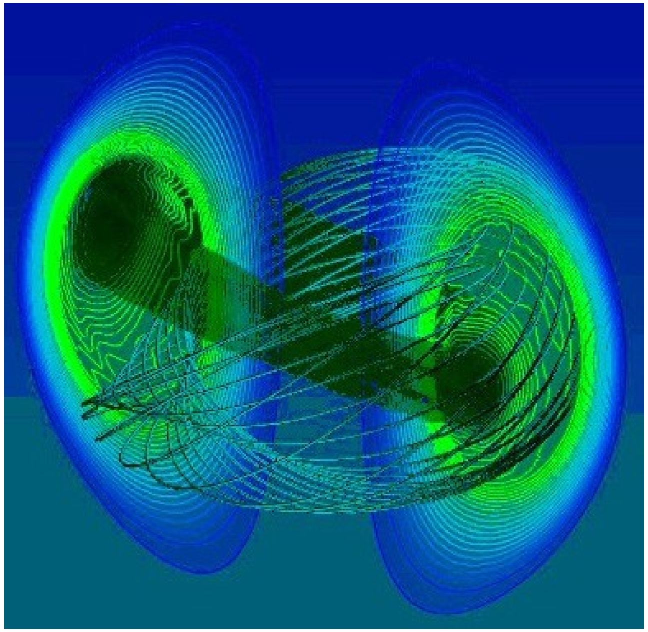
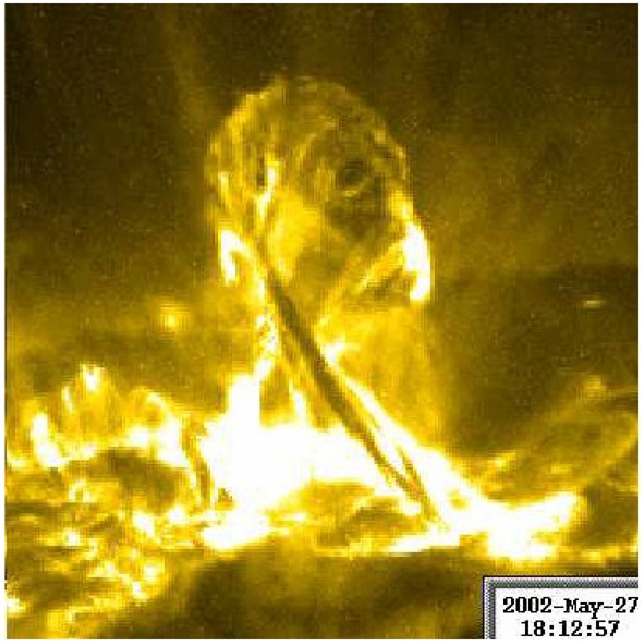
Plasma instabilities

Dr Ben Dudson, University of York





Previously...

- Plasma configurations and equilibrium
  - Linear machines, Tokamaks and Stellarators
  - Ideal MHD and the Grad-Shafranov equation
- Collisional transport in toroidal devices
  - Classical transport (small!)
  - Banana orbits and neoclassical transport
- Waves in plasmas
  - Linearisation of ideal MHD equations
  - Propagation of RF waves through plasmas

Plasma instabilities

- Plasmas exhibit a huge range of instabilities
- One of the challenges in fusion research is to find stable plasma configurations
- Understand the limits of performance

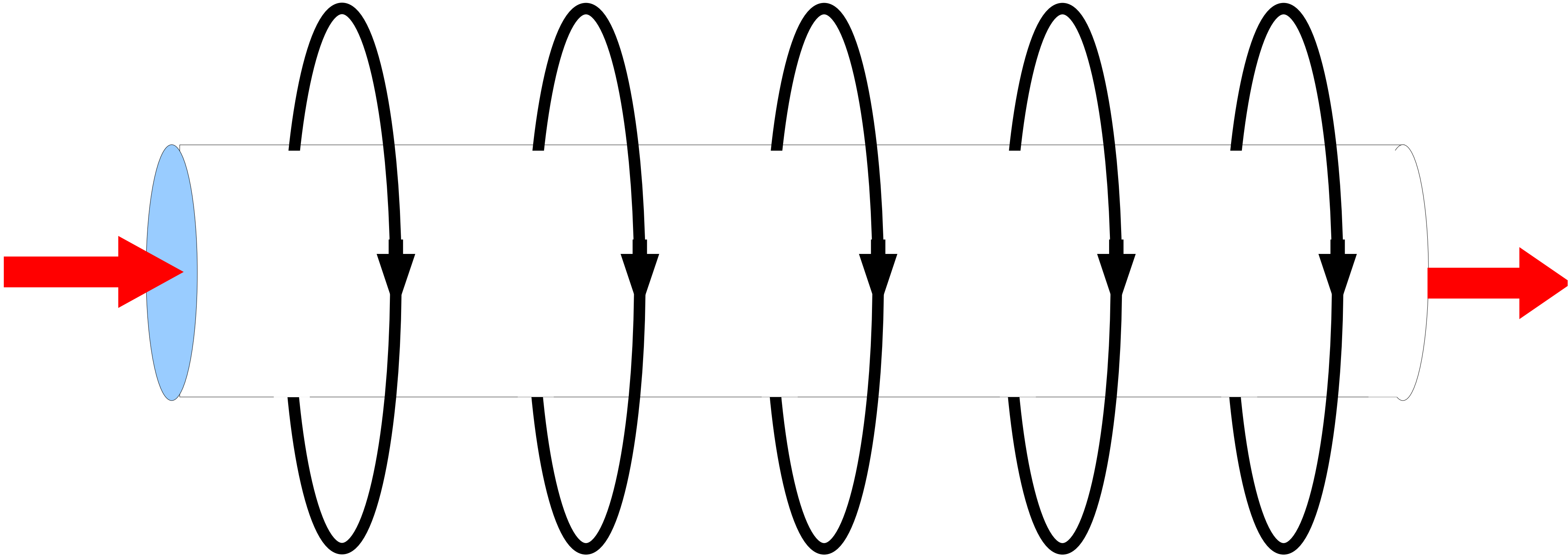
- Aim here is to:
  - Show you the basic tools and concepts used to study plasma instabilities
  - Introduce some of the jargon you'll see in papers and conferences
  - Study some common tokamak instabilities

Z-pinch

Start with simplest magnetic confinement configuration: a **Z-pinch**

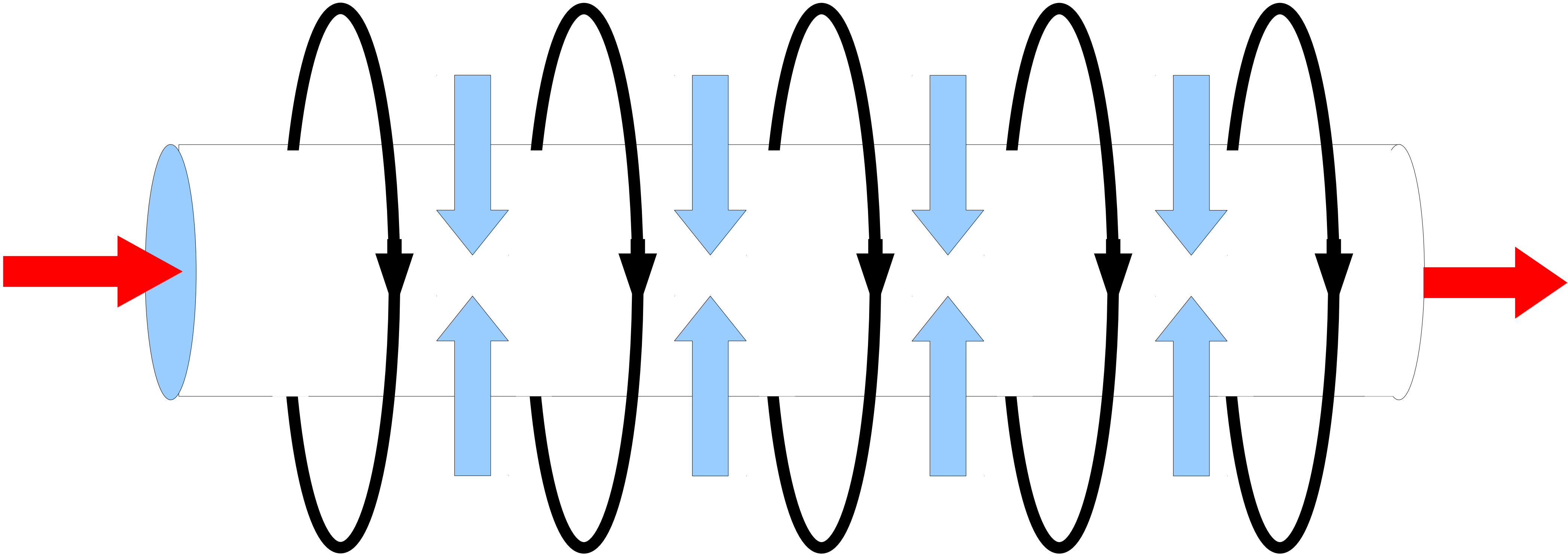
Current driven through column of plasma

Z-pinch



Generates magnetic field.

