1 GSW… Noise and IP3 in Receivers

In many cases, the designers of individual receiver components (mostly amplifiers, mixers and filters) don't know how any of their customers are going to put them together to make a receiver. However, the designer of a radio receiver is going to want to calculate the likely performance of a receiver built from these individual components.

This gives us two problems: how should the individual components be specified, and how do we combine the specifications of these individual components to produce a specification for the whole receiver?

1.1 The Parameters: Noise and IP3

In many cases, the two most critical parameters for a receiver are its noise performance (which determines the sensitivity: the smallest input signal that can be successfully receiver) and its overall third-order inter-modulation products[1] (which determines the largest input signal that can be successfully received). These parameters tend to be independent: noise is a problem for very small input signals where the inter-modulation products are so small they can be neglected; inter-modulation products are a problem for very large input signals where the signal to noise ratio is so large that noise can be neglected.

Second-order inter-modulation products are easier to prevent, since at least one of the unwanted input signals required to produce some power at the carrier frequency f_c must be at least f_c / 2 away from the carrier, and is therefore easy to filter out; and higher-order inter-modulation products tend to produce significant problems only at higher input powers that the third-order effects. In other words, the third-order effects tend to start first: get them under control and you won't have to worry too much about any higher-order effects.

There are many other important considerations when designing receivers (perhaps most notably the power consumption), but these are the two I'll consider in this chapter.

1.1.1 Noise and Noise Power Spectral Density

Having said that I'll talk about noise, I should mention that's it's often more useful to talk about the noise power spectral density. Noise (N) is measured in Watts; noise power spectral density (N_0) is measured in Watts per Hertz. N_0 is a spectral density, so by definition the amount of noise within a range of frequencies from f_1 to f_2 is given by:

$$N = \int_{f_1}^{f_2} N_0\left(f\right) df \qquad (0.1)$$

All the noise considered in this chapter is *white*, which means that the noise power spectral density is not a function of frequency, and we can simply write:

[1] Third order inter-modulation products are the result of non-linear mixing of two frequencies $f_c + \delta f$ and $f_c + 2\delta f$ (where f_c is the carrier frequency and δf is a small offset frequency, often the channel spacing of the multi-channel radio system being used). This non-linear mixing produces some power at f_c itself, which interferes with the signal the receiver is trying to receive, and since it's at the same frequency, it can't be removed by filtering. For more details, see the chapter on "Non-Linear Effects".

$$N = N_0\left(f_2 - f_1\right) = N_0 B \qquad (0.2)$$

where B is the bandwidth of interest (the frequencies from f_1 to f_2).

From the point of view of a receiver, the noise power spectral density is actually more interesting than the total noise power, since the performance of a receiver with a matched (optimum) filter is a function of E_s/N_0, where E_s is the energy in one symbol, and N_0 is the noise power spectral density.

1.2 Specifying Noise: Noise Temperature and Noise Figure

There are two common ways to specify the amount of noise that a component will introduce into the signal chain: *noise temperature* and *noise figure*. I find noise temperature more intuitive and easier to understand, but noise figure seems to be more common. They both use the same model of noise: that of treating the effect of any component as the combination of adding some noise to the input signal, followed by a perfect (noise-free) gain or loss.

For example, consider a component that when provided with an input signal S_{in} with an input noise power N_{in} produces an output signal S_{out} and output noise power N_{out}. Let the power gain of this component be G, so that:

$$\frac{S_{out}}{S_{in}} = G \qquad (0.3)$$

For those who like pictures, it looks like this:

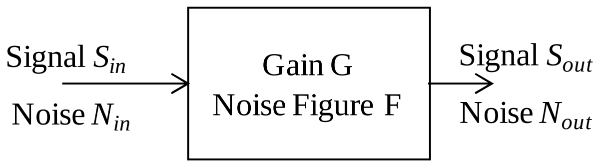

Figure 1-1 One Noisy Component with Gain

We then consider that this component behaves identically to a component that adds a noise power $N_e = N_{out} / G - N_{in}$ to the input signal, and follows this with a perfect, noise-free gain of G, like this:

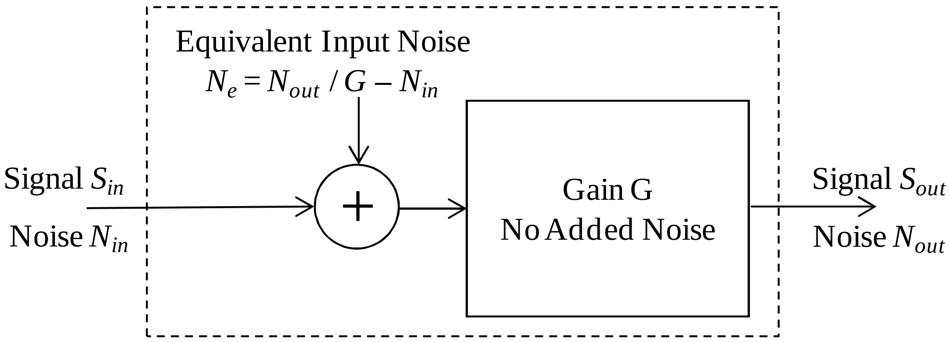

Figure 1-2 Equivalent Circuit for One Noisy Component with Gain

The amount of noise added to the input of the component in this model is called the *equivalent input noise* of the component. Since by definition:

$$G\left(N_{in}+N_{e}\right)=N_{out} \qquad (0.4)$$

it’s straightforward to derive that:

$$N_{e}=\frac{N_{out}}{G}-N_{in} \qquad (0.5)$$

1.2.1 Noise Temperature

The noise power available from a resistor at a certain temperature T is given by:

$$N=k\,T\,B \qquad (0.6)$$

where k is the Boltzmann constant (1.38 x 10^{-23} J/K), T is the temperature of the resistor (in Kelvin), and B is the bandwidth[2] of the system (in Hz).

This allows a noise power N to be associated with a temperature T. The *equivalent noise temperature* T_e is the temperature of a resistor that would provide a power equal to the equivalent input noise power for a component:

$$N_{e}=k\,T_{e}\,B \qquad (0.7)$$

Knowing the noise temperature and the gain of the component (as well as the bandwidth of the signal) together with the input noise is then enough information to calculate the total output noise power:

$$N_{out}=G\left(N_{in}+N_{e}\right)=G\left(N_{in}+k\,T_{e}\,B\right) \qquad (0.8)$$

That’s it. It’s quite simple. The only slight problem is if you’re just designing the radio-frequency stages of a receiver, you might not know what the bandwidth of the signal is likely to be. In that case we can’t calculate the absolute noise powers, the best we can do is calculate the noise power spectral densities N_0 (in W/Hz), where:

$$N_{0}B=N \qquad (0.9)$$

In this case if we know the input noise power spectral density N_{0in}, and the equivalent input noise power spectral density N_{0e}, we can calculate the output noise power spectral density N_{0out} using:

$$N_{0out}=G\left(N_{0in}+N_{0e}\right)=G\left(N_{0in}+k\,T_{e}\right) \qquad (0.10)$$

and pass this information onto the system designer. Often this is what he really wants to know anyway[3].

