# Magnetic Confinement Fusion

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# Course outline

This course is an introduction to Magnetic Confinement Fusion. I will assume some basic plasma physics, but go over lots of the background again as we go along.

- Overview and some history
- Performance limits
- Operating current and future devices

# Course outline

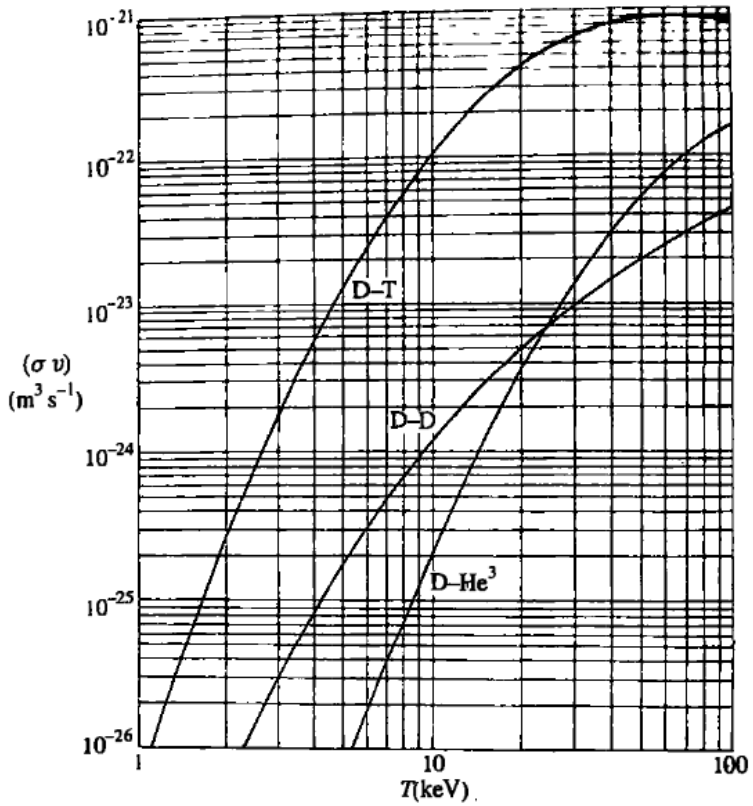
This course is an introduction to Magnetic Confinement Fusion. I will assume some basic plasma physics, but go over lots of the background again as we go along.

- Overview and some history
  - Fundamental concepts in MCF: Orbits, drifts, collisions, flux surfaces, plasma waves, ...
  - Linear machines (magnetic mirrors)
  - Stellarators
  - Tokamaks
- Performance limits
  - Pressure (beta) limits and instabilities
  - Density limits
  - Turbulence
- Operating current and future devices
  - Optimising designs and operating regimes
  - Turbulence and instability control

# Thermonuclear Fusion

Achieving fusion requires very high temperatures

- D-T has the highest cross-section, at $T \sim 10 - 20$ keV
- Other fuels possible, but require higher temperatures e.g. D-D needs $T \sim 100$ keV




The aim of Magnetic Confinement Fusion is to produce a plasma in which fusion can be sustained for long periods of time

# Thermonuclear Fusion: Alternative fuels

Some fusion fuels would avoid neutron production e.g.

$$\begin{aligned} {}^3_2He + {}^3_2He &\rightarrow {}^4_2He + 2{}^1_1p + 12.86MeV \\ {}^{11}_5B + {}^1_1p &\rightarrow 3{}^4_2He + 8.68MeV \end{aligned}$$

However: In (local) thermodynamic equilibrium all plasmas produce **Bremsstrahlung radiation**

$$P_{Br} \simeq 1.69 \times 10^{-38} Z_i^2 n_i n_e T^{1/2} \quad \mathrm{W/m^3}$$

$\Rightarrow$ High $Z$ plasmas tend to radiate more power than they produce. Out-of-equilibrium schemes have been proposed (e.g. fusors), but the power required to maintain this state has been shown to always be greater than the power produced[1]

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$$

H

pr

> For the record, the author would like to apologize for apparently killing some of the most attractive types of fusion reactors which have been proposed. He advises future graduate students working on their theses to avoid accidentally demolishing the area of research in which they plan to work after graduation. – Todd H. Rider

$\Rightarrow$ e.

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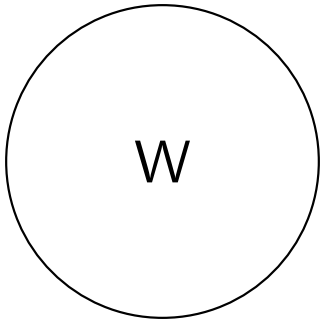
# Power balance




- For now just consider the plasma with energy content $W = \int 2 \cdot \frac{3}{2} nTdV$

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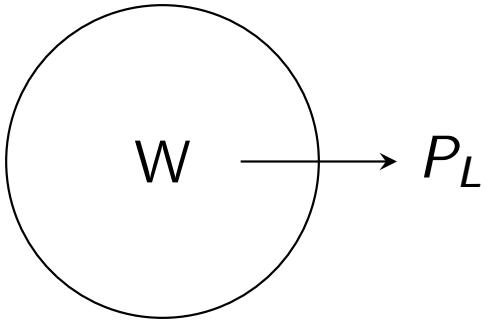


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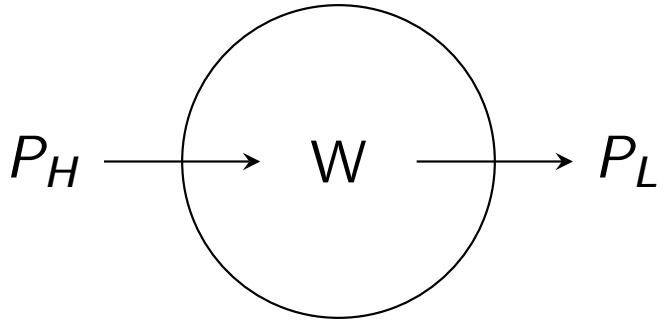
- The timescale $\tau_E$ quantifies the rate at which energy is lost from the plasma: $\boxed{P_L = W/\tau_E}$

Equations you'd be expected to know in an exam will be in a box

# Power balance

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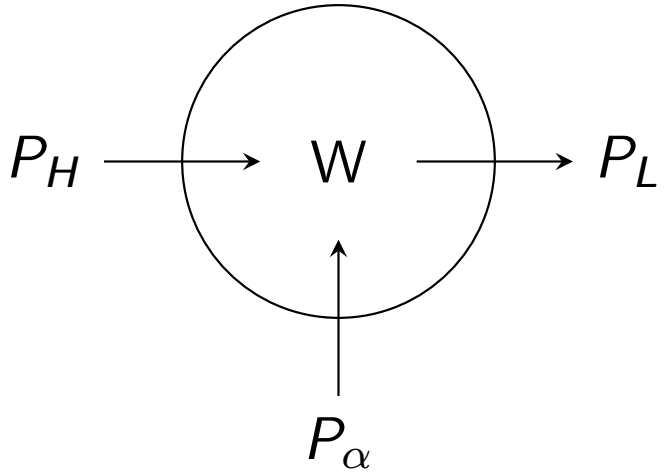


