Heating and current drive: Radio Frequency

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Previously...

- We've looked at Ohmic heating and current drive, and their limitations
- Neutral beams can be used to supply heating power and drive current
- Last lecture we studied waves in plasmas, finding 3 basic waves in ideal MHD: Shear Alfvén, fast and slow magnetosonic
- In non-homogenous plasmas, the shear Alfvén wave gives rise to Alfvén eigenmodes. These can couple together to form "gap" modes called TAEs, EAEs, ...
- This lecture we'll look at higher frequency modes, which can be coupled to inject power using radio frequency methods

Radio Frequency heating

- The general principle is to fire radio frequency waves with a frequency ω into the plasma
- These must be engineered so that they travel through the outer edge of the plasma, but are absorbed at a chosen location inside the plasma
- As a wave passes through a plasma it accelerates electrons which then collide and dissipate energy. This **collisional absorption** decreases with temperature like $T^{-3/2}$ so not effective at high temperatures
- Instead, **resonant absorption** where the frequency of the wave matches a frequency in the plasma is used. This is a collisionless process so works in high temperature plasmas

Resonant absorption

- Variations in density and temperature lead to changes in the wavevector $\underline{k}$ and so changes in the refractive index $\underline{N} = c\underline{k}/\omega$
  - $N^2 \to 0$ implies reflection (**cut off**)
  - $N^2 < 0$ implies evanescence (decaying not oscillating)
  - $N^2 \to \infty$ implies absorption (**resonance**)
- For an RF heating scheme to work, waves must be able to propagate from the antenna through the plasma without significant loss, before reaching a resonance location and giving up their energy to the plasma
- To study this process, we can use the cold plasma dispersion relations except near the resonance where thermal corrections can become important.

RF plasma waves

Maxwell's equations

$$\begin{aligned} \nabla \times \underline{E} &= -\frac{\partial \underline{B}}{\partial t} \\ \nabla \times \underline{B} &= \mu_0 \underline{J} + \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} \end{aligned}$$

Assuming uniform $\underline{B}_0$ so $\nabla \to i\underline{k}$ and $\dfrac{\partial}{\partial t} \to -i\omega$

$$\begin{aligned} \underline{k} \times \underline{E}_1 &= \omega \underline{B}_1 \\ \underline{k} \times \underline{B}_1 &= -i\mu_0 \underline{J}_1 - \frac{\omega}{c^2}\underline{E}_1 \end{aligned}$$

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where the electric permittivity tensor is $\underline{\underline{\epsilon}} = \underline{\underline{I}} + \dfrac{i}{\epsilon_0 \omega}\underline{\underline{\sigma}}$ and $\underline{\underline{\sigma}}$ is the conductivity tensor defined by

$$\boxed{\underline{J}_1 = \underline{\underline{\sigma}} \cdot \underline{E}_1}$$

RF plasma waves

$$\begin{aligned} \underline{k} \times \underline{E}_1 &= \omega \underline{B}_1 \\ \underline{k} \times \underline{B}_1 &= -\frac{\omega}{c^2} \underline{\underline{\epsilon}} \cdot \underline{E}_1 \end{aligned}$$

Substitute the first equation into the second

$$\underline{k} \times \left(\frac{1}{\omega} \underline{k} \times \underline{E}_1 \right) = -\frac{\omega}{c^2} \underline{\underline{\epsilon}} \cdot \underline{E}_1$$

using $\underline{A} \times (\underline{B} \times \underline{C}) = \underline{B} (\underline{A} \cdot \underline{C}) - \underline{C} (\underline{A} \cdot \underline{B})$ this becomes

$$\frac{1}{\omega} \left[\underline{k} (\underline{k} \cdot \underline{E}_1) - \underline{E}_1 (\underline{k} \cdot \underline{k}) \right] = -\frac{\omega}{c^2} \underline{\underline{\epsilon}} \cdot \underline{E}_1$$

which can be written as

$$\left[\underline{k}\underline{k} - k^2 \underline{\underline{I}} + \frac{\omega^2}{c^2} \underline{\underline{\epsilon}} \right] \underline{E}_1 = 0$$

Need to calculate the conductivity tensor...