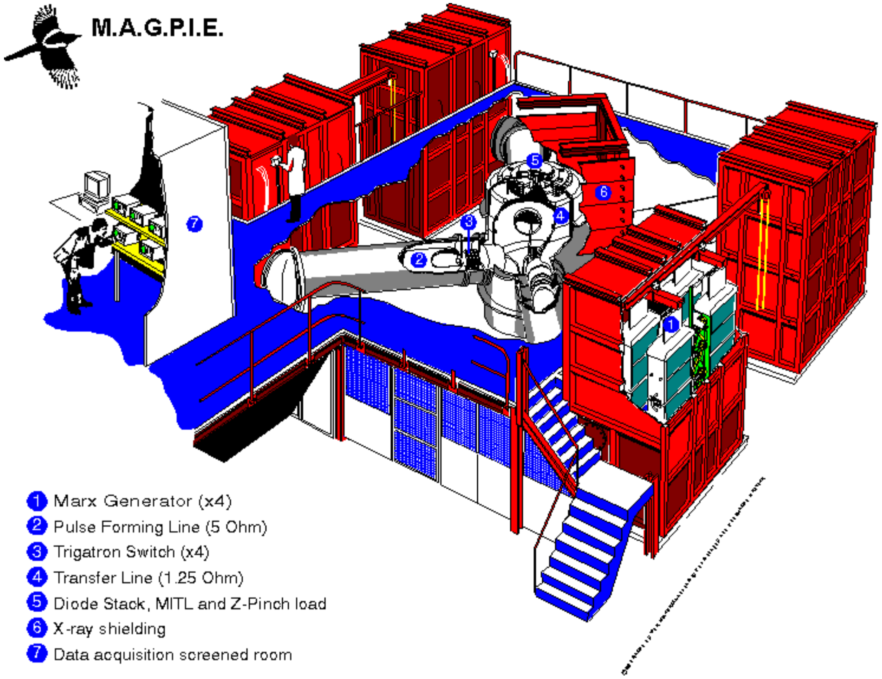
Z-pinch

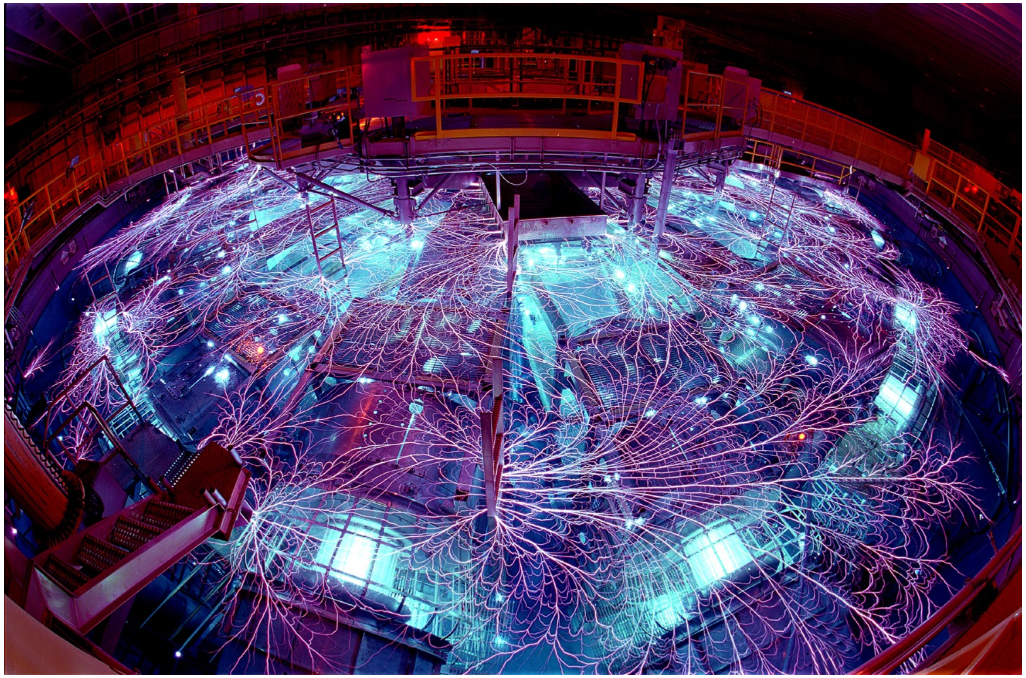


Generates magnetic field. JxB force compresses plasma

Z-pinch experiments

MAGPIE at Imperial College (UK)

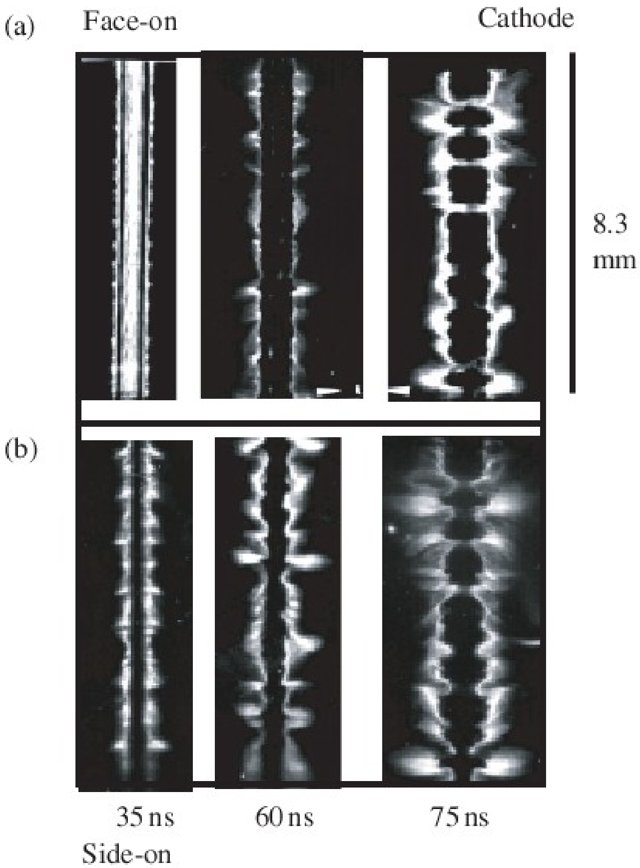


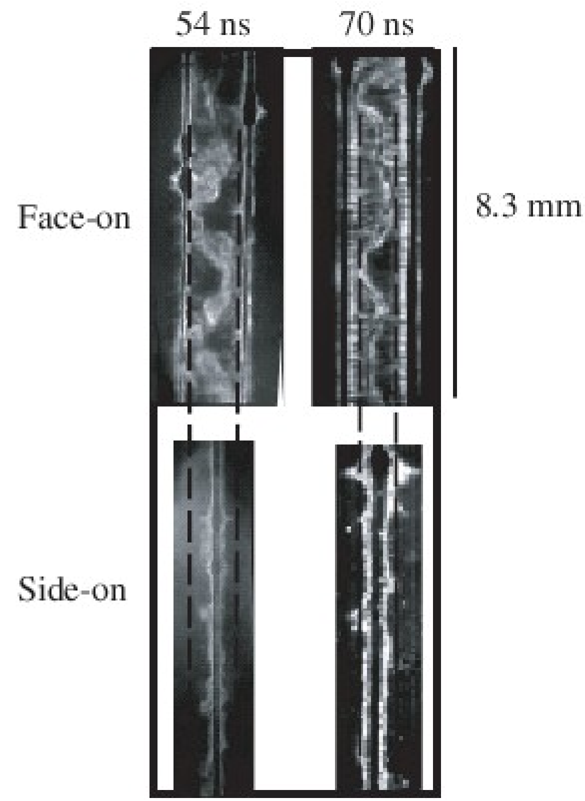


Z-machine firing at Sandia National Laboratory (USA)