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[2] Strictly speaking it’s the noise bandwidth of the system, not the 3-dB bandwidth or any other definition of bandwidth. The noise bandwidth of a system is the bandwidth of a perfect “brick-wall” filter that would let through the same amount of noise.

1.2.2 Noise Figure

The definition of the noise figure is a little less obvious. It's the ratio of the actual output noise power to the output noise power due to the input noise alone (i.e. assuming the device itself does not add any noise), *provided the input noise is due to a resistor at 290 K.*

To unpick that a bit: suppose a component has a power gain of G, and the output noise power is N_{out}. Some of this output noise power will be G times the input noise power, and some will be due to the additional noise introduced by the component itself.

In terms of the equivalent input noise, we can write:

$$N_{out} = G\left(N_{in} + N_e\right) = GN_{in} + GN_e \qquad (0.11)$$

and note that the noise figure, by definition, is then equal to:

$$F = \frac{N_{out}}{GN_{in}} = \frac{GN_{in}}{GN_{in}} + \frac{GN_e}{GN_{in}} = 1 + \frac{N_e}{N_{in}} \qquad (0.12)$$

and since for the correct value of noise figure, the input noise has to be that due to a resistor at 290 degrees Kelvin:

$$F = 1 + \frac{N_e}{N_{in}} = 1 + \frac{N_e}{290\,k\,B} \qquad (0.13)$$

Also, the amount of equivalent input noise N_e you have to add to a noise-free version of a component with a noise figure F to get the same output noise can be determined from re-arranging equation (0.12) to get:

$$N_e = N_{in}\left(F - 1\right) \qquad (0.14)$$

So things now look like this (for the equivalent circuit for the noisy component):

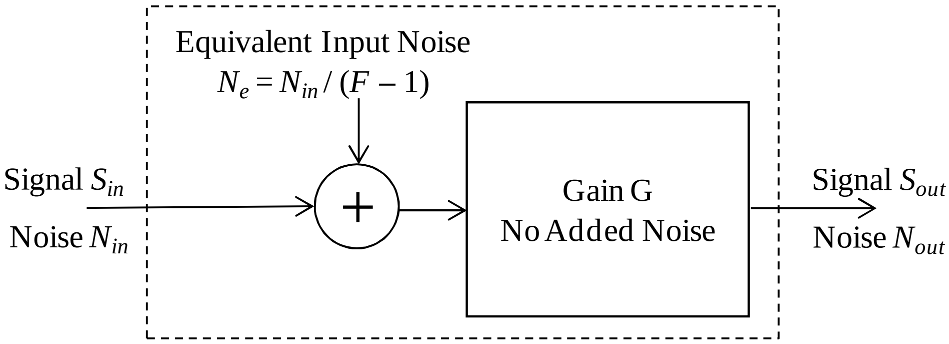


Figure 1-3 – Equivalent Input Noise in Terms of the Noise Figure

[3] As noted, it's the noise power spectral density that determines the minimum bit error rate for a link, not the noise power within the signal bandwidth (which is hard to define, since many signals have power spectra that gradually roll-off away from the main lobe, and it's not always obvious how best to define the bandwidth).

Another very useful result follows from this definition of the noise figure (in fact, this is why the noise figure is used so frequently): the noise figure is the ratio of the input signal to noise ratio to the output signal to noise ratio, again *provided the input noise is equal to that due to a resistor at 290 K*, since:

$$\frac{SNR_{in}}{SNR_{out}} = \frac{S_{in}}{N_{in}} \frac{N_{out}}{S_{out}} = \frac{S_{in}}{GS_{in}} \frac{G\left(N_{in} + N_e\right)}{N_{in}} = \frac{N_{in} + N_e}{N_{in}} = 1 + \frac{N_e}{N_{in}} = F \qquad (0.15)$$

1.2.3 The Noise Figure of Passive Components

In most cases, for passive components, the noise figure is the inverse of the gain. Or perhaps I should say the noise figure is equal to the loss[4], since passive components can't have a gain greater than one, the output is always smaller than the input.

This isn't obvious, but the result is fairly easy to derive if you think about things in terms of equivalent circuits. Consider the following filter:

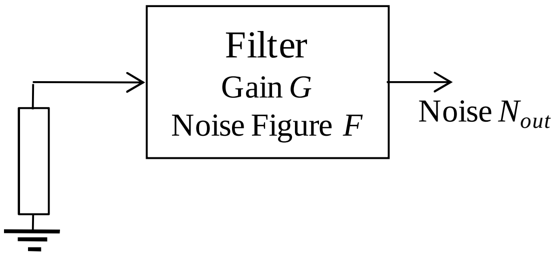


Figure 1-4 A Passive Filter with Input Tied to Ground

Clearly, the input noise going into this filter is just that due a resistor at 290 K. However, consider another component following the filter. As far as it is concerned, all it has on its input is a network of individual resistors, capacitor and inductors all at the same temperature (290 K). This network could be replaced by an equivalent network composed of an ideal resistor in series with an ideal capacitor or inductor. Since ideal capacitors and inductors are noise-free, this means from the noise point of view, the output of the passive component just looks like a resistor at 290 K.

This is exactly what the input noise looks like to this filter; so the input noise must be equal to the output noise:

$$N_{in} = N_{out} = k\,290\,B \qquad (0.16)$$

But the definition of the noise figure is:

$$F = \frac{N_{out}}{G\ N_{in}} = \frac{N_{out}}{G\ k\,290\,B} \qquad (0.17)$$

so here:

[4] Where loss is defined as the input divided by the output (in contrast to the gain being the output divided by the input).

$$F = \frac{k\,290\,B}{G\;k\,290\,B} = \frac{1}{G} \tag{0.18}$$

So, for example, a passive filter with a gain of 0.5 would have a noise figure of 2; or in decibels, a passive filter with a gain of –3 dB has a noise figure of 3 dB.

1.2.4 Noise Figures and Noise Temperatures

Since they are measuring the same thing, knowing the noise temperature of a component allows you to calculate the noise figure of the same component very easily (and vice versa). This is often useful: sometimes some components in a receiver are specified in terms of noise temperatures and others in terms of noise figures.