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- The timescale $\tau_E$ quantifies the rate at which energy is lost from the plasma: $\boxed{P_L = W/\tau_E}$
- Power input can either come from external heating $P_H$ or from alpha particle heating $P_\alpha = \int \frac{1}{4} n^2 \langle \sigma v \rangle E_\alpha dV$

where $E_\alpha$ is the energy of an alpha particle

# Power balance: Ignition

In any steady state, we need to have $P_H + P_\alpha = P_L$. To be self-sustaining, we need $P_\alpha = P_L$. This is called **ignition**

$$\int \frac{1}{4} n^2 \langle \sigma v \rangle E_\alpha dV = \frac{1}{\tau_E} \int 3nT dV$$

Important concepts and terms will be in bold

# Power balance: Ignition

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$$\int \frac{1}{4} n^2 \langle \sigma v \rangle E_\alpha dV = \frac{1}{\tau_E} \int 3nT dV$$

To estimate the conditions needed, take constant density and temperature:

$$\frac{1}{4} n^2 \langle \sigma v \rangle E_\alpha = 3nT/\tau_E$$

which rearranges to

$$n\tau_E = \frac{12T}{\langle \sigma v \rangle E_\alpha}$$

# Power balance: Ignition

To achieve self-sustaining fusion "ignition", we need to satisfy

$$n\tau_E > \frac{12T}{\langle \sigma v \rangle E_\alpha}$$

This has a minimum at $T \simeq 30$keV, but since $\tau_E$ also depends on temperature, this is probably not optimal. In the range $10 - 20$keV, $\langle \sigma v \rangle \simeq 1.1 \times 10^{-24} T^2 \mathrm{m}^3 \mathrm{s}^{-1}$ with $T$ in keV. This then gives

$$\boxed{nT\tau_E > 3 \times 10^{21} m^{-3} keVs}$$

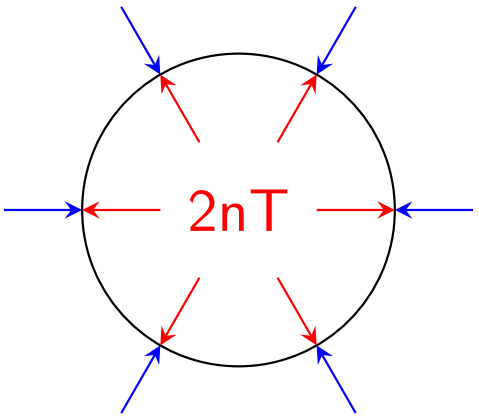
This is obviously not precise, but gives the general ballpark we need to get into

# Magnetic Confinement Fusion (MCF)

In MCF, magnetic fields are used to confine plasma. Magnetic fields behave as though they have a pressure

$$P_B = \frac{B^2}{2\mu_0}$$

In our cartoon picture of a plasma as a blob, just balance the pressure of the plasma $2nTe$ against the magnetic pressure $P_B$



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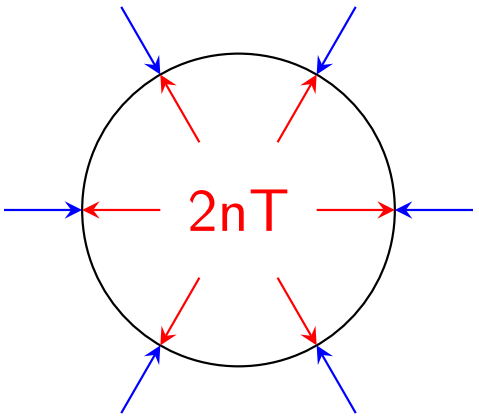


Producing more than a few Tesla using magnets is difficult, so taking $B = 1\text{T}$ and a temperature $T = 10\text{keV}$ gives

$$n \simeq 1.2 \times 10^{20} m^{-3}$$

# Magnetic Confinement Fusion (MCF)

- Though this is a gross simplification, this value for the density $n \sim 10^{20}\mathrm{m}^{-3}$ is pretty typical for an MCF device
- Putting this (along with $T = 10$ keV) into our ignition criterion $nT\tau_E > 3 \times 10^{21} m^{-3} keVs$ gives

$$\tau_E > 3s$$

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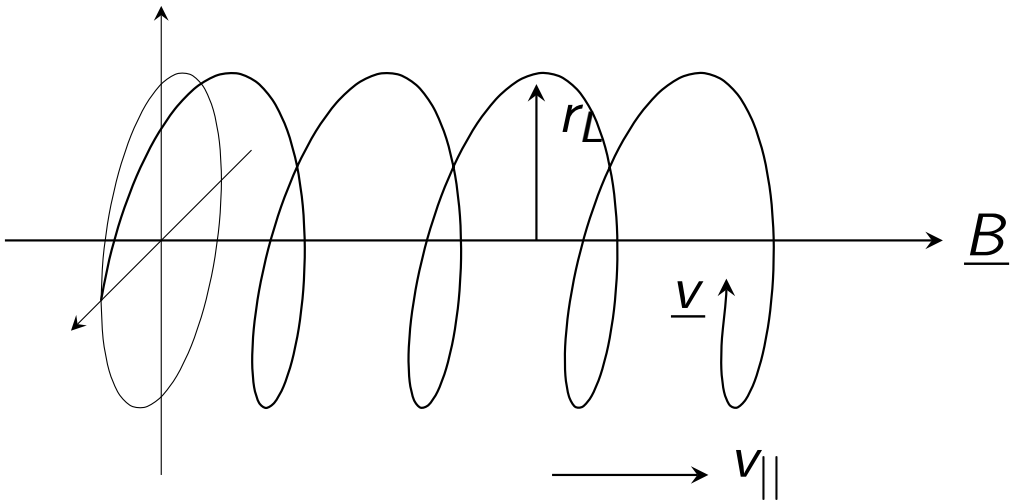
- Achieving fusion ignition in MCF means designing a plasma configuration with good insulation: it must take $\sim$ seconds for energy to leak out
- This course is about the physics behind these designs, and current areas of research
- So, starting with some basic plasma physics...