Current of several mega-Amps for a few hundred nanoseconds

X-ray observations





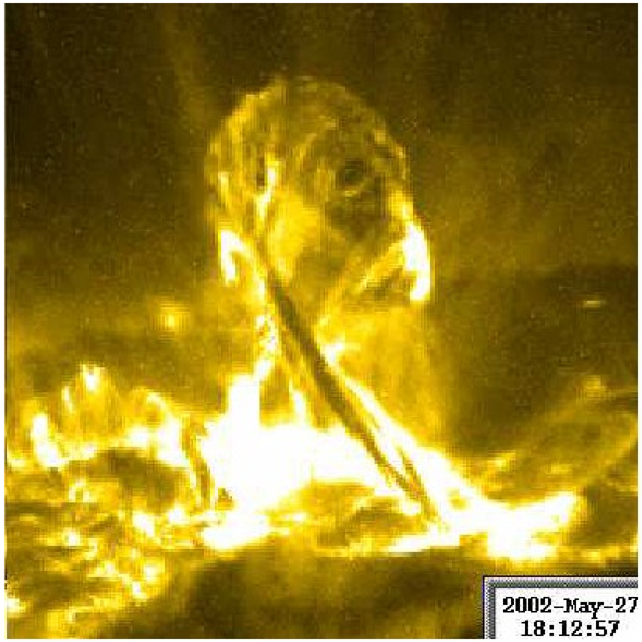
m = 0 “Sausage” instability

m = 1 “Kink” instability

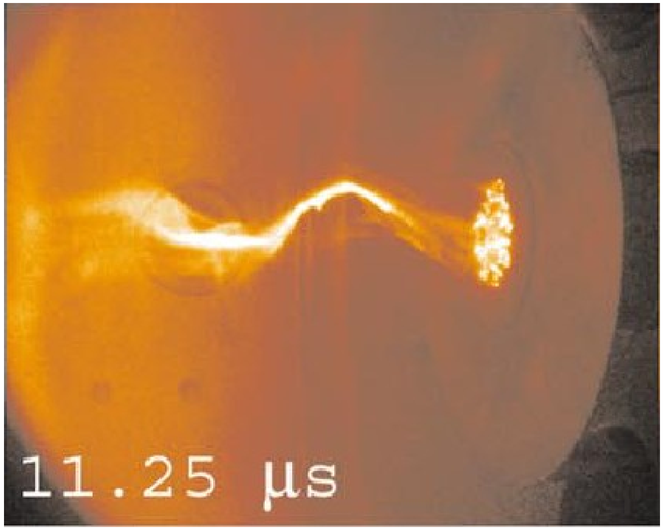
Schlieren photographs from MAGPIE: F.N.Beg *et.al.* PPCF **46** (2004) 1-10

Kink instabilities in space plasmas

Coronal loops show signs of being unstable to kink instabilities →



T.Torok and B.Kliem 2004



← Laboratory plasma jets

S.C.Hsu and P.M.Bellan
PoP 12, 032103 (2005)

Plasma modelling

Many different plasma models to choose from:

Vlasov
Fokker-Planck
6D equation

→ Take moments →

2-fluid equations
e.g. Braginskii

↓ Simplify further

Non-ideal MHD
(single fluid)

↓ Remove dissipation

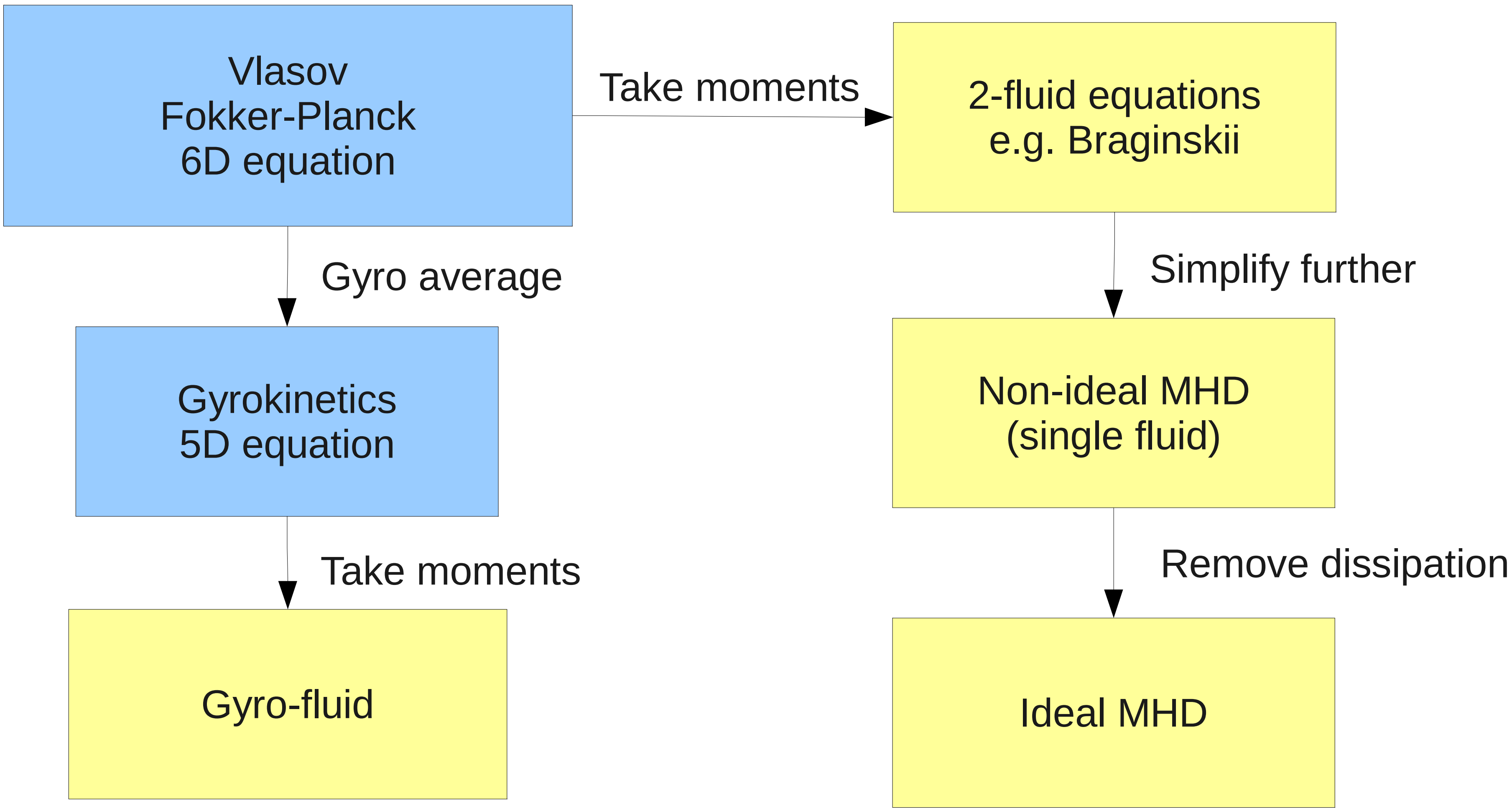
Ideal MHD

- Full Vlasov equation too difficult to solve
- Need to make approximations

- Taking moments gives set of equations for each species (electrons, ions)
- Assuming $T_i = T_e$ and $n_i = n_e$ leads to single-fluid equations
- Dropping all dissipation, and assuming slow time and space variation