The relationship between them is simple enough to work out: from equation (0.7) and equation (0.13), we get:

$$F = 1 + \frac{N_e}{N_{in}} = 1 + \frac{k\,T_e\,B}{290\,k\,B} = 1 + \frac{T_e}{290} \tag{0.19}$$

or for going the other way from F to T_e:

$$T_e = 290\left(F - 1\right) \tag{0.20}$$

So, for example, the noise temperature of a passive component (like a filter, or a length of cable with some loss) is related to its gain by:

$$T_e = 290\left(\frac{1}{G} - 1\right) \tag{0.21}$$

For the case of passive components (including cables, which introduce a loss between the antenna and the first stage of the receiver) this is sometimes expressed in terms of a transmission factor σ, defined as:

$$\sigma = G = \frac{S_{out}}{S_{in}} \tag{0.22}$$

which gives for passive components an equivalent input noise temperature of:

$$T_e = 290\left(\frac{1}{\sigma} - 1\right) \tag{0.23}$$

1.3 Adding Noise Powers

You might have noticed that in the previous sections, I've been adding powers, not amplitudes. That's because the noise sources are uncorrelated: the equivalent input noise waveform added by the component is entirely independent of the incoming noise waveform. They both have a mean power (N_{in} and N_e respectively), but the amplitude of these noise contributions at any time is an uncorrelated random variable. I'll call these noise amplitudes n_{in} and n_e respectively, with the lower-case n representing the fact that these are amplitudes not powers.

The total input noise amplitude is then:

$$n = n_{in} + n_e \tag{0.24}$$

so the mean value of the power in this total input noise is:

$$\begin{aligned} E\left\{n^2\right\} &= E\left\{\left(n_{in} + n_e\right)^2\right\} \\ &= E\left\{n_{in}^{\ 2} + 2n_{in}n_e + n_e^{\ 2}\right\} \\ &= E\left\{n_{in}^{\ 2}\right\} + E\left\{n_e^{\ 2}\right\} + 2E\left\{n_{in}n_e\right\} \end{aligned} \tag{0.25}$$

where $E\{\}$ represents the expectation (mean) value. However, since the two noise sources are completely independent and have a mean value of zero, the product of them is equally likely to be positive (when both have the same sign) or negative (when they have different signs). So the expectation value of the product $n_e\, n_{in}$ is zero. That just leaves:

$$E\left\{n^2\right\} = E\left\{n_{in}^{\ 2}\right\} + E\left\{n_e^{\ 2}\right\} \tag{0.26}$$

or in other words: the mean power in the total input noise is the sum of the mean power in the input noise with the mean power in the equivalent input noise.

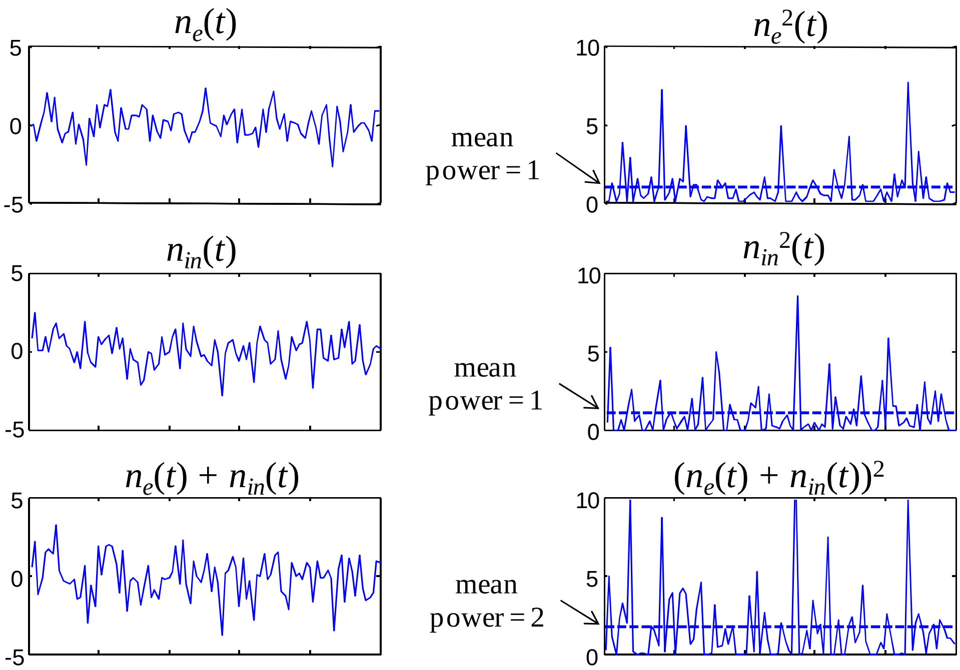


Figure 1-5 Adding Noise Contributions

This is a general rule when adding uncorrelated noise signals: always add the powers, not the amplitudes.

1.4 The Noise Performance of Receivers

To calculate the noise performance of a whole receiver involves combining the noise added by all the stages in the receiver: the various filters, mixers and amplifiers than make up the receiver signal chain. This turns out to be quite simple to do, and the resulting formula has some very important consequences.

The 'trick' is to replace all the noise sources throughout the signal path with equivalent noise sources at the beginning of the signal path. As we've seen, each component can be considered, in terms of the noise it adds, as if it had a perfect noise-free gain, but an additional noise component added into the input of $(F-1)N_i$.

We can extend this idea to more than one component:

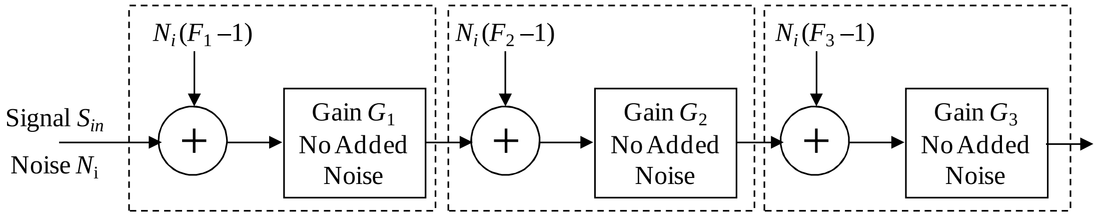


Figure 1-6 Three Noisy Components in Series

The trick now is to move the additional equivalent noise sources back to the start of the series of components. This is quite simple to do, all you have to do is divide the equivalent input noise contributions of each stage by the total gain of the components up to the start of the component. For example, we can move the second equivalent input noise contribution back to the start of the series by dividing it by G_1, and add an equivalent input noise of $N_i(F_1-1)/G_1$ to the input. For the third component, we'll need to divide the equivalent noise contribution by G_1G_2. The equivalent circuit for the series of components now looks like this:

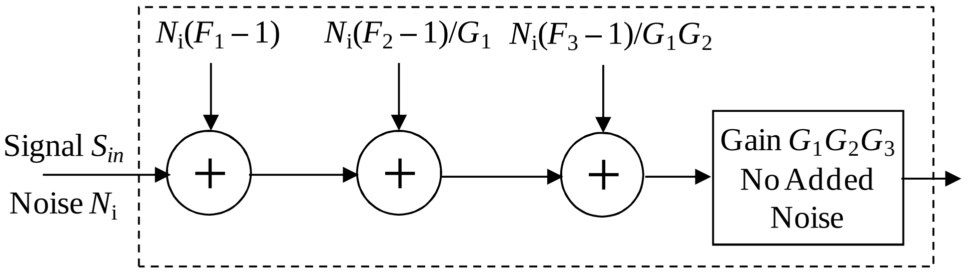


Figure 1-7 – Moving Noise Contributions to the Input

Note that in both of the above figures, the output noise power is:

$$N = G_1G_2G_3N_i + G_1G_2G_3N_i\left(F_1-1\right) + G_2G_3N_i\left(F_2-1\right) + G_3N_i\left(F_3-1\right) \qquad (0.27)$$

and the overall gain is $G_1G_2G_3$. Taking this one stage further, we can add up all these additional noise contributions, and we'll get:

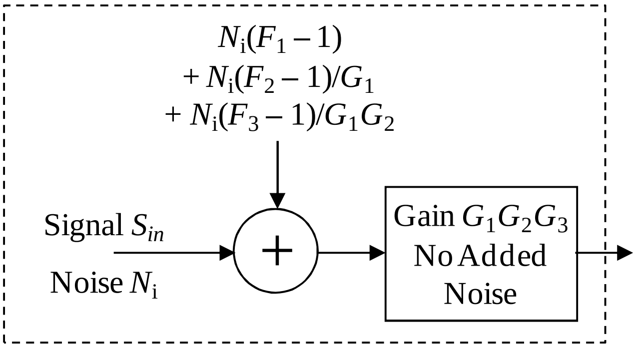


Figure 1-8 Combining Noise Contributions

This suggests we can treat the entire chain of active stages as having a gain of $G_1G_2G_3$, and a noise figure of F where:

$$\begin{gathered} N_i(F-1) = N_i(F_1-1) + N_i(F_2-1)/G_1 + N_i(F_3-1)/G_1G_2 \\ F = F_1 + (F_2-1)/G_1 + (F_3-1)/G_1G_2 \end{gathered} \tag{0.28}$$

Extending this result to more active stages, we get:

$$F = F_1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1G_2} + \frac{(F_4-1)}{G_1G_2G_3} + \ldots \tag{0.29}$$

and this is the *Friis formula* for combining the noise outputs of cascaded stages. Using this formula, and the gains and noise figures of the stages, we can calculate an overall noise figure for the whole receiver.

1.4.1 Working with Noise Temperatures

We can do the calculation equally well with noise temperatures. Using the result from equation (0.19), we can replace all the noise figures with noise temperatures since:

$$F = 1 + \frac{T_e}{290} \tag{0.30}$$

and that turns the Friis formula (equation (0.29)) into:

$$\begin{gathered} 1 + \frac{T_e}{290} = 1 + \frac{T_{e1}}{290} + \frac{\left(1 + \frac{T_{e2}}{290} - 1\right)}{G_1} + \frac{\left(1 + \frac{T_{e3}}{290} - 1\right)}{G_1G_2} + \frac{\left(1 + \frac{T_{e4}}{290} - 1\right)}{G_1G_2G_3} + \ldots \\ T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \frac{T_{e4}}{G_1G_2G_3} + \ldots \end{gathered} \tag{0.31}$$

where T_e is the overall noise temperature for the receiver, and T_{e1}, T_{e2}, T_{e3} etc are the noise temperatures of the individual stages. This is perhaps easier to remember (you don't have to remember to subtract one from all the noise figures except the first one).

1.4.2 Receiver Sensitivity

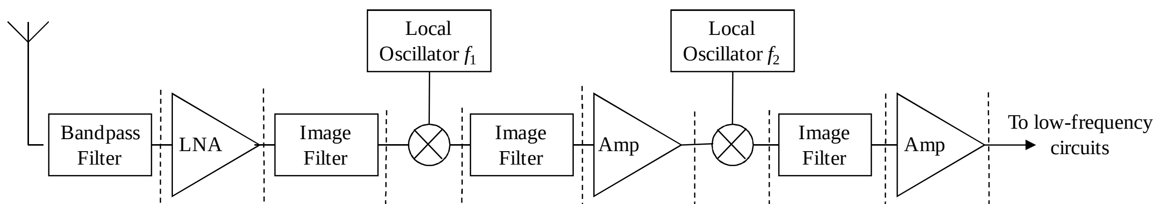
Since the noise in the original input N_i can be taken to be *kTB*, where *k* is the Boltzmann constant (1.38 x 10^{-23}), *T* is the temperature (conventionally taken to be 290 K) and *B* is the bandwidth, all we need to know is the noise bandwidth of the filters, and we can calculate the total signal to noise ratio at the output of the receiver for any level of input signal.

The smallest value of input signal which provides a certain minimum output signal to noise ratio is known as the sensitivity of the receiver. Unfortunately, there isn't a single definition of sensitivity, since the radio receiver designer often doesn't know what level of output signal to noise ratio will be required for the whole system.

A common solution is to define the sensitive of a receiver in terms of the *minimum detectable signal* (MDS). This the is the input signal level that results in a signal to noise ratio at the output of 0 dB (in other words, the same signal power and noise power). Then a system designer wanting to know what level of input signal would be required to provide a signal to noise ratio of 6 dB at the output just has to take the MDS and add 6 dB.

1.4.3 Example of Noise Figure Calculation

Consider the dual-conversion superhet receiver shown below:



-2.5	12.0	-3.0	-6.0	-2.5	20.0	18.0	-3.0	60.0	Gain (dB)
2.5	2.0	3.0	12.0	2.5	3.0	12.0	3.0	20.0	NF (dB)
	10.0		16.0		12.0	26.0			IP_3 (dBm)

Figure 1-9 - Example Dual-Conversion Super-Heterodyne Receiver

The noise figure, gain, and third-order intermodulation intercept point IP_3 is given for all the relevant components in the design[5]. (It will be assumed that the low-frequency circuits do not add significantly to the noise or non-linearity of the receiver.)

This receiver has nine component stages, and we know the noise figure and gain of all of them. However, since they are all given in decibels, and the Friis formula is in terms of real noise powers and power ratios, the first thing we need to do is take the gains and noise figures out of decibels and back into real powers and ratios. This can be conveniently done using a table like the following (spreadsheets are great at this sort of thing):

[5] You might notice that the noise figures of the passive components (filters) are all equal to the loss through the filters (and therefore minus one times the gain in dB). As discussed, this is a general result for all passive components.