# Magnetic confinement

In magnetic fields, charged particles follow helical paths




Particles are free to move along magnetic fields, but are constrained in the perpendicular direction by the Lorentz force:

$$\boxed{\underline{F} = q\underline{v} \times \underline{B}}$$

# Adiabatic invariants

- In a strong magnetic field, the particle doesn't see much change over a single orbit so the angle of a particle around a field-line doesn't matter
- This symmetry leads to a conserved quantity[2]: the **magnetic moment** $\boxed{\mu = mv_\perp^2/(2B)}$

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- This symmetry leads to a conserved quantity[2]: the **magnetic moment** $\boxed{\mu = mv_\perp^2/(2B)}$
- The energy of a particle in a static magnetic field (so there's no electric field) cannot change. Work done is

$$W = \underline{F} \cdot \underline{v} = q\left(\underline{v} \times \underline{B}\right) \cdot \underline{v} = 0$$

- Therefore, the total energy is conserved
  $\Rightarrow v^2 = v_\perp^2 + v_{||}^2$ is also conserved

[2] Nöther's theorem links symmetries to conserved quantities

# Magnetic mirrors

- The velocity of a particle parallel to the magnetic field $v_{||}$ can be written as

$$v_{||}^2 = v^2 - v_{\perp}^2 = v^2 - \frac{2\mu B}{m}$$

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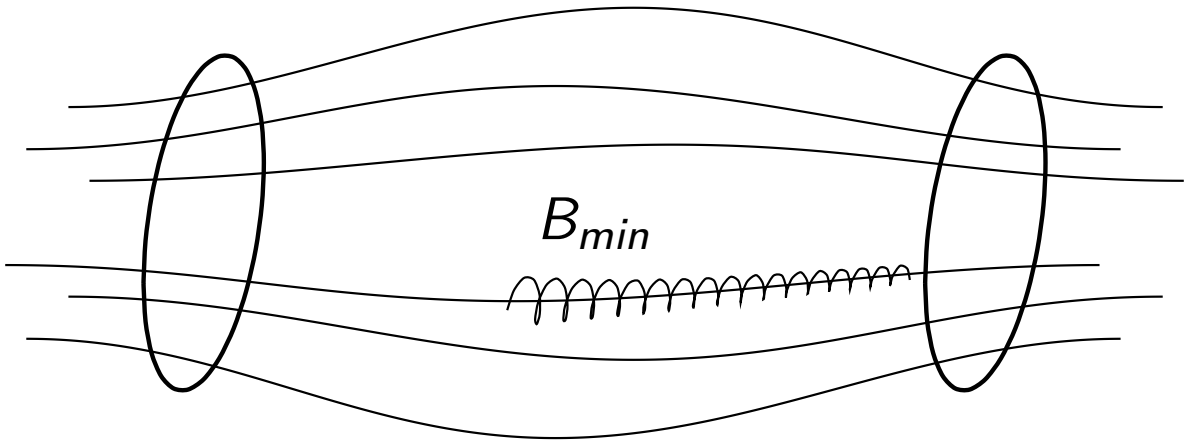
- What happens when $B$ gets too large? $v_{||}^2$ first goes to zero then becomes negative
- The parallel velocity cannot be imaginary $\Rightarrow$ the particle must reflect
- This is called a **magnetic mirror**, and was the basis of several early confinement schemes

# Tandem mirrors

The simplest magnetic confinement configuration is the axisymmetric tandem mirror




In this scheme, particles which have a magnetic moment $\mu$ such that $v_{||}^2 = 0$ at some point in the machine are confined

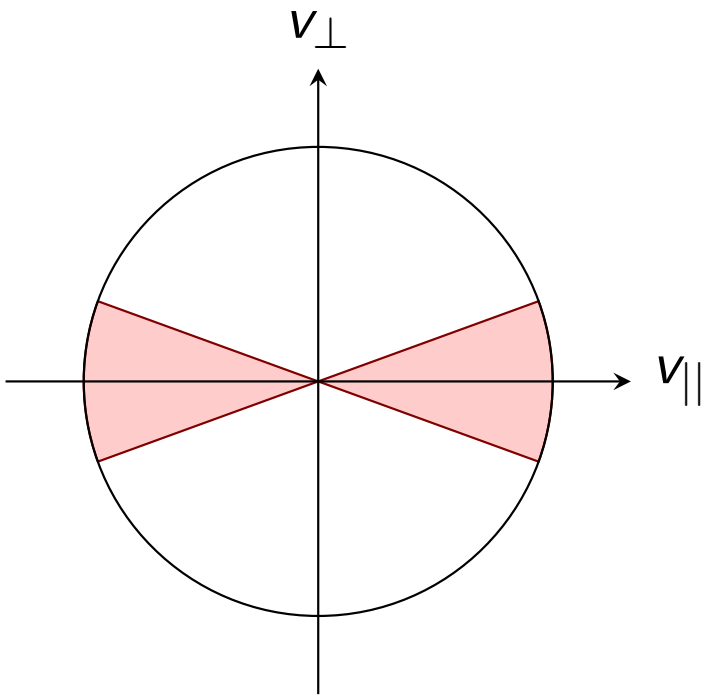
$$v_{||}^2 = v^2 - \frac{2\mu B}{m} = 0$$

Take a particle starting at the minimum B: $\mu = mv_{\perp 0}^2/2B_0$

$$v^2 - v_{\perp 0}^2 \frac{B}{B_0} = 0 \qquad \Rightarrow \frac{v_{\perp 0}^2}{v^2} = \frac{B_0}{B}$$

# Loss cone

Any particle with $\frac{v_{\perp 0}^2}{v^2} < \frac{B_0}{B_{max}}$ is lost from the machine.

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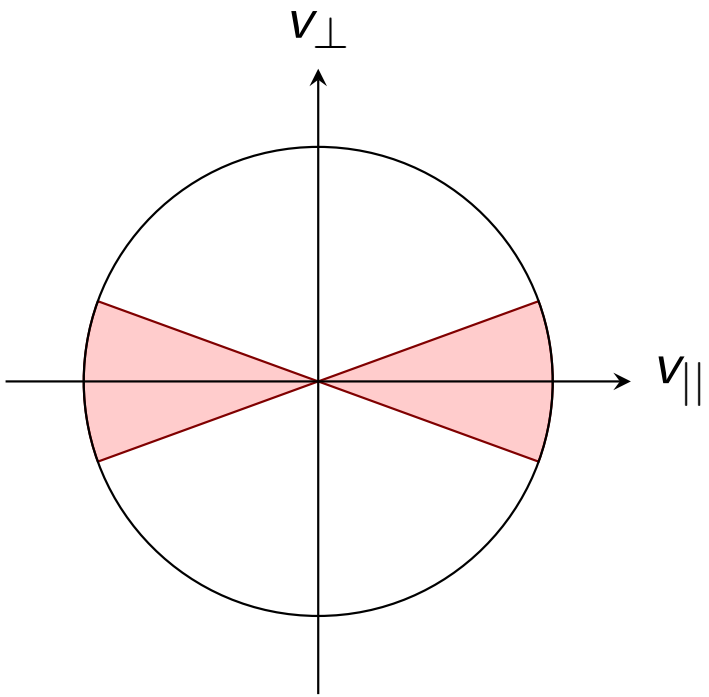


After these particles are lost, what next?
Collisions between particles change their velocities in a diffusion-like process so particles are continually lost.