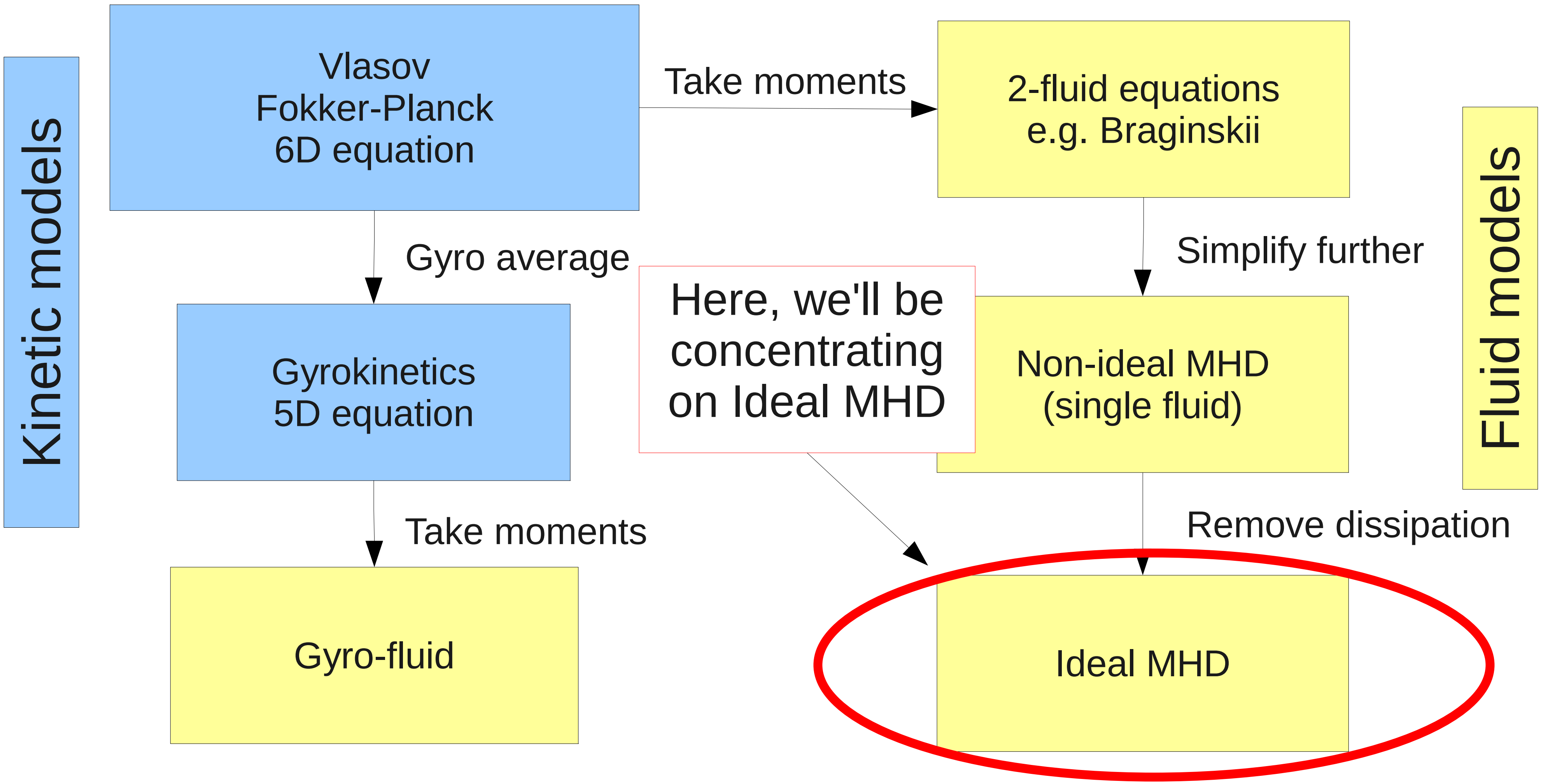
Plasma modelling

Another set of models based on averaging over gyro-orbits



Plasma modelling

Models classified into “Kinetic” and “fluid”



Ideal MagnetoHydroDynamics

- Ideal MHD makes several assumptions:
  - Length-scales >> Larmor radius
  - Length-scales >> mean free path
  - Time-scales >> Cyclotron, collision times

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  - Length-scales >> mean free path
  - Time-scales >> Cyclotron, collision times

- Assumes plasma is locally close to thermal equilibrium (i.e. Maxwellian distribution), which requires collisions
- Collisions cause dissipation (e.g. Resistivity, viscosity), which are not included in ideal MHD

Ideal MagnetoHydroDynamics

If ideal MHD makes so many assumptions, why use it?

- Ideal MHD equations include the essential physics of plasma instabilities in a (relatively) simple set of equations:

$$
\begin{aligned}
\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) &= 0 && \text{Density} \\
n\left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right] &= \mathbf{J}\times\mathbf{B} - \nabla P && \text{Momentum} \\
\frac{\partial P}{\partial t} + \mathbf{v}\cdot\nabla P &= -\gamma P \nabla\cdot\mathbf{v} && \text{Pressure} \\
\frac{\partial B}{\partial t} &= \nabla\times(\mathbf{v}\times\mathbf{B}) && \text{Magnetic field}
\end{aligned}
$$

Ideal MagnetoHydroDynamics

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- Ideal MHD equations include the essential physics of plasma instabilities in a (relatively) simple set of equations
- Additional (non-ideal) effects tend to allow new types of instability. Resistivity in particular allows field-lines to reconnect, however:
- Instabilities described by ideal MHD (“ideal” instabilities) tend to be the fastest and most violent. A plasma which is ideally unstable probably won't last long.

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- Instabilities described by ideal MHD (“ideal” instabilities) tend to be the fastest and most violent. A plasma which is ideally unstable probably won't last long.
- Many non-ideal instabilities are variations on ideal instabilities, and lots of the jargon is from ideal MHD

Ideal MagnetoHydroDynamics

Main reason to use ideal MHD is: It works much better than it "should" do, even in hot (i.e. nearly collisionless) plasmas

- Perpendicular to the B field, movement is restricted and the effective mean-free-path is approximately the gyro-radius
=> As long as perpendicular length-scales are long compared with the gyro-radius then the fluid approximation is ok

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- Parallel to the field,
  - the mean-free-path is very long
  - However, gradients in this direction also tend to be very small.
  - Kinetic modifications to MHD primarily modify parallel dynamics.
  - As we shall see, parallel dynamics are not very important for determining when *linear* instabilities start.

Magnetic field curvature and pressure

Ideal MHD momentum equation

$$n\left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right] \;=\; \mathbf{J}\times\mathbf{B} - \nabla P$$

JxB term can be written as:

$$\begin{aligned}\mathbf{J}\times\mathbf{B} &= \frac{1}{\mu_0}\left(\nabla\times\mathbf{B}\right)\times\mathbf{B} \\ &= \frac{1}{\mu_0}\left(\mathbf{B}\cdot\nabla\right)\mathbf{B} - \frac{1}{2\mu_0}\nabla B^2\end{aligned}$$

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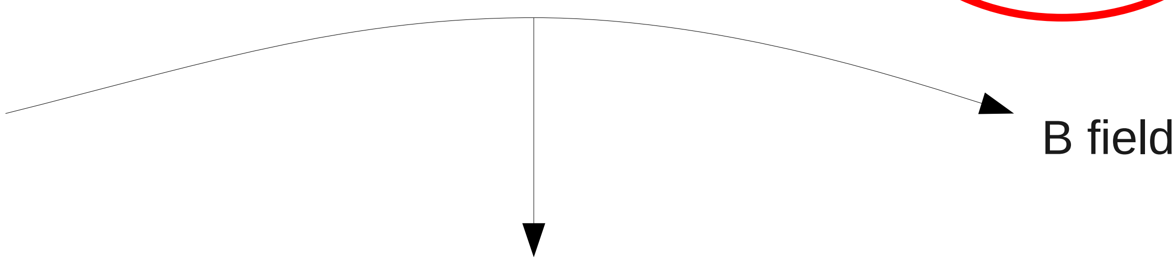

Curvature vector $\kappa = \left(\mathbf{b}\cdot\nabla\right)\mathbf{b} = \left(\nabla\times\mathbf{b}\right)\times\mathbf{b}$