Conductivity tensor

Recall the equations of motion for electrons and ions

$$
\begin{aligned}
M_i n \frac{\partial \underline{v}_i}{\partial t} &= +en\left(\underline{E} + \underline{v}_i \times \underline{B}\right) - \nabla p_i + P_{ie} \\
m_e n \frac{\partial \underline{v}_e}{\partial t} &= -en\left(\underline{E} + \underline{v}_e \times \underline{B}\right) - \nabla p_e + P_{ei}
\end{aligned}
$$

Define mass density ρ, fluid velocity $\underline{v}$ and current $\underline{J}$

$$
\boxed{
\begin{aligned}
\rho &= n\left(M_i + m_e\right) \\
\underline{v} &= \frac{M_i \underline{v}_i + m_e \underline{v}_e}{M_i + m_e} \\
\underline{J} &= ne\left(\underline{v}_i - \underline{v}_e\right)
\end{aligned}
}
$$

Conductivity tensor

Adding the equations of motion gives the MHD momentum equation

$$\rho \frac{\partial \underline{v}}{\partial t} = \underline{J} \times \underline{B} - \nabla p$$

Multiply the ion equation by m_e, the electron equation by M_i and subtract:

$$\begin{aligned} M_i m_e n \frac{\partial}{\partial t} \left(\frac{\underline{J}}{n} \right) =& e\rho \underline{E} + en \left(m_e \underline{v}_i + M_i \underline{v}_e \right) \times \underline{B} \\ & - m_e \nabla p_i + M_i \nabla p_e - \left(M_i + m_e \right) \underline{P}_{ei} \end{aligned}$$

Ohm's law

The exchange of momentum between electrons and ions due to collisions $\underline{P}_{ei}$ is given by the resistivity:

$$\underline{P}_{ei} = \eta e^2 n^2 \left(\underline{v}_i - \underline{v}_e\right) = \eta e n \underline{J}$$

and we can write

$$m_e \underline{v}_i + M_i \underline{v}_e = \frac{\rho}{n}\underline{v} - \left(M_i - m_e\right)\frac{\underline{J}}{ne}$$

to obtain **Ohm's law** for plasmas:

$$\begin{aligned}
\underline{E} + \underline{v} \times \underline{B} =& \eta \underline{J} && \text{Resistivity} \\
&+ \frac{1}{e\rho}\left[\frac{M_i m_e n}{e}\frac{\partial}{\partial t}\left(\frac{\underline{J}}{n}\right)\right. && \text{Electron inertia} \\
&+ \left(M_i - m_e\right)\underline{J} \times \underline{B} && \text{Hall term} \\
&+ \left. m_e \nabla p_i - M_i \nabla p_e\right]
\end{aligned}$$

Cold plasma $T \to 0$

- When studying RF waves in plasma, thermal corrections can be neglected except when the phase velocity becomes comparable to the thermal speed near resonant surfaces. $\Rightarrow T \to 0$
- Neglecting collisional damping, we can consider ideal plasma $\Rightarrow \eta \to 0$

This simplifies Ohms law to:

$$\underline{E} + \underline{v} \times \underline{B} = \frac{1}{e\rho}\left[\frac{M_i m_e n}{e}\frac{\partial}{\partial t}\left(\frac{\underline{J}}{n}\right) + (M_i - m_e)\,\underline{J} \times \underline{B}\right]$$

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Linearising the velocity equation and this Ohm's law gives:

$$M_i n_o \frac{\partial \underline{v}_1}{\partial t} = \underline{J}_1 \times \underline{B}_0$$

$$\underline{E}_1 = -\underline{v}_1 \times \underline{B}_0 + \frac{1}{n_0 e}\underline{J}_1 \times \underline{B}_0 + \frac{m_e}{n_0 e^2}\frac{\partial \underline{J}_1}{\partial t}$$

