Turbulence and transport



`Turbulence is the most important unsolved problem of classical physics.'

- Richard Feynman -

Da Vinci 1452-1519

Reynolds, Richardson, Kolmogorov



Dr Ben Dudson, University of York. http://www-users.york.ac.uk/~bd512/

Turbulence

- Ubiquitous phenomenon in fluids on all scales, from coffee cups to galactic clouds
- Heisenberg is reported to have said that he'd ask God two questions: why relativity and why turbulence? "I really think he may have an answer to the first question" [A variant of this is attributed to Horace Lamb]
- Navier-Stokes equations governing fluids known since 1845
 - The Clay institute has a \$1 million millennium prize for proving whether a wellbehaved solution to 3D Navier-stokes equation exists.
- No general theory of turbulence known. It's possible that none exists.

<u>Nevertheless</u>

Turbulence is one of the most intriguing and important areas of classical physics.

Great progress has been made in developing statistical and computational models of turbulence.

Here we'll look at some of the most important ideas in this field, and (eventually) how it applies to tokamaks

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Tokamak turbulence

- Neoclassical theory gives a minimum for the transport of heat and particles in a tokamak
- In real tokamaks, transport is often observed to be ~10x neoclassical levels
- Fluctuations are observed in density and temperatures
- This transport is thought to be due to turbulence

Non-linear evolution of plasma instabilities such as ballooning and drift waves leads to complicated "turbulent" behaviour. Mid-plane probe, 2 cm from LCFS



Turbulence oooooooo Cascades

Flows 0000 Plasma 0000000

What is turbulence?

Many different definitions of turbulence have been proposed

From the general...

"Turbulent fluid motion is an irregular condition of the flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned."

- J. O. Hinze. Turbulence, 1959.

...to the highly specific:

"Turbulence is any chaotic solution to the 3-D Navier–Stokes equations that is sensitive to initial data and which occurs as a result of successive instabilities of laminar flows as a bifurcation parameter is increased through a succession of values."

- G. T. Chapman and M. Tobak, 1985

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When you are a Bear of Very Little Brain, and you Think of Things, you find sometimes that a Thing which seemed very Thingish inside you is quite different when it gets out into the open and has others looking at it

- Winnie-the-Pooh, A.A. Milne, 1928

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What is turbulence?

A minimal set of properties of fluid turbulence might be:

- Disorganised, chaotic, <u>apparently</u> random behaviour
- Time-dependent
- Sensitive to initial conditions (i.e non-repeatable)
- Large range of length- and time-scales
- Results in enhanced mixing and dissipation

Deterministic, not random

<u>N.B.</u> there is a debate as to whether turbulence needs to be strictly 3D. 2D solutions can exhibit the above properties, but cannot share an important property of 3D turbulence (vortex stretching).

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Onset of turbulence

Useful to look at what happens as the velocity of a fluid passing around an obstacle is slowly increased

At low velocity, flow is symmetric and smooth (laminar)



As the velocity is increased, vortices form at the trailing edge



Increasing still further, up-down symmetry is broken, and vortices peel off



Karman vortex street



At large velocities the flow becomes irregular and unpredictable i.e. <u>turbulent</u>

Images: Bijan Mohammadi, Uni. Montpellier 2

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Onset of turbulence



Karman vortex street

Seen in weather (here Rishiri Island in Japan), chimneys, bridges, car aerials etc.

Images: Bijan Mohammadi, Uni. Montpellier 2

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Reynolds number

Experiments by Reynolds (1883) on flow in a pipe showed that the transition between smooth (laminar) and turbulent flow was controlled by the dimensionless number

Low viscosity fluids (e.g. air) tend to be more turbulent than high viscosity fluids (e.g. water, honey).

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Eddy picture

Energy

`Big whorls have little whorls, which feed on their velocity, And little whorls have lesser whorls, and so on to viscosity.'

- Lewis Richardson (1922) -

Fluid is 'stirred' on a large scale to produce large eddies

These are unstable and break up into smaller eddies

In turn, these smaller eddies are unstable and break into yet smaller eddies

Eventually, viscosity becomes important, and the energy is dissipated as heat.

Dissipated by viscosity

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Energy cascade

This idea of energy being transferred from large-scale eddies to small scales is called a "**cascade**", and is one of the most influential in this field

This is still pretty vague. Can we fill in some details? ...

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How small do eddies get?

We know that when Re~1 viscosity dominates, so for the smallest scale eddies

$$Re_{small} = vl/\nu \sim 1$$

In statistical steady state, energy flow is the same at all scales: $~E\sim\epsilon$

Putting these together gives

$$l \sim LRe^{-3/4}$$

$$v \sim u R e^{-1/4}$$

How small do eddies get? (cont.)