Magnetic field curvature and pressure

Ideal MHD momentum equation

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Plasma pressure

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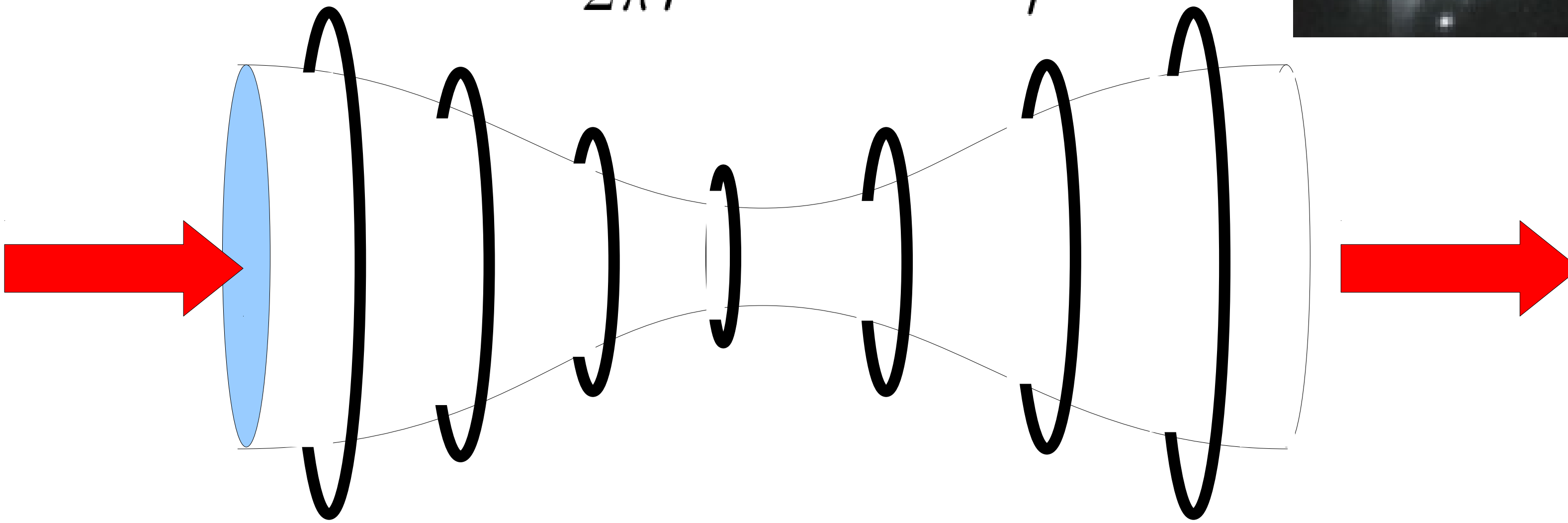
Magnetic pressure

Sausage instability

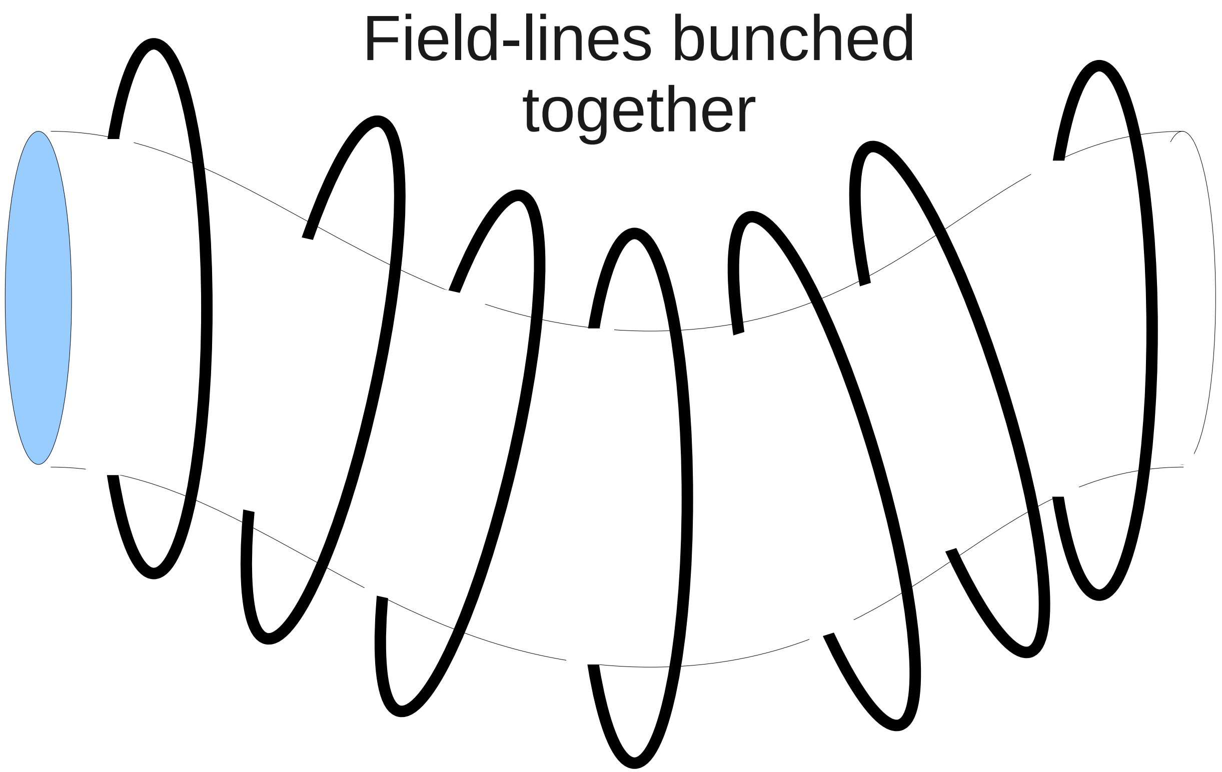
$$\mathbf{B} = \frac{\mu_0 I}{2\pi r} \qquad B^2 \propto \frac{1}{r^2}$$



Current constant through plasma column

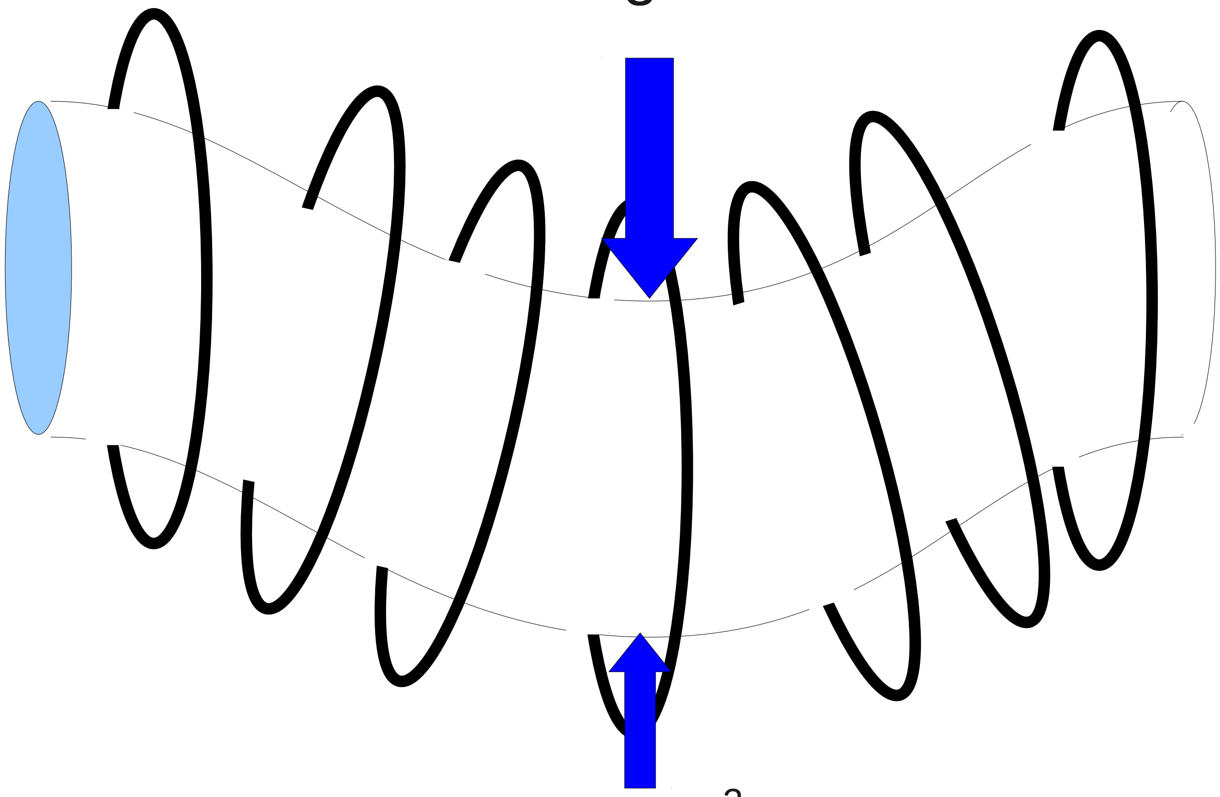
Where plasma gets narrower, B field gets stronger. Squeezes plasma out of narrow region and so enhances the instability

Kink instability



Kink instability

Magnetic pressure again enhances any initial perturbation. Hence unstable.