Cold plasma $T \to 0$

Using $\dfrac{\partial}{\partial t} \to -i\omega$ gives

$$\underline{v}_1 = \frac{i}{\omega M_i n_0} \underline{J}_1 \times \underline{B}_0$$

$$\underline{E}_1 = -\underline{v}_1 \times \underline{B}_0 + \frac{1}{n_0 e} \underline{J}_1 \times \underline{B}_0 - \frac{i\omega m_e}{n_0 e^2} \underline{J}_1$$

and eliminating $\underline{v}_1$ gives

$$\underline{E}_1 = -\frac{i}{\omega M_i n_0} (\underline{J}_1 \times \underline{B}_0) \times \underline{B}_0 + \frac{1}{n_0 e} \underline{J}_1 \times \underline{B}_0 - \frac{i\omega m_e}{n_0 e^2} \underline{J}_1$$

which then gives us the conductivity tensor $\underline{J}_1 = \underline{\underline{\sigma}} \cdot \underline{E}_1$

Cold plasma dispersion relation

This conductivity tensor can be put back into our expression for the linearised electric field:

$$\left[\underline{kk} - k^2 \underline{\underline{I}} + \frac{\omega^2}{c^2} \underline{\underline{\epsilon}}\right] \underline{E}_1 = 0$$

w.l.o.g. take $\underline{B}_0$ to be in the z direction, and $\underline{k}$ to be in the x-z plane i.e.

$$\underline{k} = \begin{pmatrix} k_x \\ 0 \\ k_z \end{pmatrix} = \begin{pmatrix} k_\perp \\ 0 \\ k_{||} \end{pmatrix} = \begin{pmatrix} k \sin\theta \\ 0 \\ k \cos\theta \end{pmatrix}$$

where θ is the angle between $\underline{k}$ and $\underline{B}_0$

Cold plasma dispersion relation

All this gives the following expression:

$$\begin{pmatrix} S - N_{||}^2 & -iD & N_{||}N_{\perp} \\ iD & S - N^2 & 0 \\ N_{||}N_{\perp} & 0 & P - N_{\perp}^2 \end{pmatrix} \begin{pmatrix} E_{1x} \\ E_{1y} \\ E_{1z} \end{pmatrix} = 0$$

where $\boxed{\underline{N} = c\underline{k}/\omega}$ is the **refractive index**, $N_{||}$ and $N_{\perp}$ are the components parallel and perpendicular to $\underline{B}_0$ respectively.

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This equation contains the plasma frequency for electrons and ions ($s = i, e$) normalised to the wave frequency ω

$$\Pi_s = \sqrt{\frac{n_0 q_s^2}{\epsilon m_s}} \qquad X_s = \frac{\Pi_s}{\omega}$$

and the electron and ion cyclotron frequencies

$$\Omega_s = \frac{q_s B_0}{m_s} \qquad Y_s = \frac{\Omega_s}{\omega}$$

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These then form the permittivities ($\underline{\underline{\epsilon}}$ components) for Right and Left handed circularly polarised waves and Parallel permittivity

$$R = 1 - \frac{X_e^2}{1 + Y_e} - \frac{X_i^2}{1 + Y_i} \simeq 1 - \frac{X_e^2}{1 + Y_e + Y_e Y_i}$$

$$L = 1 - \frac{X_e^2}{1 - Y_e} - \frac{X_i^2}{1 - Y_i} \simeq 1 - \frac{X_e^2}{1 - Y_e + Y_e Y_i}$$

$$P = 1 - X_e^2 - X_i^2 \simeq 1 - X_e^2$$

which in cartesian coordinates appear as the Sum and Difference

$$S = \frac{R + L}{2} \qquad D = \frac{R - L}{2}$$

Parallel propagation ($N_\perp = 0$)