Stage	Gain (dB)	Gain up to this stage (dB)	Gain up to this stage (linear)	Noise Figure (dB)	Noise Figure (linear)	Contribution to Overall Noise Figure
Bandpass Filter	–2.5	0.0	1.0	2.5	1.78	1.78
First Amplifier (LNA)	12.0	-2.5	0.562	2.0	1.59	(1.585 – 1) / 0.562 = 1.04
First Image Filter	–3.0	9.5	8.913	3.0	2.00	(2.0 –1) / 8.913 = 0.11
First Mixer	–6.0	6.5	4.467	12.0	15.85	(4.57 – 1) / 4.467 = 3.32
Second Image Filter	–2.5	0.5	1.122	2.5	1.78	(1.78 – 1) / 1.122 = 0.69
Second Amplifier	20.0	–2.0	0.631	3.0	2.00	(2.00 – 1) / 0.631 = 1.58
Second Mixer	18.0	18.0	63.1	12.0	15.85	(15.85 – 1) / 63.1 = 0.23
Third Image Filter	–3.0	36.0	3981	3.0	2.00	(2.0 – 1) / 3981 = 0.00025
Third Amplifier	60.0	33.0	1995	20.0	100	(100 – 1) / 1995 = 0.05

Then add up all the noise contributions to the overall noise figure:

$$\begin{aligned} F &= 1.78+1.04+0.11+3.32+0.69+1.58+0.23+0.00025+0.05 \\ &= 8.81 \end{aligned} \quad (0.32)$$

so this receiver has an overall noise figure of 8.81 (which is 9.45 dB).

If the channel bandwidth is 200 kHz, then the noise power at the input is:

$$kTB = 1.38\times10^{-23}\times290\times200\times10^{3} = 8\times10^{-16}\text{ W} = -121\text{ dBm} \quad (0.33)$$

and if we happen to know that the minimum signal-to-noise ratio required to maintain an acceptable bit error rate at the output of the receiver (the input to the detector stage) is 6 dB (or 3.98 in linear terms), then we can work out the sensitivity of this receiver (and it's easier to work in dB here, so we can subtract rather than having to divide):

$$\begin{aligned} F(\text{dB}) &= SNR_{in}(\text{dB}) - SNR_{out}(\text{dB}) = 9.45 \\ S_{in}(\text{dBm}) - N_{in}(\text{dBm}) - SNR_{out}(\text{dB}) &= 9.45 \\ S_{in}(\text{dBm}) &= N_{in}(\text{dBm}) + SNR_{out}(\text{dB}) + 9.45 \\ S_{in}(\text{dBm}) &= -121+6+9.45 = -105.55\,\text{dBm} \end{aligned} \quad (0.34)$$

If we didn't know what minimum signal-to-noise ratio was required for acceptable performance, all we could do is consider the sensitivity to be the minimum detectable signal MDS), which is the input signal that gives an output signal power equal to the output noise power. Using this definition, the sensitivity would be 6 dB less, at –111.55 dBm.

1.4.4 Comparative Noise Contributions

Looking at the contributions that each individual stage makes to the overall noise figure for the receiver, the contribution of each stage (except the first stage) could be written as:

$$\frac{F_i - 1}{\prod_{j<i} G_j} \tag{0.35}$$

in other words the contribution of each stage is its own noise figure minus one, divided by the total gain up to the input of the stage. After the second amplifier, the combined gain is so high that it doesn't really matter whether subsequent stages are low-noise or not, it won't make much difference to the overall noise performance of the receiver. Of much more concern are the first few stages in the receiver chain, where the signal level is low, and there hasn't been much gain. That's where low-noise design is much more important.

In this example, the largest single contribution to the overall noise figure is from the first mixer, and the second-largest contribution is from the bandpass filter before the front-end low-noise amplifier. To improve the noise performance of the receiver it would be nice to get rid of this filter, but that would allow the entire output of the receive antenna to be fed straight into the input of the sensitive low-noise amplifier: and if there are any strong emitters around (for example someone with a mobile phone walks past) they might block the wanted signal.

In general, for low-noise design you need as much low-noise gain as early as possible in the receiver chain. Once the signals are large, you can usually forget about noise.

Exactly the opposite is true for the other problem with receivers discussed in this chapter: the 3rd-order intermodulation products.

1.5 Calculating Intermodulation Distortion

The power of the 3rd-order intermodulation product I_d in the output of a non-linear stage fed by three equal-power inputs is given by[6]:

$$I_d = \frac{G}{IP_3^2} S_{in}^{\ 3} \tag{0.36}$$

where S_{in} is the power in the input signal (in Watts), G is the power gain of the stage, and IP_3 is the third-order intermodulation intercept point (also in Watts).

If this is the power, then the rms amplitude of this distortion must be:

$$d_{rms} = \sqrt{I_d} = g\left(\frac{1}{IP_3}\right) S_{in}^{\ 3/2} \tag{0.37}$$

where g is the amplitude gain of the stage, rather than the power gain G. Since the power gain is the square of the amplitude gain, this means:

[6] For the derivation of this formula, see the chapter on non-linear effects.

$$g^2 = G \tag{0.38}$$

A radio receiver with several non-linear elements in series will have contributions to the total amount of intermodulation distortion from each non-linear stage. Just as before, we can readily calculate the amplitude of the overall intermodulation product by using the technique of replacing every real component with an ideal linear component, with an additional intermodulation term added to the input, replacing something like this:

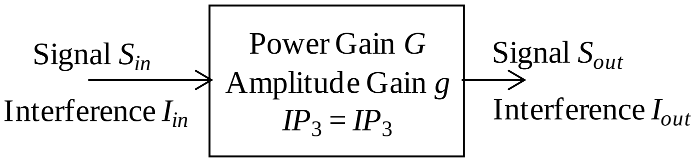


Figure 1-10 Non-Linear Component Adding Intermodulation Product

with something like this:

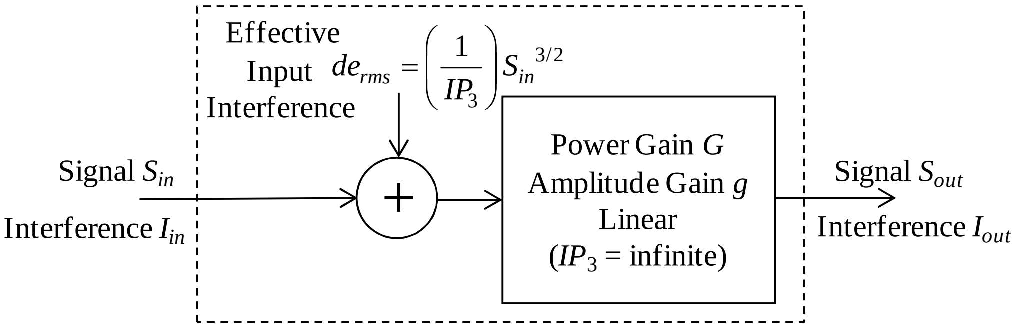


Figure 1-11 Equivalent Model of Non-Linear Component

Note that the amplitude of the required effective input intermodulation product term that's added to the input to the non-linear component is:

$$de_{rms} = \frac{d_{rms}}{g} = \left(\frac{1}{IP_3}\right) S_{in}^{3/2} \tag{0.39}$$

so that when it is multiplied by the amplitude gain of the non-linear stage, we end up with the result for the output term, given in equation (0.37).