Calculating growth-rates

- Same as calculating wave dispersion relations
- Linearise the ideal MHD equations by splitting into equilibrium and a small perturbation:

$$
\begin{aligned}
n &= n_0 + \delta n \\
P &= P_0 + \delta P \\
\mathbf{v} &= \mathbf{v}_0 + \delta \mathbf{v} \\
\mathbf{B} &= \mathbf{B}_0 + \delta \mathbf{B}
\end{aligned}
$$

- Substitute into the equations e.g. Density equation:

$$
\frac{\partial}{\partial t}\left(n_0 + \delta n\right) = -\nabla \cdot \left[\left(n_0 + \delta n\right)\left(\mathbf{v}_0 + \delta \mathbf{v}\right)\right]
$$

Calculating growth-rates

$$\frac{\partial}{\partial t}\left(n_0 + \delta n\right) = -\nabla \cdot \left[\left(n_0 + \delta n\right)\left(\mathbf{v}_0 + \delta \mathbf{v}\right)\right]$$

Expand terms:

$$\begin{aligned}
\frac{\partial n_0}{\partial t} + \frac{\partial \delta n}{\partial t} = & - & \nabla \cdot \left(n_0 \mathbf{v}_0\right) \\
& - & \nabla \cdot \left(n_0 \delta \mathbf{v}\right) - \nabla \cdot \left(\delta n \mathbf{v}_0\right) \\
& - & \nabla \cdot \left(\delta n \delta \mathbf{v}\right)
\end{aligned}$$

$$\frac{\partial}{\partial t}\left(n_0 + \delta n\right) = -\nabla\cdot\left[\left(n_0 + \delta n\right)\left(\mathbf{v}_0 + \delta\mathbf{v}\right)\right]$$

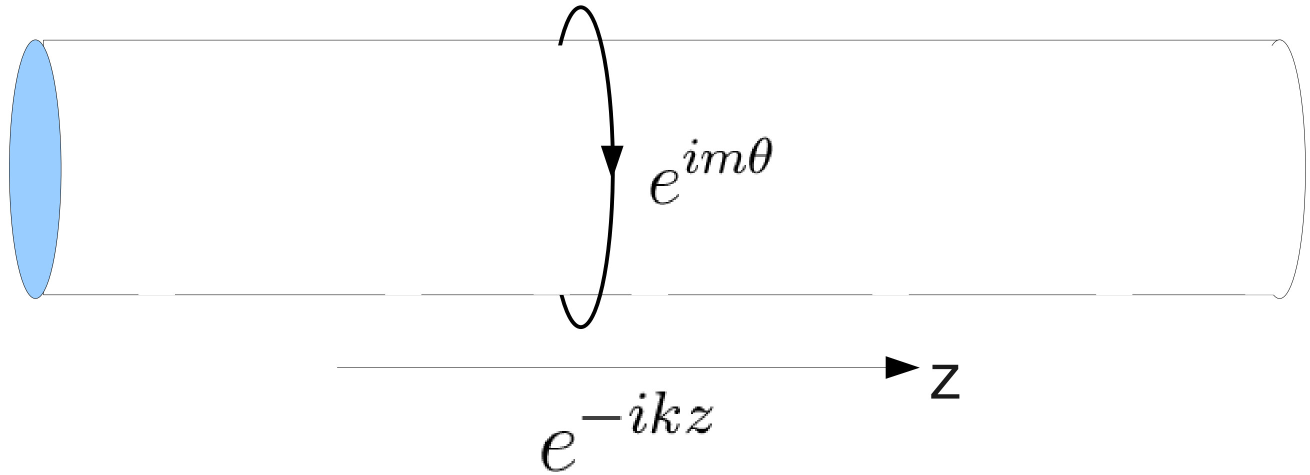
Expand terms:

$$\begin{aligned}\cancel{\frac{\partial n_0}{\partial t}} + \frac{\partial \delta n}{\partial t} = & - \quad \cancel{\nabla\cdot\left(n_0\mathbf{v}_0\right)} \\ & - \quad \nabla\cdot\left(n_0\delta\mathbf{v}\right) - \nabla\cdot\left(\delta n\mathbf{v}_0\right) \\ & - \quad \cancel{\nabla\cdot\left(\delta n\delta\mathbf{v}\right)}\end{aligned}$$

Remove all terms with only equilibrium terms, or more than one perturbed quantity (nonlinear terms)

Calculating growth-rates

- Often useful to Fourier analyse



NB: In a torus, e^{-ikz} becomes $e^{-in\phi}$ where $\phi = z/R$

Write perturbations of the form:

$$\delta n\left(r,\theta,z\right) = \delta n\left(r\right) e^{im\theta} e^{-ikz}$$

Calculating growth-rates

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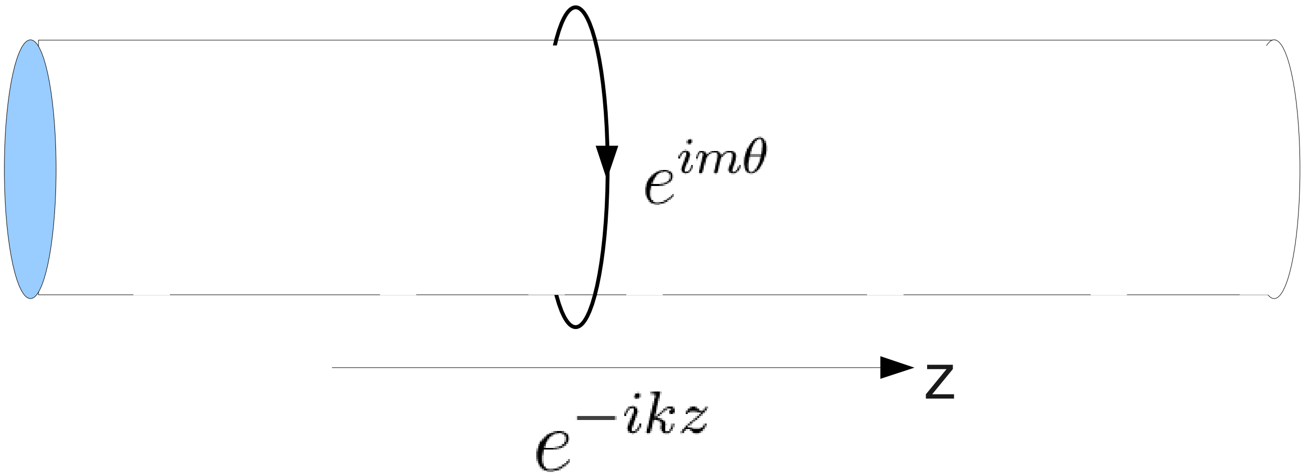