To find solutions, we're looking for where the determinant of this matrix is zero. For parallel propagation, $\underline{k} = k_{||}\underline{b}$ and the dispersion equation becomes

$$\begin{pmatrix} S - N_{||}^2 & -iD & 0 \\ iD & S - N^2 & 0 \\ 0 & 0 & P \end{pmatrix} \begin{pmatrix} E_{1x} \\ E_{1y} \\ E_{1z} \end{pmatrix} = 0$$

The determinant of this is

$$\left(S - N_{||}^2\right)\left(S - N^2\right)P - \left(-iD\right)\left(iD\right)P = 0$$

$$\Rightarrow \left[\left(S - N_{||}^2\right)^2 - D^2\right]P = 0$$

Either $P \simeq 1 - X_e^2 = 0$ or $\left(\frac{R+L}{2} - N_{||}^2\right)^2 = \left(\frac{R-L}{2}\right)^2$

Parallel propagation ($N_\perp = 0$)

Therefore three solutions:

- Electrostatic plasma wave $\omega^2 \simeq \Pi_e^2$ (no dispersion)
- Right handed circularly polarised wave
  - $N_{||}^2 = R \simeq 1 - \dfrac{\Pi_e^2}{(\omega + \Omega_e)(\omega + \Omega_i)}$
  - Electron cyclotron resonance (absorption) at $\omega = -\Omega_e$
- Left handed circularly polarised wave
  - $N_{||}^2 = L \simeq 1 - \dfrac{\Pi_e^2}{(\omega - \Omega_e)(\omega - \Omega_i)}$
  - Ion cyclotron resonance (absorption) at $\omega = \Omega_i$
- Cut-offs (reflections) at $R = 0$ and $L = 0$

Perpendicular propagation ($N_{||} = 0$)

For propagation perpendicular to $\underline{B}_0$, set $N_{||} = 0$ to get:

$$\begin{pmatrix} S & -iD & 0 \\ iD & S - N^2 & 0 \\ 0 & 0 & P - N_\perp^2 \end{pmatrix} \begin{pmatrix} E_{1x} \\ E_{1y} \\ E_{1z} \end{pmatrix} = 0$$

which has a determinant

$$S\left(S - N^2\right)\left(P - N_\perp^2\right) - (-iD)(iD)\left(P - N_\perp^2\right) = 0$$

$$\Rightarrow \left[S\left(S - N_\perp^2\right) - D^2\right]\left(P - N_\perp^2\right) = 0$$

so either $N_\perp^2 = P \simeq 1 - X_e^2$ or $SN_\perp^2 = D^2 - S^2$

Perpendicular propagation ($N_{||} = 0$)

These two solutions correspond to:

- Ordinary O-mode $\omega^2 = \Pi_e^2 + k_\perp^2 c^2$ (light like)
  - Electric field parallel to $\underline{B}_0$
  - Density cut-off (reflection) at $\omega = \Pi_e$
- Extraordinary X-mode $SN_\perp^2 = D^2 - S^2$
  - Electric field perpendicular to $\underline{B}_0$
  - Resonances at:
    Upper hybrid frequency $\omega_{UH} = \sqrt{\Pi_e^2 + \Omega_e^2}$
    Lower hybrid frequency $\omega_{LH} = \Omega_e \sqrt{\dfrac{\Omega_i^2 + \Pi_i^2}{\Omega_e^2 + \Pi_e^2}}$
  - Cut-offs (reflections) at $R = 0$ and $L = 0$

RF injection in tokamaks

Three resonances are commonly used to inject RF power into tokamak plasmas

- Ion Cyclotron Resonance Heating (ICRH) $\omega \sim \Omega_i$
  Resonance only occurs when two or more ion species are present at an ion-ion hybrid, or Buchsbaum, resonance.
  Typical frequencies $\sim 30 - 120$MHz