1.5.1 Adding Intermodulation Product Terms

At this point it might be worth pausing to consider why we always work with powers in the case of noise, but I'm talking about envelopes and rms amplitudes in the case of the intermodulation products.

The reason is that the intermodulation products are not independent between the different stages; they all result from the same signals passing through the components. So they all have the same shape, and that means I have to add them coherently. Consider adding two signals that are the same shape, $s_e(t)$ and $s_{in}(t)$. The result is a signal with a magnitude of $s_e(t) + s_{in}(t)$, and that has a mean power of:

$$E\left\{\left(s_e(t) + s_{in}(t)\right)^2\right\} = E\left\{s_e^2(t)\right\} + E\left\{s_{in}^2(t)\right\} + E\left\{2 s_e(t) s_{in}(t)\right\} \tag{0.40}$$

and this time the final term is not zero (see the figure below). If the signals are exactly the same shape, then we could write:

$$s_e(t) = k\, s_{in}(t) \qquad (0.41)$$

where k is a constant. In this case, we get:

$$E\left\{\left(s_e(t)+s_{in}(t)\right)^2\right\} = E\left\{(1+k)^2\, s_e^{\,2}(t)\right\} = (1+k)^2\, E\left\{s_e^{\,2}(t)\right\} \qquad (0.42)$$

You can't just add the powers any more, you have to add the amplitudes. That's why I'm working with amplitudes here.

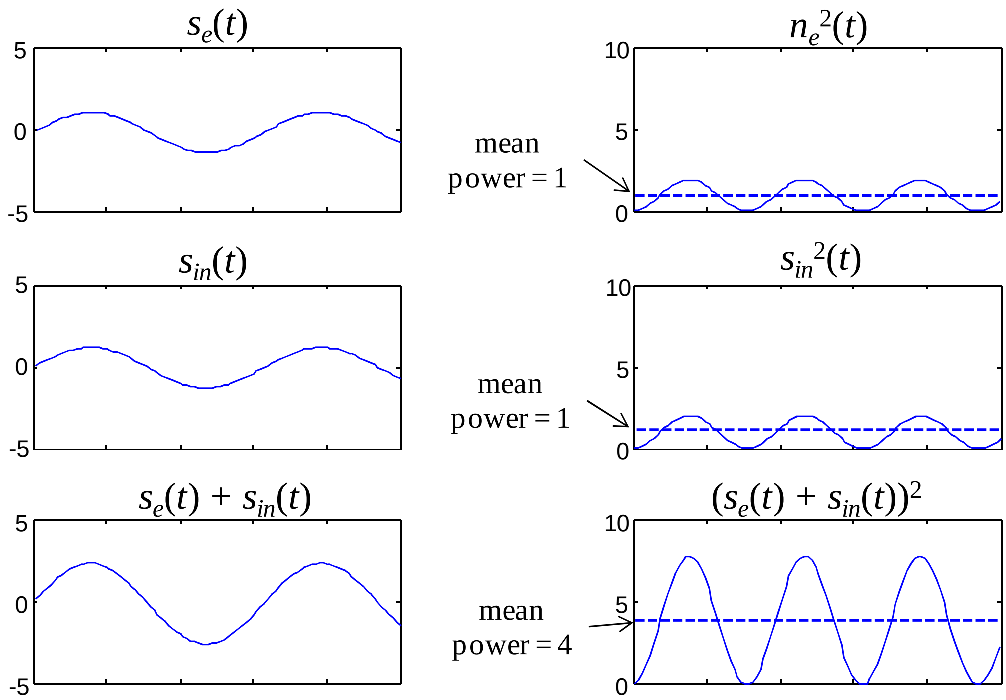


Figure 1-12 Adding Intermodulation Product Contributions

1.5.2 Intermodulation Products and Multi-stage Receivers

We can extend this idea (of replacing real components with ideal linear components with additional equivalent input intermodulation product terms) to a series of components, just like we did for noise. For example, suppose we had a chain of components, and two of them had significant non-linearities:

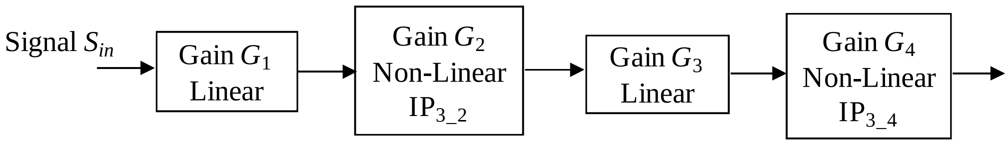


Figure 1-13 Sample System with Four Stages, Two Non-Linear

Expressing the non-linear stages in terms of an ideal linear stage and an additional contribution from the distortion gives:

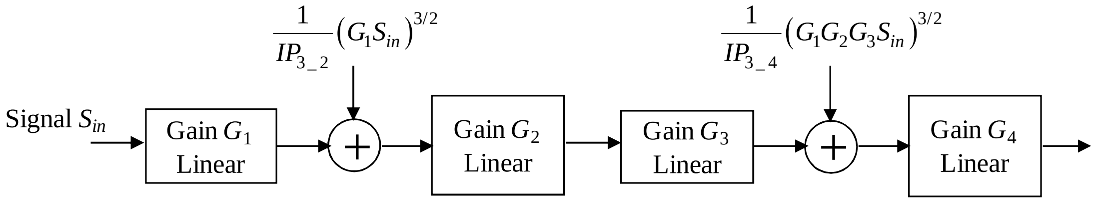


Figure 1-14 Equivalent System with Two Stages of Non-Linearity

Note the equivalent distortion input to the second stage is:

$$\frac{1}{IP_{3\_2}}\left(G_1S_{in}\right)^{3/2} \tag{0.43}$$

since the signal input at this stage has a power of G_1S_{in} (it's G_1 times more powerful than the input to the whole chain, since it's just come through the first stage with a gain of G_1). Similarly, the signal input to the last stage has an equivalent input distortion component of:

$$\frac{1}{IP_{3\_4}}\left(G_1G_2G_3S_{in}\right)^{3/2} \tag{0.44}$$

since the input to this stage has a power of $G_1G_2G_3S_{in}$.