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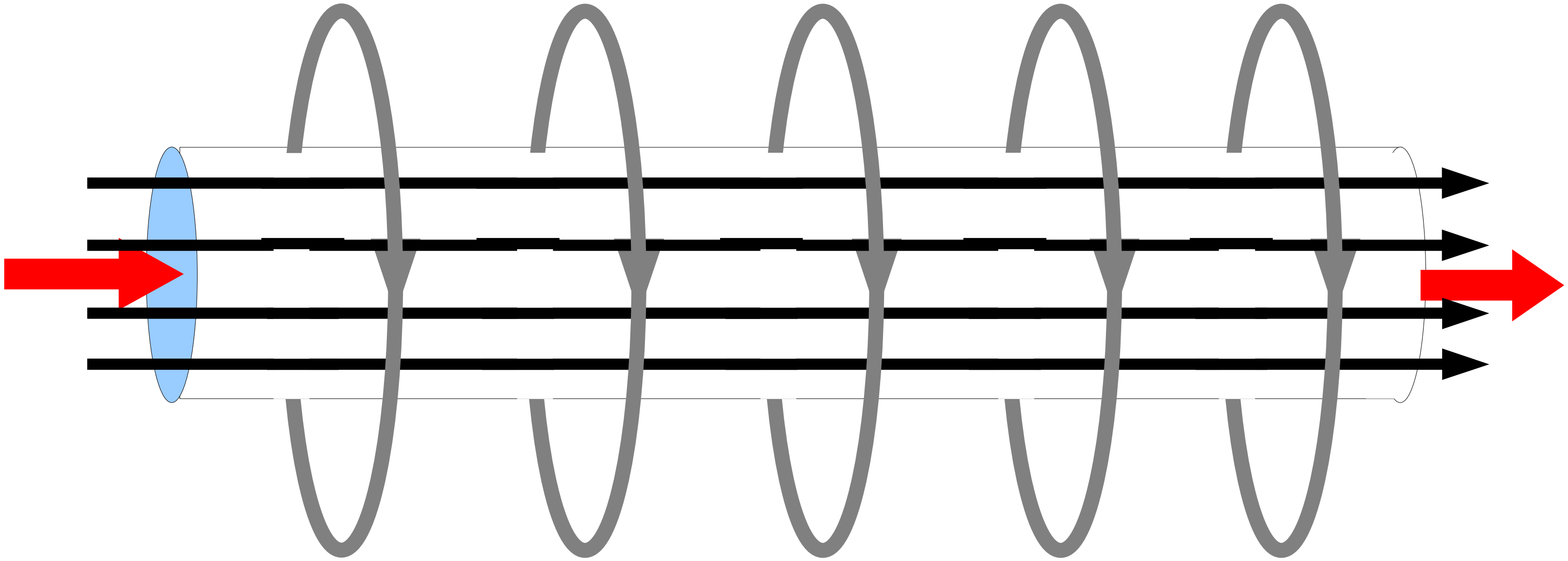
$$\delta n\left(r,\theta,z\right) = \delta n\left(r\right)e^{im\theta}e^{-ikz}$$

Need to solve this throughout plasma, then match solutions in the vacuum region outside. Long and messy calculation...

Stabilising kink modes

Add a magnetic field along the plasma:



As magnetic field has a pressure (i.e. Energy per unit volume), it takes energy to compress magnetic field-lines

Stabilising kink modes

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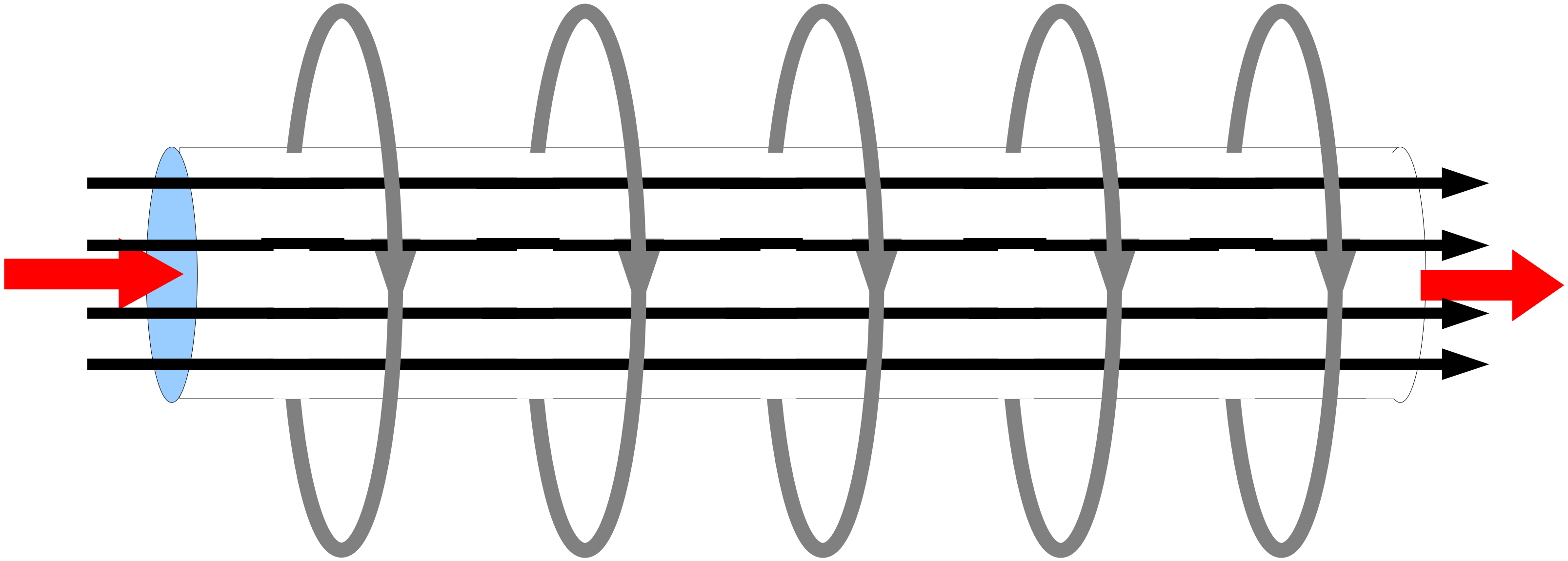


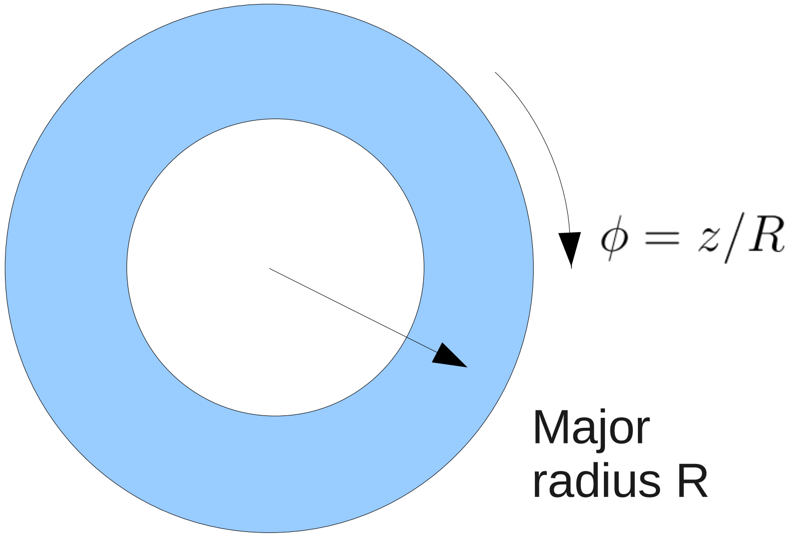
<u>Kruskal-Shafranov condition</u>

Adding a field along the plasma column (B_Z) stabilises sausage and kink instabilities with wavelengths less than:

$$L_s = \frac{2\pi r B_Z}{B_\theta}$$

Tokamaks

Wrap a z-pinch into a torus



Longest length is $2\pi R$

Tokamaks

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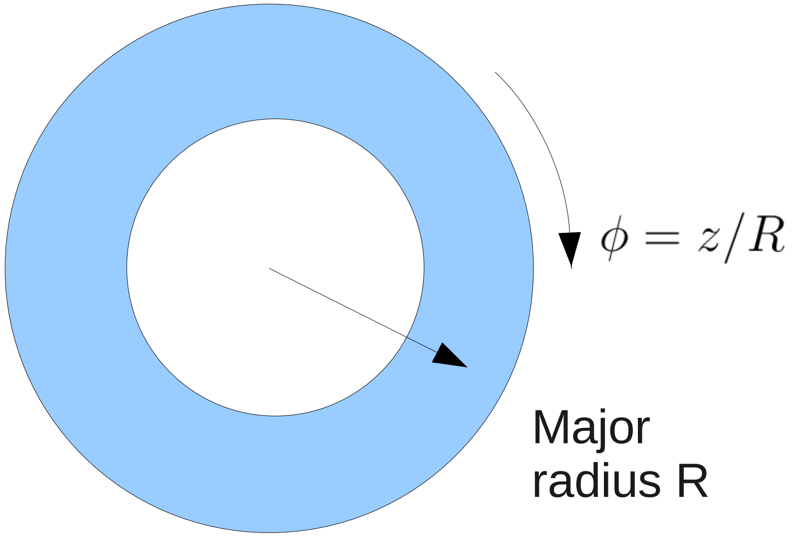