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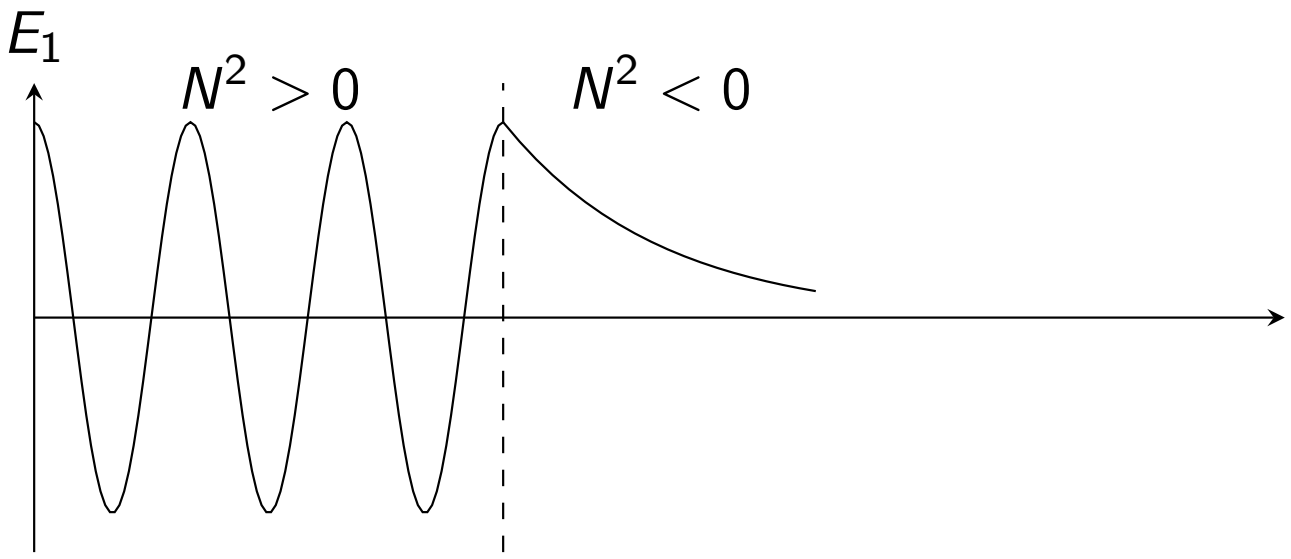
- Lower Hybrid (LH). Lies between the electron and ion cyclotron frequencies
  Typical frequencies $\sim 1 - 8$Ghz
- Electron Cyclotron Resonance Heating (ECRH) $\omega \sim \Omega_e$
  For perpendicular propagation this resonates at the Upper Hybrid frequency
  Typical frequencies $\sim 100 - 200$GHz
- These need to be efficiently coupled into the plasma

Wave propagation

Away from resonant surfaces, we can use the cold plasma relations to calculate wave propagation through the plasma from the antenna to the resonant location.

