{B_\theta} > 2\pi R$$

- And so

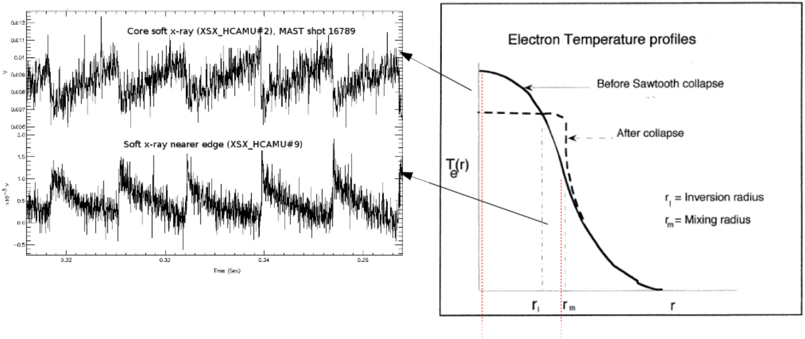
$$q = \frac{rB_\phi}{RB_\theta} > 1$$



Internal kink modes: Sawteeth in tokamaks

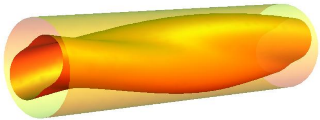
Kink modes with $m = 1$, $n = 1$ can also occur when the boundary of the plasma is held fixed: **Internal kinks**.

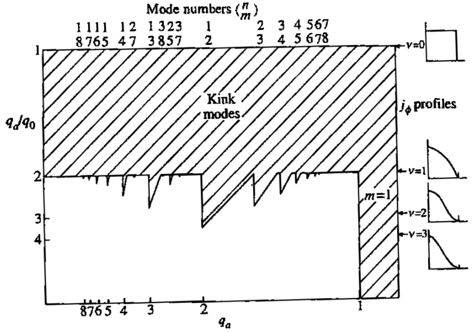
- If $q < 1$ in the core an instability can occur
- Caused by too high current density on axis
- Result in repetitive drops in core temperature and density
- Clearly seen on soft x-ray signals as a sawtooth pattern



- Stability of modes with $m > 1$ depends on the current profile
- Unstable modes are aligned with magnetic field (resonant) just outside the plasma edge

$m = 2$ perturbation





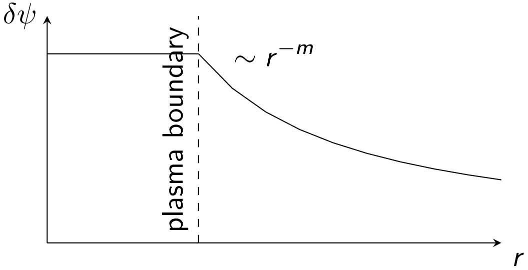
Current profiles

$$
j = j_0 \left(1 - (r/a)^2\right)^{\nu}
$$

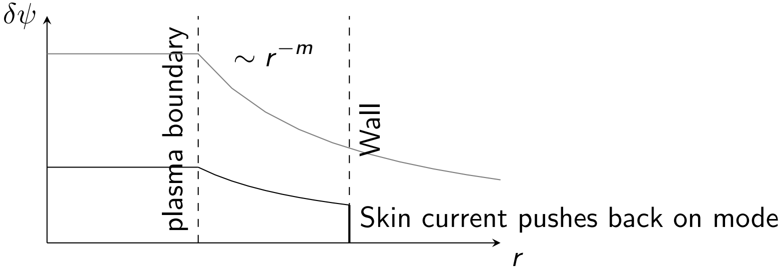
$$
q_a/q_0 = \nu + 1
$$

Need $q_a > 2$ and preferably $q_a > 3$ for stability

["Tokamaks" by J.Wesson, "Introduction to Kink Modes" by S.Cowley]

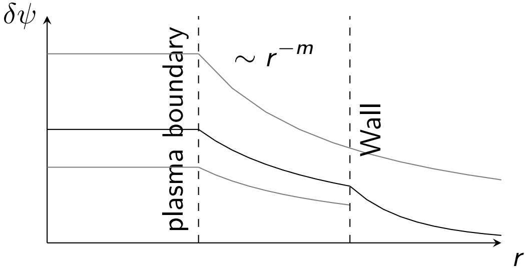


- If there is no wall (wall is far away) then the flux-surface perturbation goes like $\delta\psi \sim r^{-m}$



- If there is no wall (wall is far away) then the flux-surface perturbation goes like $\delta\psi \sim r^{-m}$
- An ideal wall is a superconductor which forces the perturbation to zero at the boundary. This corresponds to a current sheet which pushes back on the mode.

Wall stabilisation of external kink modes



- If there is no wall (wall is far away) then the flux-surface perturbation goes like $\delta\psi \sim r^{-m}$
- An ideal wall is a superconductor which forces the perturbation to zero at the boundary. This corresponds to a current sheet which pushes back on the mode.
- In real machines, the vessel walls always have some finite resistivity. The current and hence radial magnetic field can diffuse into the wall

Wall stabilisation of external kink modes

The resistive diffusion is the same as we've seen before

$$\frac{\partial \underline{B}}{\partial t} = \frac{\eta}{\mu_0}\nabla^2 \underline{B}$$

and since we're interested in diffusion into the wall:

$$\frac{\partial \underline{B}}{\partial t} = \frac{\eta}{\mu_0}\nabla^2 \underline{B}$$

If the thickness of the wall is L and the time for diffusion into the wall is τ_W then

$$\frac{\underline{B}}{\tau_w} \simeq \frac{\eta}{\mu_0}\frac{1}{L_w^2}\underline{B}$$

and so the **wall time** is $\tau_w \simeq \dfrac{L_w^2 \mu_0}{\eta}$. This is typically ~ 10ms.