# Collisions

How long does it take to change the velocity of a particle in a plasma?

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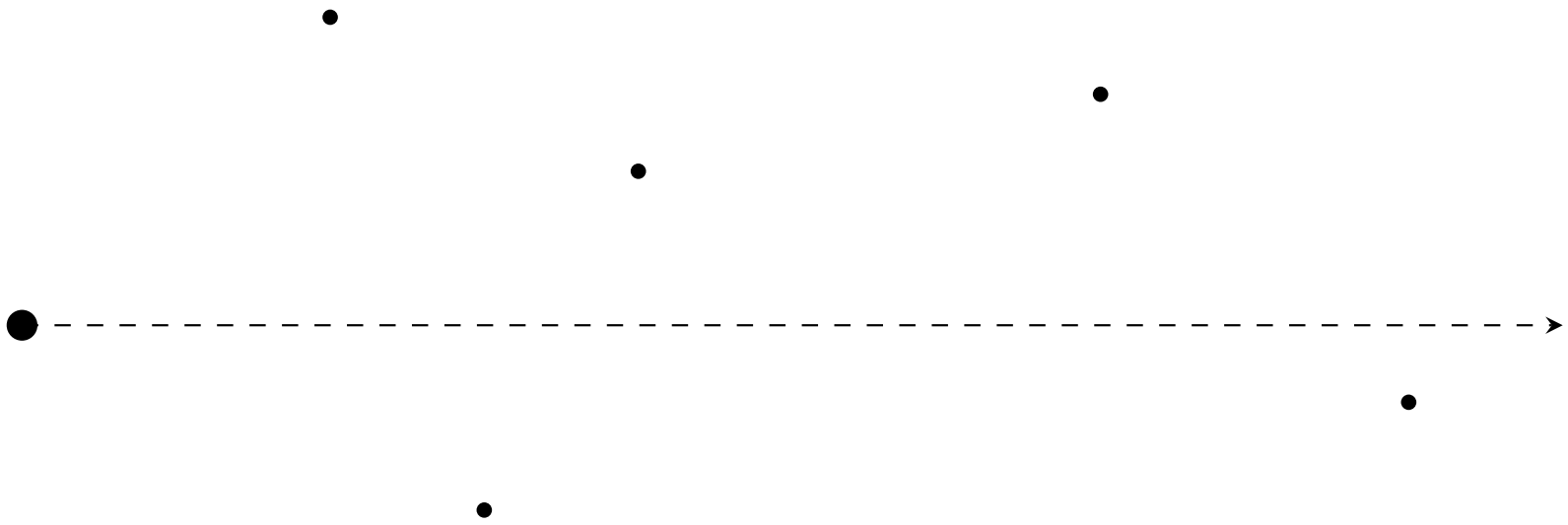
$$\nu \sim n$$

Units:

$$\left[T^{-1}\right] = \left[L^{-3}\right]$$



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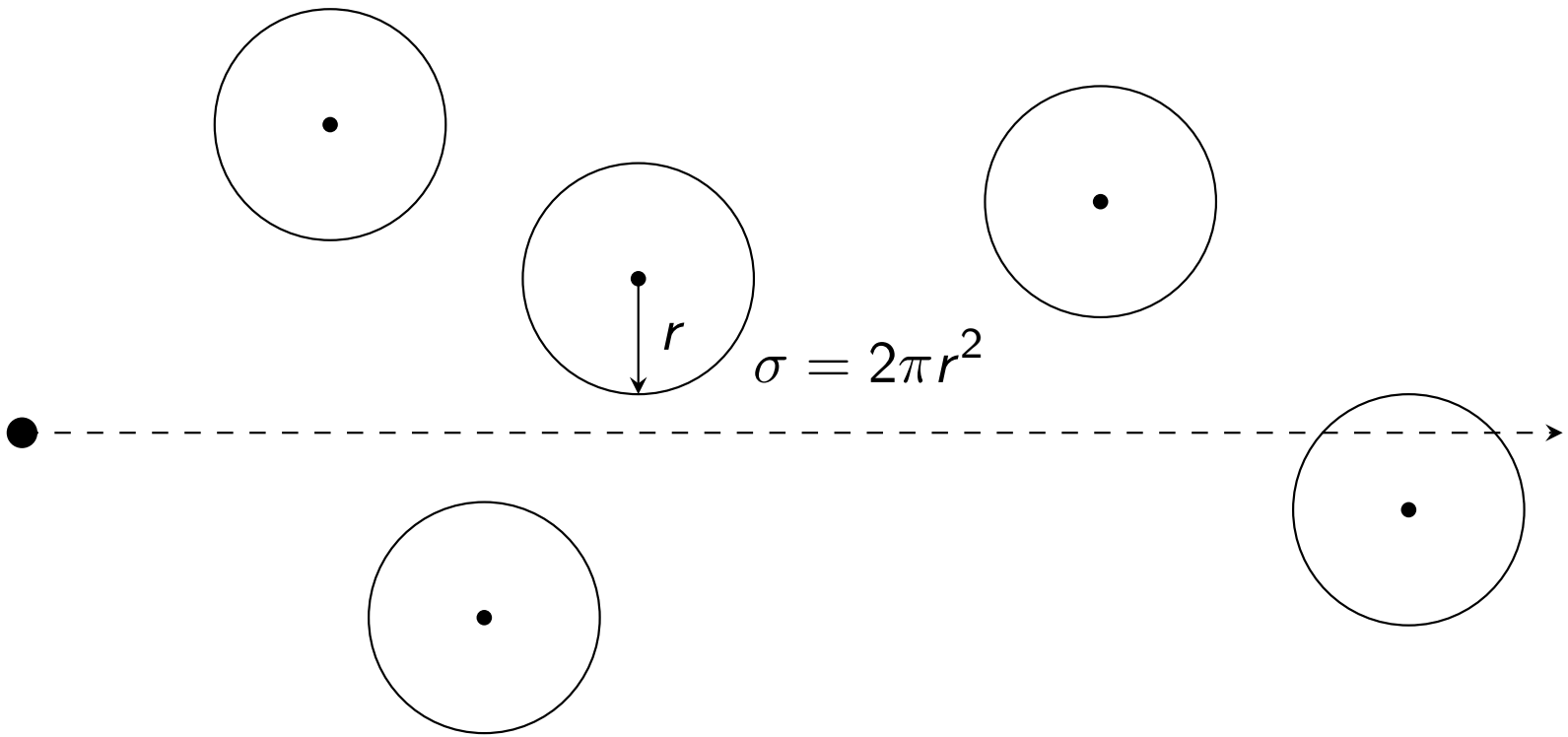