Longest length is $2\pi R$

Need to stabilise all wavelengths shorter than this, so

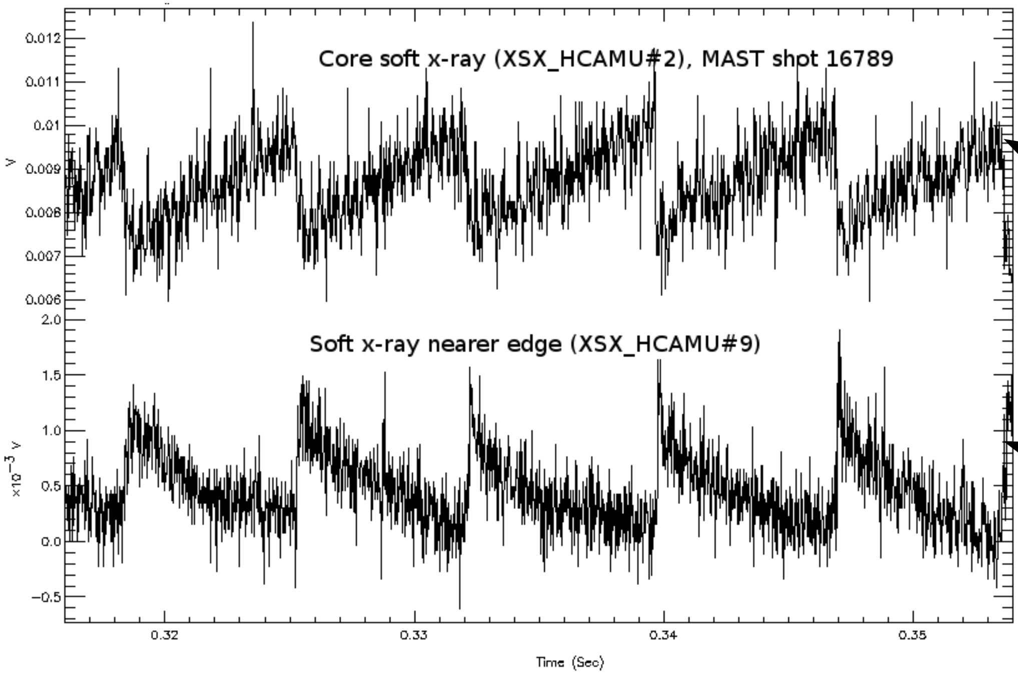
$$L_s > 2\pi R \Rightarrow \frac{2\pi r B_z}{B_\theta} > 2\pi R \qquad \text{i.e.} \qquad \frac{r B_z}{R B_\theta} > 1$$

Called “Safety factor”, and in general written as:

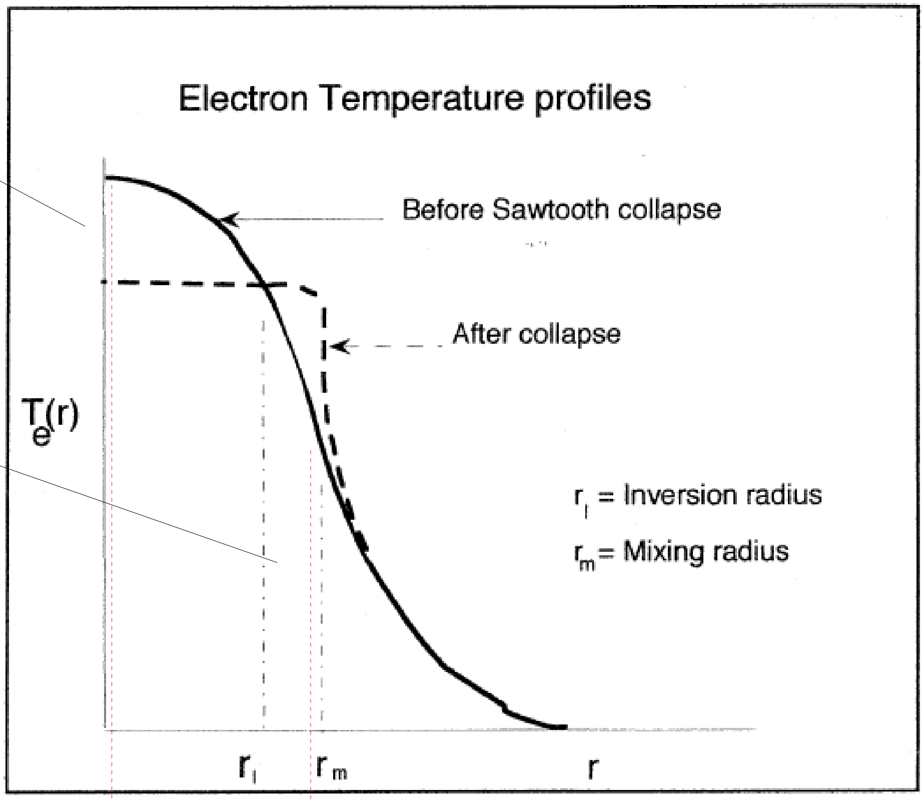
$$\boxed{q = \frac{1}{2\pi} \oint \frac{r B_z}{R B_\theta} d\theta > 1}$$

Sawteeth in tokamaks

- What happens when $q < 1$ in tokamaks?
- Repetitive drops in core temperature and density
- First reported by von Goeler et al 1974
- Clearly seen on soft x-ray signals as a sawtooth pattern



MAST soft X-ray signals

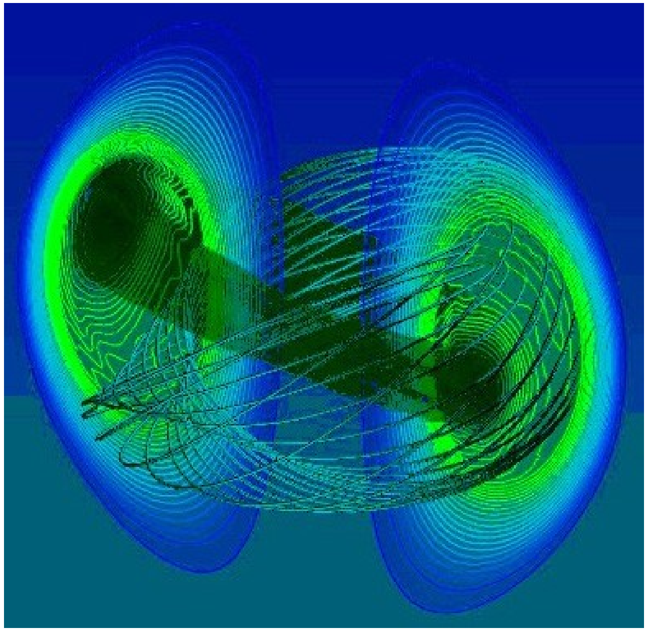


Sawteeth in tokamaks

- When the plasma current becomes too peaked on axis, the safety factor q can drop below 1 in the core

Sawteeth in tokamaks

- When the plasma current becomes too peaked on axis, the safety factor q can drop below 1 in the core
- This is then thought to lead to a kink-type instability
  NB: Called “internal” kink since the boundary is not affected
  Contrast with (much more dangerous) “external” kink where the boundary does move
- A reconnection then occurs (non-ideal effects important here)
- The hot core is expelled along with some current, bringing q up again



"Fusion Simulation Project Workshop Report" by Arnold Kritz and David Keyes

Summary and key points

- Ideal MHD provides a convenient model to study plasma instabilities. Works surprisingly well for linear problems, but should always keep in mind the assumptions made
- Z-pinch plasmas are susceptible to sausage (m = 0) and kink (m = 1) instabilities.
- Bending or compressing field-lines takes energy, and magnetic fields behave as if they have a pressure.
- Too much current in the core of a tokamak produces a "Sawtooth" instability when the "safety factor" q drops below 1

Next time: Pressure driven instabilities...