Then we move these additional contributions back to the start of the whole system. They are amplitudes, so moving them back through a component with a power gain of G will reduce the amplitude by a factor $\sqrt{G}$. Hence the contribution of the second stage becomes:

$$\frac{1}{\sqrt{G_1}}\frac{1}{IP_{3\_2}}\left(G_1S_{in}\right)^{3/2} = \frac{G_1}{IP_{3\_2}}\left(S_{in}\right)^{3/2} \tag{0.45}$$

and the contribution of the fourth stage becomes:

$$\frac{1}{\sqrt{G_1G_2G_3}}\frac{1}{IP_{3\_4}}\left(G_1G_2G_3S_{in}\right)^{3/2} = \frac{G_1G_2G_3}{IP_{3\_4}}\left(S_{in}\right)^{3/2} \tag{0.46}$$

and that produces an equivalent system that looks like this:

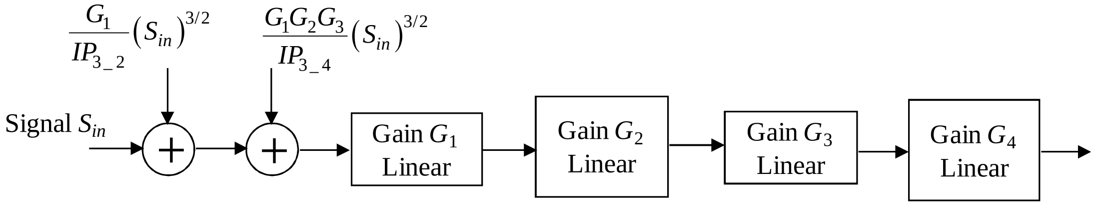


Figure 1-15 Combining Non-Linear Stages

Comparing this to a one-stage system with intermodulation distortion added to the input:

$$d_{in} = \left(\frac{1}{IP_3}\right)S_{in}^{\ 3/2} \tag{0.47}$$

suggests that the total amount of intermodulation products added by all the stages is equivalent to a perfectly linear system with an intermodulation product term added to the input of the whole series of components, of amplitude:

$$\begin{aligned} d_{in} &= \frac{G_1}{IP_{3\_2}} S_{in}^{3/2} + \frac{G_1 G_2 G_3}{IP_{3\_4}} S_{in}^{3/2} \\ &= \left(\frac{G_1}{IP_{3\_2}} + \frac{G_1 G_2 G_3}{IP_{3\_4}} \right) S_{in}^{3/2} \end{aligned} \tag{0.48}$$

This is exactly what would happen if the system were replaced by a single non-linear component with a gain of $G_1G_2G_3G_4$ and an IP_3 of:

$$IP_{3system} = \frac{1}{\dfrac{G_1}{IP_{3\_2}} + \dfrac{G_1 G_2 G_3}{IP_{3\_4}}} \tag{0.49}$$

In other words, we can derive an equivalent IP_3 for the whole system, and use this to calculate the total amount of intermodulation product introduced by all the components in the system.

Note that this formula could be expressed in the form:

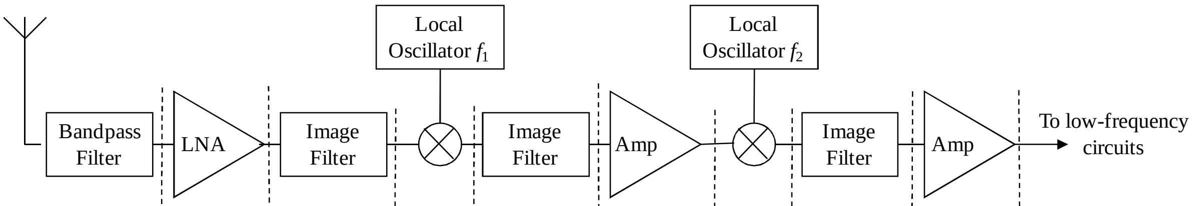
$$IP_{3system} = \frac{1}{\dfrac{\text{Gain up to stage with } IP_{3\_2}}{IP_{3\_2}} + \dfrac{\text{Gain up to stage with } IP_{3\_4}}{IP_{3\_4}}} \tag{0.50}$$

which suggests how it can be expanded for use with any number of non-linear terms:

$$IP_{3system} = \frac{1}{\sum_i \dfrac{\text{Gain up to stage with } IP_{3\_i}}{IP_{3\_i}}} \tag{0.51}$$

1.5.3 *Example of Intermodulation Calculation*

Consider the superhet receiver from section 1.4.3:



-2.5	12.0	-3.0	-6.0	-2.5	20.0	18.0	-3.0	60.0	Gain (dB)
2.5	2.0	3.0	12.0	2.5	3.0	12.0	3.0	20.0	NF (dB)
	10.0		16.0		12.0	26.0			IP_3 (dBm)

Figure 1-16 - Example Dual-Conversion Super-Heterodyne Receiver (Again)

This receiver has four stages with significant non-linearities. It's usually easier to keep the gain and IP_3 in dB and dBm respectively initially since the division then becomes a subtraction; it's only when combining the contributions from the different stages that things have to be taken out of dB so that the contributions can be added together.

Just like with noise, this sort of calculation can be readily done with a spreadsheet, and in this case the results might look something like this:

Stage	Gain up to this stage (dB)	IP_3 of this stage (dBm)	$10\log_{10}\dfrac{G_{upto\_i}}{IP_{3i}}$	$\dfrac{G_{upto\_i}}{IP_{3\_i}}$
First LNA	–2.5	10.0	–12.5	0.056
First Mixer	6.5	16.0	–9.5	0.112
Second Amplifier	–2.0	12.0	–14.0	0.040
Second Mixer	18.0	26.0	–8.0	0.158

hence the resultant IP_3 of the whole system is:

$$\begin{aligned} IP_{3system} &= \frac{1}{0.056+0.112+0.040+0.158} \\ &= 2.73 \text{ mW} \\ &= 4.37 \text{ dBm} \end{aligned} \qquad (0.52)$$

(Unlike the noise analysis earlier in the chapter, I've only considered here the components with a significant non-linearity. If you were designing a spreadsheet you might have to consider all the components, and give a value of infinity to the IP3 of the linear components, but if doing the calculation by hand we can be a bit more intelligent and just consider the non-linear components.)