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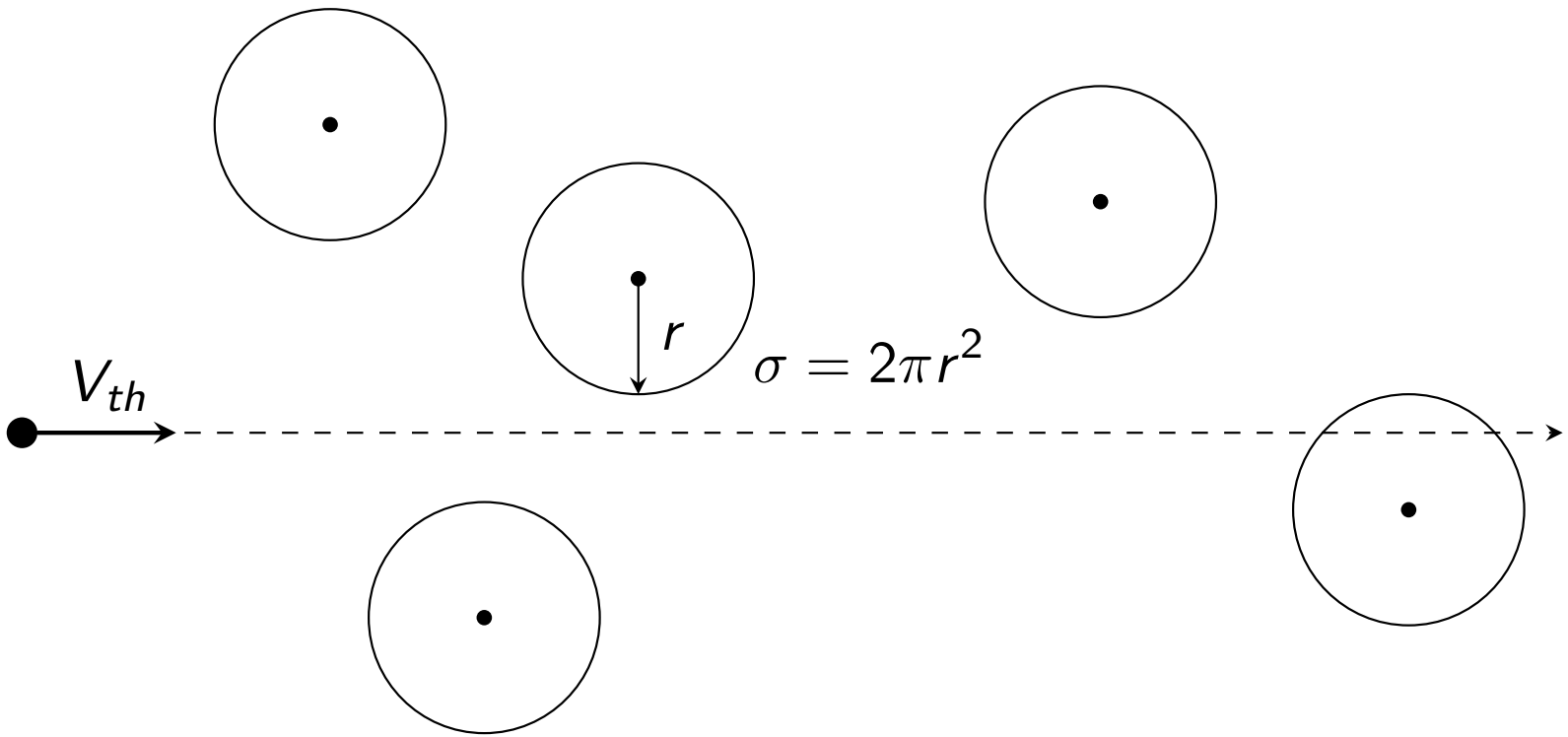

# Collisions in a plasma

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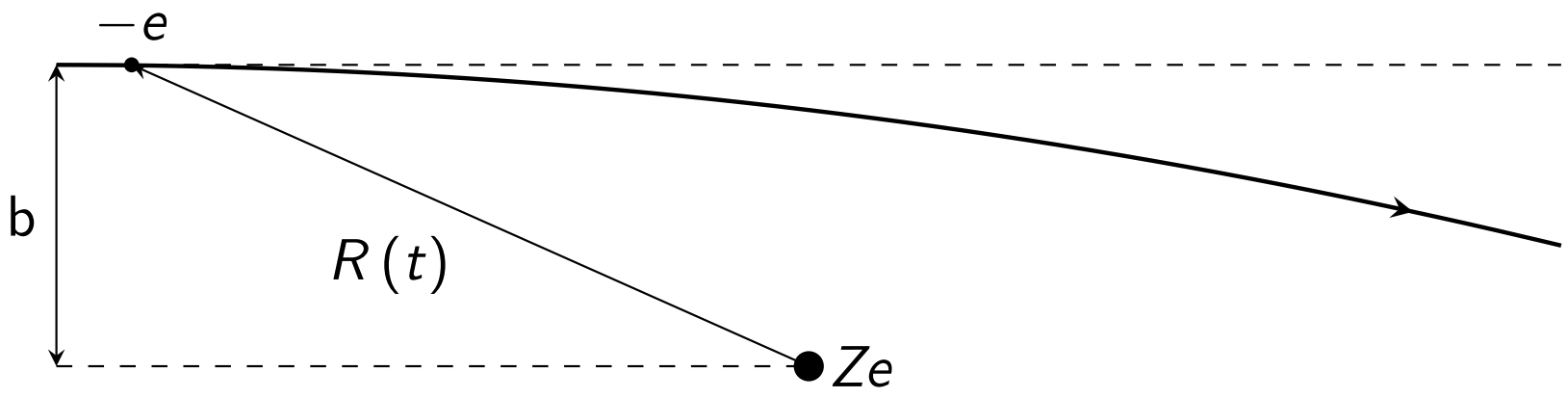
It turns out that most of the change in velocity of a particle is due to lots of small deflections. Consider an electron passing by an ion:

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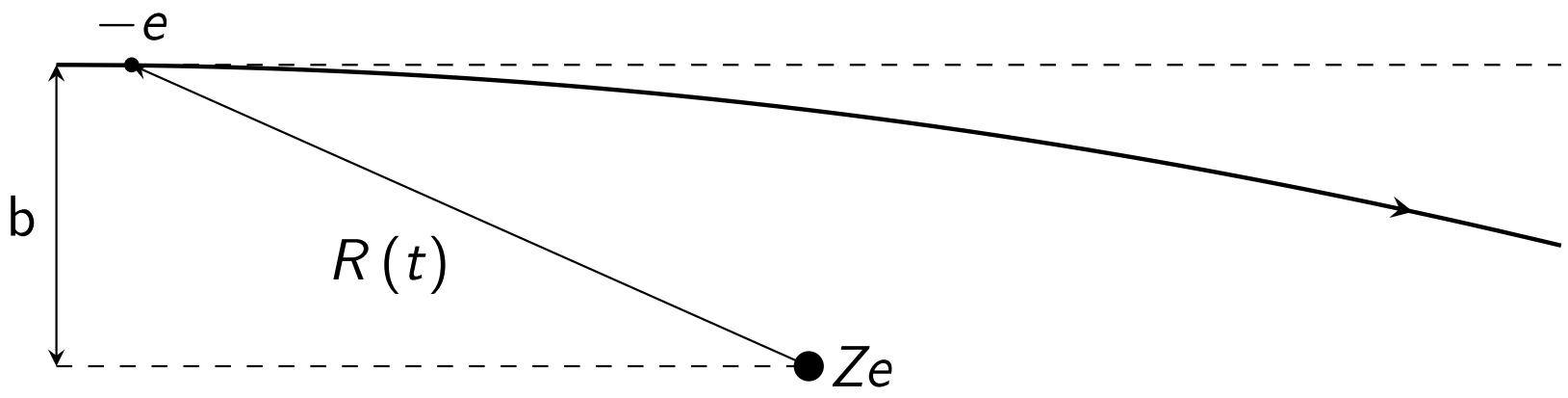
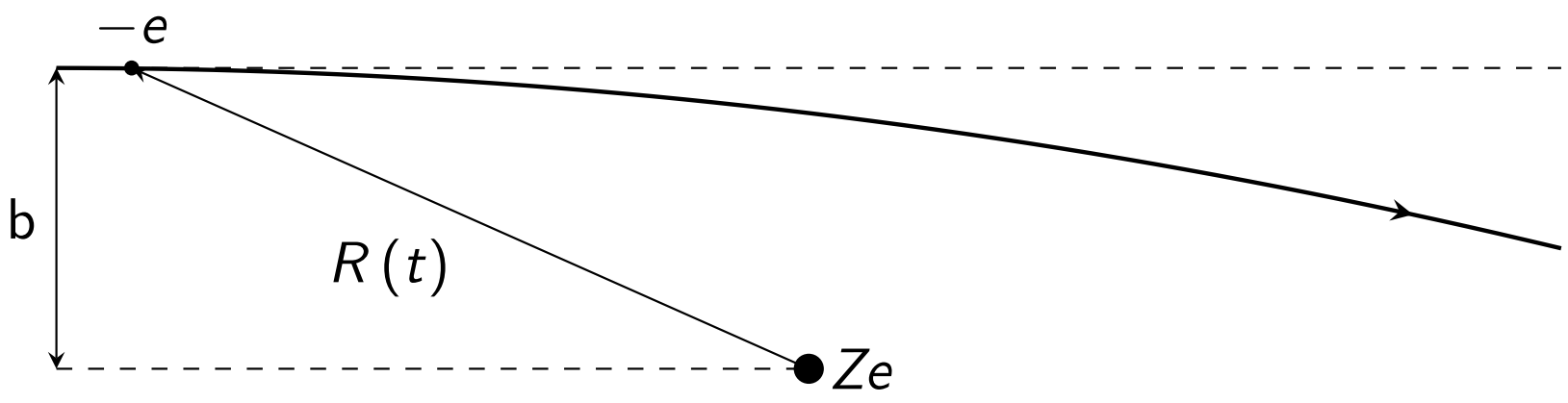



How much is the electron velocity changed by? Assume a small deflection, so the radius $R(t)$ follows the dashed line

$$R(t) \simeq \sqrt{b^2 + (vt)^2}$$

so that the electron passes the ion at $t = 0$

# Collisions in a plasma



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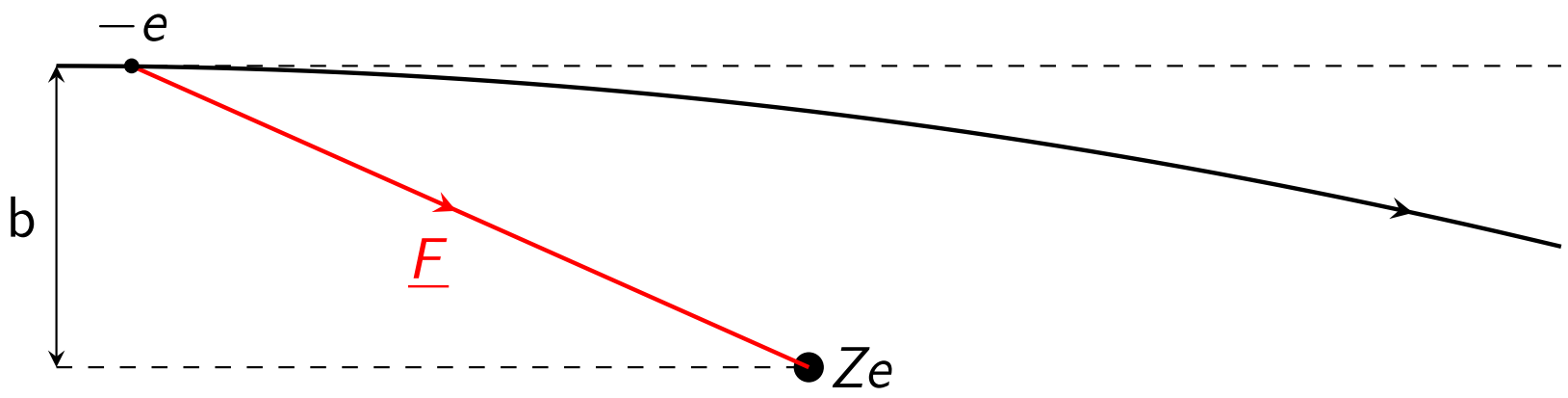
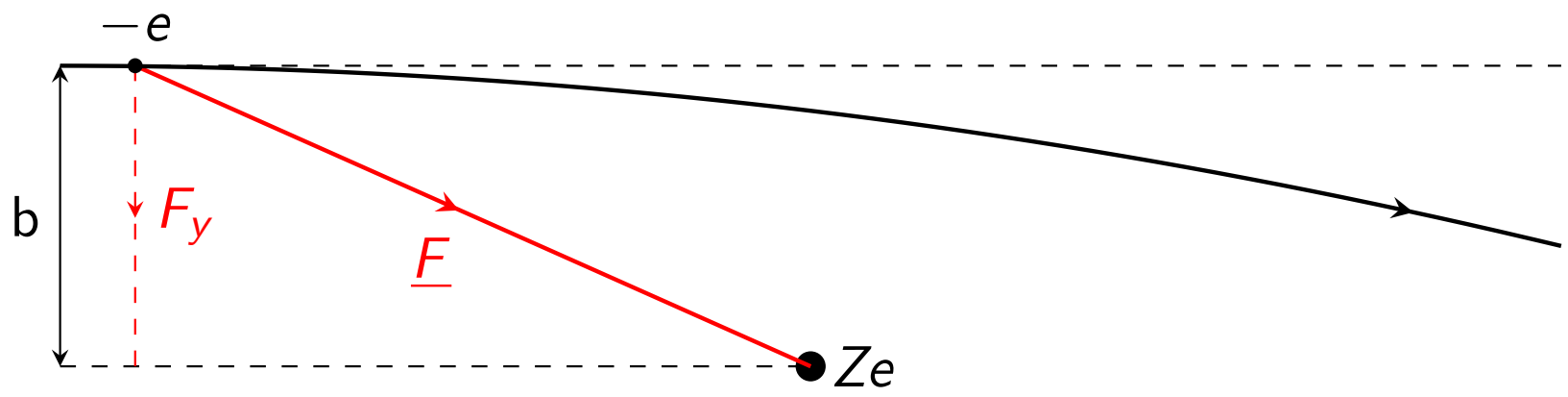