Typical values in wind tunnels $Re \sim 10^3 - 10^5$

Note that dissipation is predominantly at small scales

 $\epsilon_{small}/\epsilon_{large} \sim Re$

Smallest eddies are approximately 1000x smaller than the large-scale features

NB: This has implications for simulations

Energy cascade II

Now we have an (approximate) idea of the scales and energies involved:

What about the middle bit? ...

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Kolmogorov spectrum

What might affect the energy at a given scale? $E=F\left(\epsilon,
u,k,t,BC,l,\ldots
ight)$

Boundary conditions Consider eddies much smaller than the system size. Kolmogorov argued (1941) that these eddies only feel the effect of large scales by the energy cascade rate.

$$E = F\left(\epsilon, \nu, k, t\right)$$

For small eddies, turnover time is small compared with system evolution timescales. Hence system is in statistical equilibrium and time dependence can be dropped.

$$- - E = v^2 F(kl) \quad \text{"Un}$$

Iniversal equilibrium range" $1/k \ll L$

What about boundaries, anisotropy (e.g. Jets)? For small enough eddies, their local environment looks homogeneous and isotropic.

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Kolmogorov spectrum II

For very large Reynolds numbers, we can imagine eddies which are too small to feel the effect of large-scale structures, but too small to be affected by viscosity. For these,

$$E = F\left(\epsilon, k\right)$$

These eddies have no way of "knowing" what size they are: a self-similar system where the dynamics of each scale is essentially the same

$$E = \epsilon^{\alpha} k^{\gamma} \qquad \qquad \left[\frac{L^2}{T^2}\right] = \left[\frac{L^2}{T^3}\right]^{\alpha} \left[\frac{1}{L}\right]^{\gamma}$$

Dimensional analysis gives one solution:

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Energy cascade III

Fig. 5.5. log-log plot of the energy spectrum in the time domain and enlargement of the beginning of the dissipation range for tidal channel data (Grant, Stewart and Moilliet 1962).

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Turbulence and flows

In many situations we are interested in how turbulence affects flows (e.g. flow over wings etc.)

Apparently random behaviour, so try statistical approach $\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v'}_{\text{Fluctuations due}}_{\text{to turbulence}}$ Time-average flow Velocity evolution equation: $\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nabla \cdot \Pi$ Stress tensor $\Pi_{ij} = \nu \frac{1}{2} \left| \frac{\partial v_i}{\partial x_i} + \frac{\partial v_j}{\partial x_i} \right|$ Note: ignoring off-diagonal terms gives $\nabla \cdot \Pi \simeq \nu \nabla^2 \mathbf{v}$ Turbulence Plasma Cascades Flows www-users.

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Turbulence and flows II

Time-average the velocity equation:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \overline{\rho \mathbf{v} \cdot \nabla \mathbf{v}} = -\overline{\nabla p} + \nabla \cdot \overline{\Pi}$$

$$\overline{\mathbf{v} \cdot \nabla \mathbf{v}} = \overline{\mathbf{v}} \cdot \nabla \overline{\mathbf{v}} + \overline{\overline{\mathbf{v}} \cdot \nabla \mathbf{v}'} + \overline{\mathbf{v}' \cdot \nabla \overline{\mathbf{v}}} + \overline{\mathbf{v}' \cdot \nabla \mathbf{v}'}$$

$$\overline{\mathbf{v}' \cdot \nabla \mathbf{v}'} = \overline{\nabla \cdot (\mathbf{v}' \mathbf{v}')}^{0} - \overline{\mathbf{v}' \nabla \cdot \mathbf{v}'}$$

$$\Rightarrow \rho \overline{\mathbf{v}} \cdot \nabla \overline{\mathbf{v}} = -\nabla \overline{p} + \nabla \cdot \left(\Pi - \overline{\rho \mathbf{v}' \mathbf{v}'}\right)$$
Beyindly stress

Turbulence acts on mean flows like a fictional viscous stress. NOTE: In contrast to viscosity, this stress is <u>not dissipative</u>. Represents transfer of momentum between mean flows and turbulence.

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Reynolds stress

Transfer of momentum Mean flow
Turbulence
(and energy)

But what is it?

$$\frac{\partial}{\partial t} \left(\rho \mathbf{v}' \mathbf{v}' \right) = 2 \mathbf{v}' \rho \frac{\partial \mathbf{v}'}{\partial t} = 2 \mathbf{v}' \left[-\rho \mathbf{v} \cdot \nabla \mathbf{v}' - \mathbf{v}' \cdot \nabla \mathbf{v}' - \nabla p - \nabla \cdot \Pi \right]$$

Time-average to get

$$\mathbf{v} \cdot \nabla \left(\overline{\rho \mathbf{v}' \mathbf{v}'} \right) = \nabla \cdot \left(-\overline{\rho \mathbf{v}' \mathbf{v}' \mathbf{v}'} \right) + \cdots$$

End up with an equation involving 3^{rd} order correlations. Equation for 3^{rd} order involves 4^{th} order and so on

Always have more unknowns than equations!