Resistive Wall Modes (RWM)

- In a real machine the walls are not ideal: they always have some finite resistivity
- A kink instability which would be stable for an ideal wall but unstable without a wall can grow (relatively) slowly
- Growth is limited by the rate at which magnetic field perturbations can diffuse through the wall: the wall's resistive timescale
- Hence called a Resistive Wall Mode (RWM)
- These modes set limits on the pressure (β)

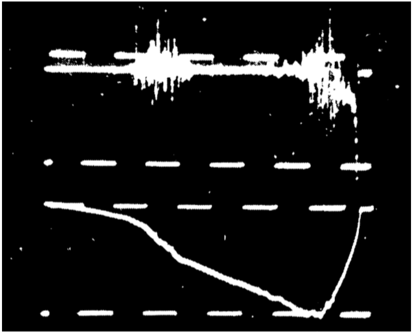
Disruptions

So what happens when a tokamak hits one of these limits?

- In a tokamak the position of the plasma is controlled using a feedback system (see this week's problem sheet)
- If an event is violent enough it can move the plasma too quickly for the system to respond. Distortions to the plasma can also confuse the system so that it makes things worse
- At this point the control system may fail and give up ("FA cutout")
- The plasma then typically hits either the top or bottom of the vessel. This is called a **disruption**.
- These must be avoided in large tokamaks, so ways to arrange a "soft landing" are being developed e.g. Massive Gas Injection.

ZETA quiescent period

- In experiments on ZETA, it was noticed that under some circumstances fluctuations were reduced for a period
- During these "quiescent" periods, confinement time was improved



Plasma current I and dI/dt traces from ZETA.
Time marks are 1ms.
[Bodin and Newton Nucl. Fusion 20 (1980) 1255,

A.Gibson *et al.* Plasma Physics 9 (1967) 1]

J.B.Taylor studied how an unstable equilibrium might relax to a minimum energy state, subject to constraints.

- In ideal MHD, magnetic fields are "frozen" into the plasma, and each field line is constrained.
- If small deviations from ideal MHD occur, then field lines may reconnect, and Taylor argued that there is then only a single constraint on the magnetic field
- This leads to a unique solution for the constrained minimum energy ("Taylor") state, in which

$$\nabla \times \mathbf{B} = \mu \mathbf{B}$$

where μ is a constant which depends on the pinch ratio $\Theta = \mu a/2$.

"Relaxation of Toroidal Plasma and Generation of Reverse Magnetic Fields" by J.B.Taylor (1974)

In a circular cross-section, large aspect-ratio torus, this relaxed state has the solution

$$B_r = 0 \qquad B_\theta = \alpha J_1(\mu r) \qquad B_\phi = \alpha J_0(\mu r)$$

where J_0 and J_1 are Bessel functions. Toroidal field reverses if $\Theta > 1.2$. Field-reversal ratio $F = B_\phi(a) / \langle B_\phi \rangle$

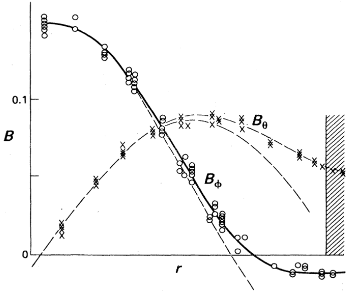


FIG. 2. Experimental and theoretical magnetic field profiles. HBTX-1A (from Bodin, 1984).

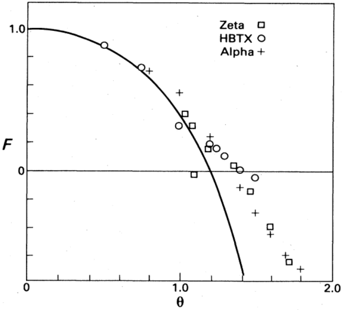
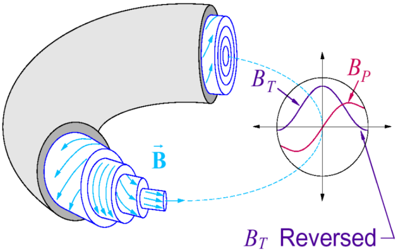


FIG. 3. F-θ diagram. Data from HBTX1, ALPHA, and ZETA and theoretical curve (from Bodin and Newton, 1980).

Reversed Field Pinches

- Internal kink modes stable
- Require a close-fitting copper shell, or active feedback, to stabilise external kinks
- Smaller external field relative to tokamaks simplifies construction



- Tend to be turbulent, and have poor confinement
- At high pinch ratio, the plasma can form a helical equilibrium (SHAx) with better confinement

Lorenzini et al., Nature Phys. 5, 570 (2009)

Summary

- **Kink instabilities** are driven by plasma current, and stabilised by an applied toroidal magnetic field
- **Internal kinks** occur even if the boundary is fixed when $q < 1$ in the core of tokamak plasmas, and limit the current on axis.
- **External kinks** require a distortion to the plasma edge, and require $q > 2$ and preferably $q > 3$ at the edge for stability. These limit the total plasma current and often the maximum plasma pressure.
- Perfectly conducting walls can stabilise external kink modes
- Resistive walls allow kink modes to grow on wall timescales: Resistive Wall Modes (RWMs)
- In tokamaks RWMs must be avoided at high β.
  In RFPs external kinks must be controlled at all pressures