The electron feels a force towards the ion

$$\underline{F} = \frac{Ze^2}{4\pi\epsilon_0 R(t)^2}$$

# Collisions in a plasma




The electron feels a force towards the ion and so a vertical force

$$F_y = \frac{Ze^2}{4\pi\epsilon_0 R(t)^2}\frac{b}{R(t)}$$

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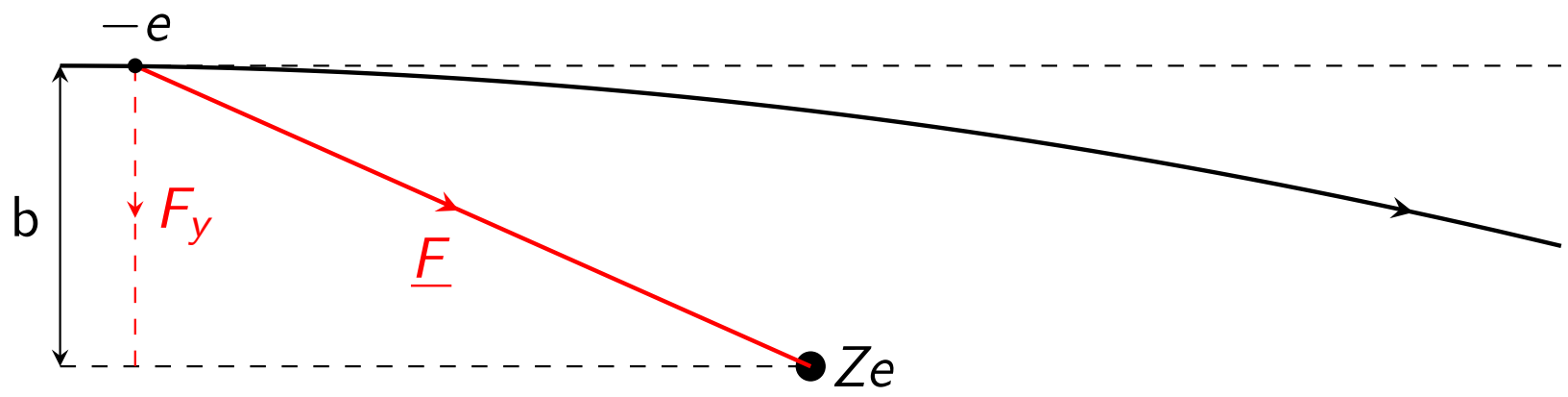


The electron feels a force towards the ion and so a vertical force

$$F_y = \frac{Ze^2}{4\pi\epsilon_0 R(t)^2}\frac{b}{R(t)}$$

We can get the total change in vertical momentum by integrating this force over time:

$$m_e\frac{dv_y}{dt} = F_y \qquad \Rightarrow m_e\Delta v_y = \int_{-\infty}^{\infty}\frac{Ze^2}{4\pi\epsilon_0 R(t)^2}\frac{b}{R(t)}dt$$

# Collisions in a plasma

Hence

$$
\begin{aligned}
m_e \Delta v_y &= \frac{Ze^2}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{b}{\left(b^2 + v^2 t^2\right)^{3/2}} dt = \frac{Ze^2}{4\pi\epsilon_0} \left[ \frac{t}{b\sqrt{v^2 t^2 + b^2}} \right]_{-\infty}^{\infty} \\
&= \frac{Ze^2}{4\pi\epsilon_0} \left[ \frac{1}{\pm b\sqrt{v^2 + b^2/t^2}} \right]_{-\infty}^{\infty} \\
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# Collisions in a plasma

Hence

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\end{aligned}
$$

The deflection angle is therefore

$$
\alpha = \frac{\Delta v_y}{v} = \frac{Ze^2}{2\pi\epsilon_0 b m_e v^2} = \frac{b_{min}}{b}
$$

where

$$
b_{min} \equiv \frac{Ze^2}{2\pi\epsilon_0 m_e v^2}
$$

# Collisions in a plasma

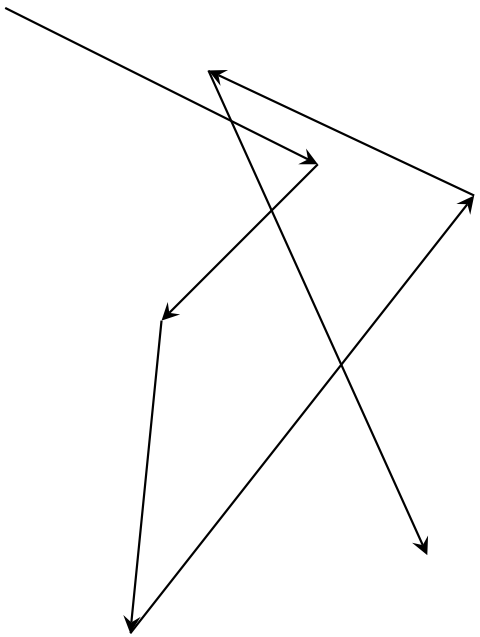
- This length $b_{min}$ is the impact parameter which produces large deflections
- Typical collisions have $b \sim \lambda_D$, the Debye length since the charge is shielded for $b \gg \lambda_D$

# Collisions in a plasma

- This length $b_{min}$ is the impact parameter which produces large deflections
- Typical collisions have $b \sim \lambda_D$, the Debye length since the charge is shielded for $b \gg \lambda_D$
- The ratio $\boxed{\Lambda = \lambda_D / b_{min}}$ needs to be large for small-angle approximations to work
- The logarithm $\ln \Lambda$ is called the **Coulomb logarithm**, and is around $10 - 20$ in fusion plasmas
- The small angle approximation is therefore a very good one