As when deriving fluid equations, the non-linear advection term leads to an infinite series of coupled equations

This is the turbulence closure problem

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Turbulence closure

We can write down a deterministic set of equations (Navier-Stokes)

- Solution is chaotic and hard/impossible to find
- Statistical averages are non-random, and are experimentally reproducible
 - Cannot find a closed set of equations to describe them

Need more information to close the set of equations

In engineering, a variety of heuristic models are used (most common are **"mixing-length" models**), which work reasonably well in some circumstances

No satisfactory solution yet found

"It must be admitted that the principal result of fifty years of turbulence research is the recognition of the profound difficulties of the subject" – S A Orszag (1970)

So, how does all this relate to plasma turbulence? ...

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Plasma turbulence

The equations governing plasmas are much more complicated than fluids

<u>BUT</u> in some ways plasma turbulence is simpler than fluid turbulence because the magnetic field imposes a structure on the eddies.

In some situations, plasma turbulence can be treated as a 2D or quasi 2D problem

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Plasma and fluid turbulence

Differences

Plasmas support many more linear waves than fluids

Plasma turbulence tends to be driven by small-scale instabilities, rather than large-scale stirring

Gyro-orbits tend to limit the smallest scales in plasmas

Electrons and ions respond differently. Like two coupled fluids

Similarities

The same difficulties apply, including statistical closure

Many of the same ideas / concepts are used, including

- Spreading of energy in k (c.f. cascades)
- Coupling between flows and turbulence (Reynolds stress)

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Plasma turbulence

Many different areas, but two main categories are:

MHD turbulence

Electromagnetic phenomenon

Solution to resistive MHD Most applicable to high-beta situations (e.g. astrophysical plasmas)

Images: NASA

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Magnetic Reynolds number

Reminder: In fluids, $\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nu \nabla^2 \mathbf{v}$

In plasmas, dynamics of magnetic field can be more important:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B}$$

Analogously to fluids, define

Plasma turbulence

Many different areas, but two main categories are:

MHD turbulence

Electromagnetic phenomenon

Solution to resistive MHD

Most applicable to high-beta situations (e.g. astrophysical plasmas)

Drift-wave turbulence

Images: NASA

Essentially electrostatic fluctuations

Driven by linear drift instabilities (e.g. ITG, ETG)

Most relevant to tokamak core turbulence

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Drift wave turbulence and Zonal

The original view of drift-wave turbulence was much like fluid turbulence:

However, coupling between turbulence and flows (Reynolds stress) is a key part of drift-wave turbulence:

"Zonal flows in plasma – a review" by P.H.Diamond, S-I Itoh, K Itoh and T.S. Hahm PPCF 47 (2005) R35 - R161

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Zonal flows

(a)

What are "Zonal" flows?

- n = 0 electrostatic potential fluctuations
- Small or no parallel variation
- Finite radial wavenumber
- Zero frequency

Self-regulating loop

Figure 4. Zonal electric field and zonal flow. The poloidal cross-section of toroidal plasma is illustrated. The hatched region and the dotted region denote the positive and negative charges, respectively: (*a*) The flow perturbation in the poloidal cross-section. (*b*) A birds-eye view of the net flow associated with the zonal perturbation is illustrated.

"Zonal flows in plasma – a review" by P.H.Diamond, S-I Itoh, K Itoh and T.S. Hahm PPCF 47 (2005) R35 - R161

Flows

Cascades

Turbulence

(b) (a) V_{θ} , V_{ξ}

Plasma

Summary

- Turbulence occurs widely in fluids, and (unfortunately) also in plasmas
- Popular model is that energy is transferred across scales as a "cascade" of eddy sizes
- Dimensional arguments lead to the Kolmogorov scaling of energy with length (k). Closest thing to a "universal" theory of turbulence.
- Turbulence is chaotic (sensitive to initial conditions), and becomes more so with increasing Reynolds number
- Statistical averages are reproducible, but the equations which govern them are not closed (more unknowns than knowns)
- Turbulence in fluids and plasmas interacts with the mean flow through Reynolds stress. In plasmas this can lead to a self-regulating interaction between "zonal" flows and drift-wave turbulence.

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