- In regions where $N^2 < 0$, the wave cannot propagate



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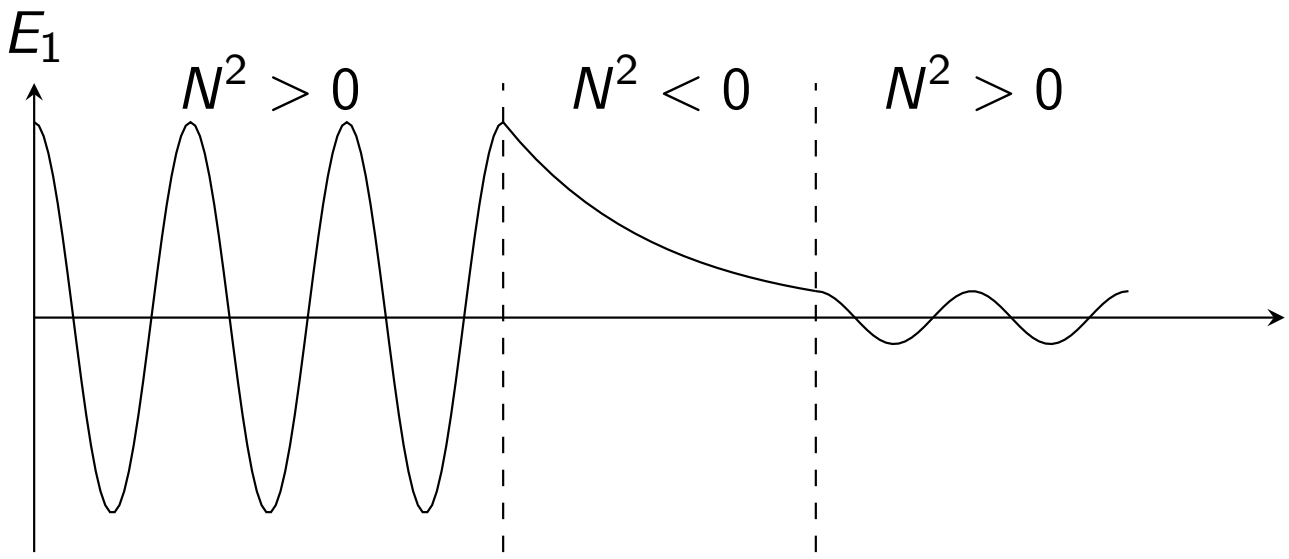

- Where N^2 goes through 0 defines a cut-off location beyond which the wave becomes evanescent
- If the region with $N^2 < 0$ is thin, some wave energy can leak or "tunnel" through

Wave absorption at resonant layers

At resonant surfaces, the phase velocity slows and can become comparable to the particle thermal speeds. At this point there are two (collisionless) ways for a wave to lose energy:

- Some energy can be transferred to another wave which is resonant in the same region. This is called **mode conversion**
- Collisionless resonant wave-particle interactions transfers energy from the wave to the particles with a resonance condition

$$\omega - k_{||} v_{||j} - l \left|\Omega_j\right| = 0$$

where $l = 0, 1, 2, 3, \ldots$

  - $l = 0$ is the Landau damping resonance
  - $l \neq 0$ are the electron or ion cyclotron resonances

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  where $l = 0, 1, 2, 3, \ldots$
  - $l = 0$ is the Landau damping resonance
  - $l \neq 0$ are the electron or ion cyclotron resonances
- Only a small population of particles are affected by the wave
  - Collisions tend to drive back to a Maxwellian
  - Strong heating can produce high energy "tail" of energetic particles e.g. ICRH can lead to $\sim$ 1MeV ions

Electron cyclotron propagation

For an X-mode wave with $\omega \sim \Omega_e$ the refractive index varies across the plasma:

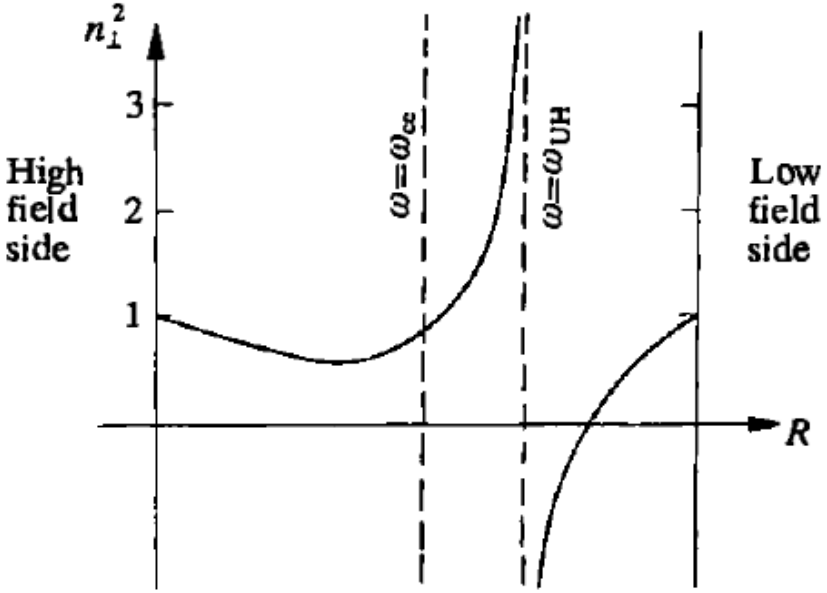

Figure : electron cyclotron X-mode wave with $n_{||} = 0$. [Wesson fig 5.7.1]

- There is a cut-off on the low field side so the wave must tunnel through to reach the resonance on the high field side
- The higher the density (or lower the field) the further apart the cut-off and resonance are
- This poses a problem for spherical tokamaks

Bernstein wave heating

- In spherical tokamaks, the magnetic field is low compared to conventional tokamaks. In this case the density cut-off can prevent RF waves from reaching the plasma core.
- For cold plasmas we found that the electrostatic plasma wave does not propagate as $\omega \simeq \Pi_e$ and so the group velocity $\frac{\partial \omega}{\partial k} = 0$
- When thermal effects are taken into account, electrostatic waves can propagate at harmonics of the plasma frequency. These are called **Bernstein waves**
- Only exist in a hot plasma, so cannot propagate in vacuum
- Injection requires mode conversion O→X→B

Lower hybrid current drive

The most effective current drive presently is LH current drive.

- A phased array antenna couples energy to Lower Hybrid (X-mode) waves with phase velocity parallel to the magnetic field
- These waves resonate with electrons and transfer energy through Landau damping ($\omega - k_{||} v_{||j} = 0$)
- The phased array couples preferentially to waves travelling in one toroidal direction so electrons are driven in one direction
- The accelerated electrons become less collisional, so electrons travelling in one toroidal direction are preferentially heated.
- The effective resistivity is lowered for these electrons, and this "asymmetric resistivity" accounts for around 75% of the driven current.

Ray tracing

How do we follow an EM wave through a plasma?
Assume plasma equilibrium quantities vary on long lengthscales (relative to the wavelength); then the wave phase Φ is well defined

- Frequency $\omega = \dfrac{\partial \Phi}{\partial t}$
- Wavevector $\underline{k} = \nabla \Phi$

Suppose we have a dispersion relation $\omega = \omega\left(\underline{x}, \underline{k}\right)$ then along a ray:

- Phase is constant i.e. $\dfrac{d\Phi}{dt} = \dfrac{\partial \Phi}{\partial t} + \dfrac{d\underline{x}}{dt} \cdot \nabla \Phi = 0$

  Therefore $\dfrac{d\underline{x}}{dt} \cdot \underline{k} = -\omega\left(\underline{x}, \underline{k}\right)$

Additional issues

- Electron Cyclotron RF waves can also be used to drive currents (ECCD). Current research topic on controlling instabilities (NTMs) using this
- Wave orientation is never quite parallel or perpendicular
- Finite temperature complicates the dispersion relations
- Particle response can be relativistic

Summary

- The cold plasma dispersion relation with $T \to 0$ and $\eta \to 0$ gives an approximation for wave propagation away from resonant regions
- Parallel waves are the electrostatic plasma wave, the right and left circularly polarised waves
- Perpendicular waves are the O-mode (light like) and the X-mode Upper and Lower hybrid waves
- At resonant surfaces wave-particle interactions lead to damping. Landau damping resonance can lead to current drive
- In tokamaks, RF waves propagating from the low field side must tunnel through a cut-off to get to the resonant surface. This is prohibitive in Spherical Tokamaks (STs)
- At finite temperatures, Electrostatic Bernstein Waves (EBW) propagate and can be used to inject energy into STs