# Collision times

Because most of the deflection of a particle in a plasma is due to lots of small collisions, the path of a particle looks quite different to in a gas



Molecule path in a gas

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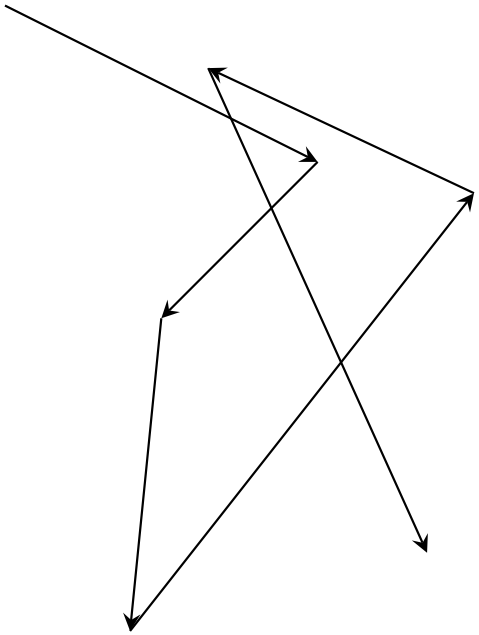

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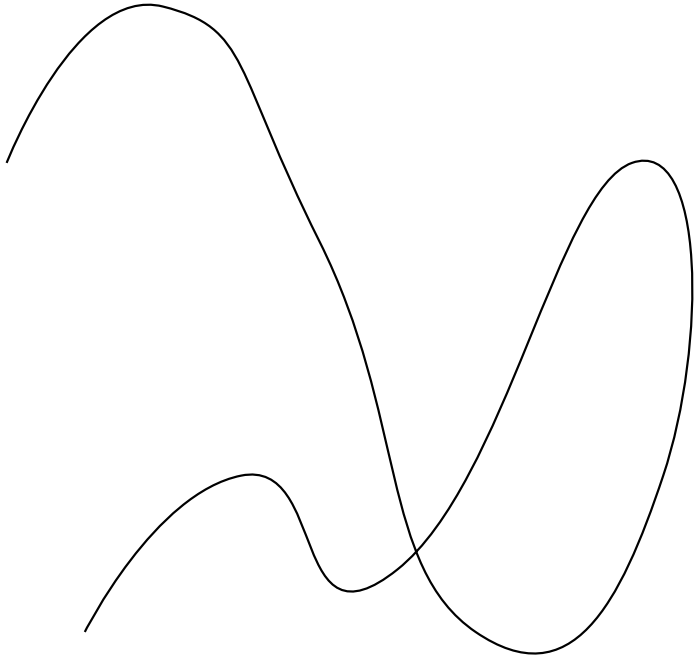

Path in a plasma

How to define a collision time when particles are always colliding?

# Collision times

In a plasma, the **collision time** is defined as the average time it takes a particle to be deflected by $90^o$.

$$\tau_{ei} = \frac{12\pi^{3/2}}{2^{1/2}} \frac{m_e^{1/2} T_e^{3/2} \epsilon_0^2}{n_i Z^2 e^4 \ln \Lambda}$$

$$\tau_{ei} = 3.44 \times 10^{11} \left( \frac{1m^{-3}}{n_e} \right) \left( \frac{T_e}{1eV} \right)^{3/2} \frac{1}{Z_i \ln \Lambda}$$

Putting in $T_e = 10\text{keV}$, and a density of $n_e = 10^{20}\text{m}^{-3}$ gives $\tau_{ei} = 1.7 \times 10^{-4}\text{s}$.

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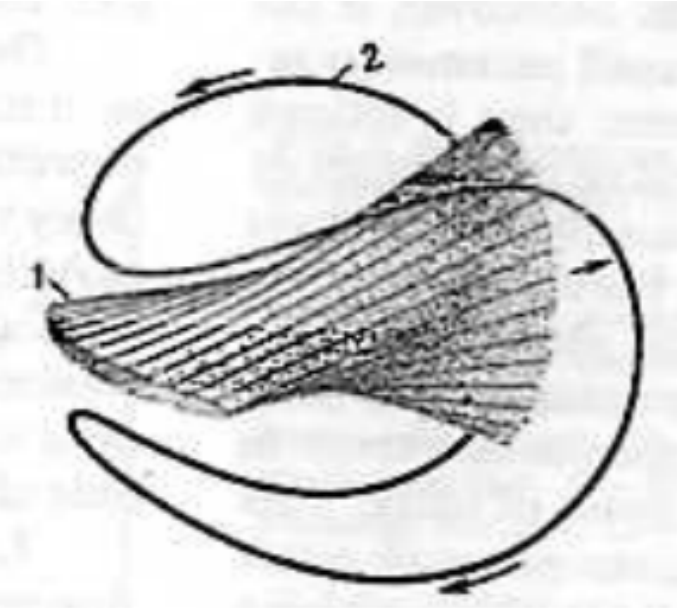
- This implies that even at high temperatures, electrons will be scattered and lost from magnetic mirrors quite quickly

# Mirror variants and modern designs

- There were several problems with axisymmetric mirrors, which which we'll study in more detail later
- A big problem was that they were MHD unstable, and so couldn't contain high plasma pressure
- Variants are minimum-B machines, which use baseball coils to confine particles. This is more stable, but has worse individual particle confinement



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- Variants are minimum-B machines, which use baseball coils to confine particles. This is more stable, but has worse individual particle confinement
- In the late '70s, the tokamak was found to be more immediately promising for getting to fusion conditions
- Research on magnetic mirrors basically ceased in the 1980s in Europe and the US, but has continued in Japan and Russia[3]
- Modern designs (such as Gas Dynamic Traps[4]) improve on the basic mirror, but a reactor would need to be $\sim$ 1km long.

[3] LLNL report TR-408176 "The Axisymmetric Tandem Mirror" 2008 `https://e-reports-ext.llnl.gov/pdf/366958.pdf`

[4] Ivanov, Prikhodko Plasma Phys. Control. Fusion **55** (2013) 063001

# Summary

- Charged particles can be confined using strong magnetic fields
- Conservation of energy and **magnetic moment** $\mu$ leads to particles bouncing off strong $B$ fields and so **trapping**
- This can be used to make a simple confinement device: the axisymmetric tandem mirror
- In a plasma, collisions consist of lots of small deflections
- **Small angle deflections** can be used to derive expressions for the **collision times**: the timescale over which the velocity is changed by $90^o$
- In tandem mirrors, collisions lead to losses of particles and energy from the ends of the machine

**Next time**: Toroidal machines