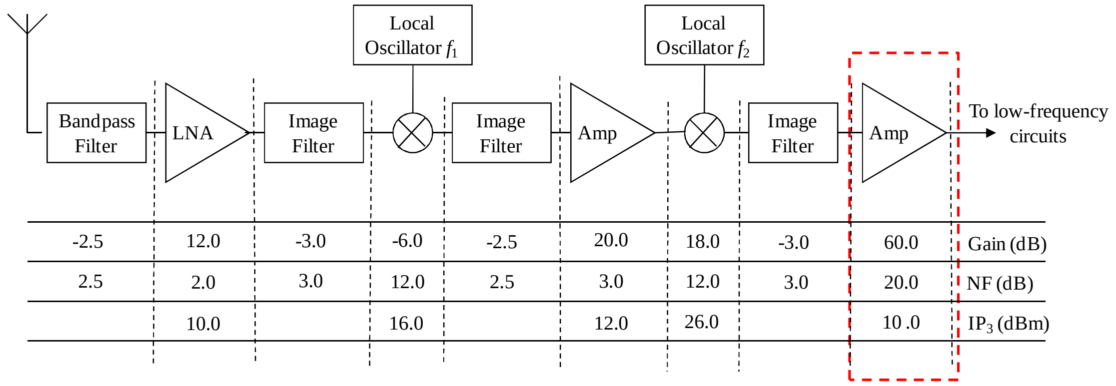
1.5.4 Comparative Intermodulation Contributions

Looking at the contributions that each individual stage makes to the overall IP_3 intercept point for this receiver, the single largest contribution is from the second mixer, even though this component has by far the largest IP_3. This isn't surprising: the power in the intermodulation product generated at each stage varies with the cube of the signal power, and it's the later stages which tend to have the largest signal powers.

1.5.5 IP_3 and Channel Selecting Filters

There is one potential trap when working out the overall IP_3 of a receiver, and that is to remember that in order to generate any intermodulation product at the signal frequency, you need two other frequencies to be present.

If you've filtered out all of the other frequencies, then further non-linear components in the receiver aren't going to make any difference. If the third amplifier in the receiver above actually had an IP_3 of 10 dBm:



then you might think that this would give the entire receiver a very low IP_3, since the signal would be so large at this stage. However, if the image filter just before this amplifier gets rid of all the signals except the wanted signal, then there aren't any other signals left to produce any intermodulation products, and the IP_3 of this component can be neglected.

It's only the non-linear stages up to the filter than selects an individual channel or an individual carrier frequency that need to be considered when calculating an overall IP_3 (after this point there won't be any power left at adjacent carrier frequencies that can be mixed together to create any distortion in the final signal).

1.6 Receiver Dynamic Range

The dynamic range of a receiver is the range of input signals that give acceptable performance. In general, this is impossible to calculate without knowing what the receiver is going to be used for, but if we define the lowest signal input power as the minimum detectable signal (the signal that provides an equal amount of signal power and noise power in the output), and we define the highest input signal power to be the input signal power that produces a third-order intermodulation product power equal to the output noise power, then we can produce a figure-of-merit for the receiver. This is known as the *spurious-free dynamic range* (SFDR).

It turns out that there is a very simple and memorable formula for the spurious-free dynamic range in terms of the overall IP3 and the minimum detectable signal (MDS). Equation (0.36) gives power in the intermodulation product in terms of the input signal powers S_{in} and the gain G, and putting this equal to the output given by the MDS (which is just the MDS times the gain):

$$I_d = \frac{G}{IP_3^{\,2}} S_{in}^{\;3} = G \times MDS \tag{0.53}$$

and a bit of algebra on this then gives:

$$S_{in} = \left(IP_3^{\,2} \times MDS \right)^{1/3} \tag{0.54}$$

This is the maximum input power for the spurious free dynamic range. The minimum is just the MDS itself. So the ratio of the two is:

$$\frac{\text{Max Power}}{\text{Min Power}} = \frac{\left(IP_3^{\,2} \times MDS \right)^{1/3}}{MDS} = \frac{IP_3^{\,2/3}}{MDS^{2/3}} = \left(\frac{IP_3}{MDS} \right)^{2/3} \tag{0.55}$$

which in terms of dB is:

$$SFDR(\text{dB}) = \frac{2}{3}\left(IP_3(\text{dBm}) - MDS(\text{dBm})\right) \quad (0.56)$$

For the example receiver considered above, this works out to be:

$$= \frac{2}{3}\left(4.37 - (-111.52)\right) = 77.25 \text{ dB} \quad (0.57)$$

1.7 Tutorial Questions

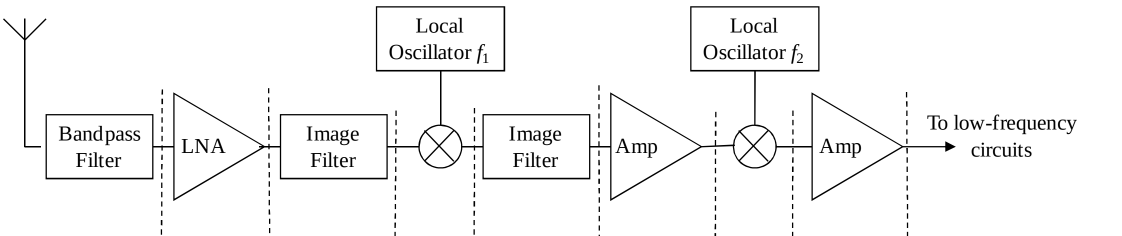
1) In the example given in the notes, by how much would the noise performance improve if the second amplifier and the second image filter were the other way around (i.e. the amplifier came first)?

What would be the disadvantage (if any) of doing this?

2) A passive filter has an insertion loss of 2 dB. What can you deduce about its noise figure and third-order intercept point?

3) What would happen to the spurious-free dynamic range of this receiver if the gain of the second amplifier were reduced to 10 dB?

4) Consider the single-conversion superhet design shown below:



-1.5	10.0	-4.0	-8.0	-2.5	20.0	10.0	60.0	Gain (dB)
???	3.0	???	13.0	???	3.0	20.0	20.0	NF (dB)
	10.0		6.0		10.0	20.0	40.0	IP_3 (dBm)

The receiver noise bandwidth is 200 kHz. Calculate:

a) The equivalent receiver noise figure;
b) The expected output noise power if the receive antenna is pointing at the ground;
c) The receiver's third-order intermodulation intercept point;
d) The spurious-free dynamic range.

Assume all the filters are passive (i.e. they contain no active components).

Why are the first three components in the order they are in? Why not combine the bandpass filter with the image filter to save costs?

5) Consider the super-heterodyne receive in the example above. If you were asked to change any one component in order to improve this dynamic range, which component should you choose?