1 GSW… Time-Variant Multipath Channels

Despite my ambition that chapters should be stand-alone, I've had to make a few exceptions, and this chapter is one of them. I'm going to assume you've read the previous chapter on Multipath Channel Models, and know what the rms delay spread and the coherence bandwidth of a multipath channel are, and how to calculate them.

The delay spread of a multipath channel was calculated in the last chapter from the mean power delay profile (averaged over time); and the coherence bandwidth was also determined as a function of time, although it is more usually quoted as an average value, with the average again being over time. Neither parameter contains any information about how quickly the impulse response of the channel is changing, and yet this is a very important characteristic of a channel: slowly changing channels present a different set of problems to more quickly-changing channels.

This chapter discusses the time-variant nature of multipath channels: what causes the impulse response of the channels to change, what problems this causes, and what simple parameters can be used to characterise the rate of these changes.

1.1 A Quick Recap of Important Results

A time-variant mobile radio channel can be characterised either by a time-dependent impulse response $h(\tau, t)$ or a time-dependent frequency response $H(\omega, t)$. This is a simple extension of the usual representation of linear time-invariant systems to the case where the system isn't time-invariant.

For time-variant systems, the output of the system for any given input signal $x(t)$ can be determined using the time-dependent impulse response:

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t)\, x(t-\tau)\, d\tau \tag{0.1}$$

and in the general case, $h(\tau, t)$ can be anything. The time-dependent frequency response $H(\omega, t)$ is the gain of the channel experience by energy leaving the transmitter at time t with frequency ω.

1.1.1 *The Problems of Time-Varying Channels*

Knowing how fast the channel is changing can be very useful for several reasons. For example, anyone designing an equaliser (a circuit at the receiver that tries to work out the impulse response of the channel, so it can undo the effects the multipath interference in the received signal) needs to know how fast the channel is changing. These circuits typically work by requiring the transmitter to transmit a known series of bits (known as a *training sequence* or as *pilot symbols*), and using the received signal (complete with echoes) to work out the impulse response of the channel. The receiver then designs an equaliser based on this channel impulse response.

One problem with using a training sequence is that the receiver works out what the channel impulse response was during the time when the training sequence is transmitted, not when the information is being transmitted: and the channel is changing all the time. Know how fast the impulse response of a channel is changing, and you know how often you need to transmit these training sequences so that the receiver can keep its equaliser up-to-date.

Another problem is causes by channels that fade very slowly, and information is lost during the fades. One possible solution when using such channels is to re-transmit the lost information after the channel has come out of the fade. Know how fast the channel is fading, and you know how long you have to wait before a re-transmission of the data has a good chance of getting through.

There are two interesting questions to answer about these time-varying channels: for how long does the channel remain reasonably constant in terms of its impulse response, and what is the spectrum of the signal that is received?

1.1.2 The Channel Model

Again, I'll assume for most of this chapter a channel model typical of many urban mobile radio channels: a series of groups of rays, each group reflected from a different distant reflector, and each group arriving with a different delay, with the power in each individual ray in a group equally likely to arrive from any direction. A simple physical justification for this model is a channel of the form shown below, with a large number of local scatterers evenly spread all around the receiver, and a smaller number of large reflectors spaced further away:

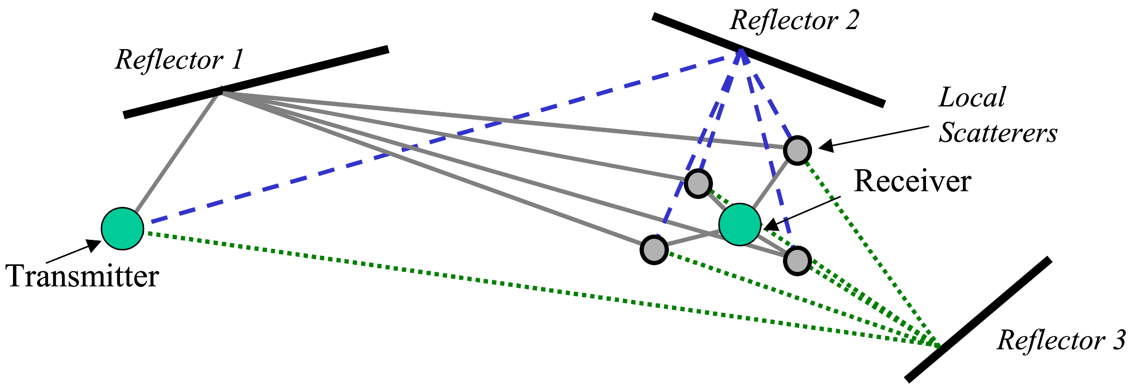

Figure 1-1 Model with Local Scatterers and Distant Reflectors

The difference in delays between the rays in each group is too small for the receiver to resolve, so as far as the receiver is concerned, the impulse response looks something like this:

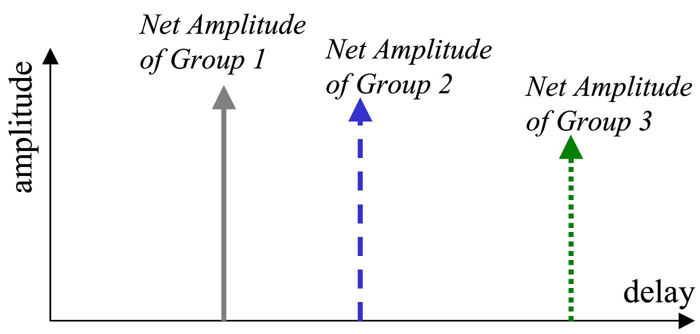

Figure 1-2 Impulse Response for Groups of Scatterers

although the amplitudes (and phases) of these resolved rays changes with time. This is caused by the motion of the receiver (or the local scatterers), causing the different rays that contribute to each group to arrive with different phases: see the chapter on Fading Distributions for more details. (The delays of the rays change with time as the receiver moves as well, but much more slowly: this effect isn't usually a problem.)

Plotting the impulse response against time results in plots such as that shown below, and this chapter is all about characterising the rates at which the amplitudes and phases of each of these rays change with time.

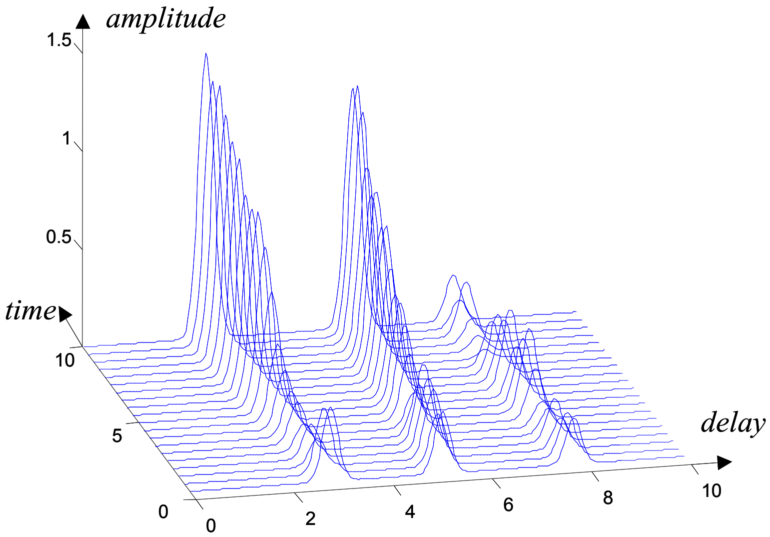

Figure 1-3 Example Time-Variant Three-Ray Impulse Response

1.2 The Doppler Spectrum

If we use this model of a series of groups of rays arriving with slowly-changing delays, and only consider short periods of time during which the delays of these rays can be considered to be constant (i.e. the receiver hasn't moved very far towards or away from the transmitter), we can write the time-variant impulse response in the simplified form:

$$h(\tau,t)=\sum_i h_i(t)\,\delta(\tau-\tau_i) \tag{0.2}$$

where the i^{th} group arrives after a delay τ_i with an amplitude $h_i(t)$.

First, consider just the i^{th} group of these rays, all arriving with a delay of around τ_i. If the transmitter transmits a single frequency at ω_c, then the receiver will receive a delayed version of this signal, with an amplitude that's constantly changing, following the variations of $h_i(t)$.

If the transmitter transmits a single frequency signal at a carrier frequency ω_c:

$$\cos(\omega_c t) \tag{0.3}$$

then the signal arriving in this i^{th} ray can be written as an attenuated version of this signal, with an amplitude $h_i(t)$ delayed by τ_i:

$$r_i(t)=h_i(t)\cos\left(\omega_c(t-\tau_i)\right) \tag{0.4}$$

As in the chapter on Multipath Channel Models (see that chapter for more details), I'll represent the amplitude and phase of a constant-frequency oscillation in terms of a complex number, with a phase angle equal to the relative phase of the oscillation and some reference (in this case the transmitted signal). So the signal arriving in this ray would be represented by a complex number with amplitude $h_i(t)$ and phase angle $-\omega_c \tau_i$:

$$h_i(t)\exp(-j\omega_c\tau_i) \tag{0.5}$$

The real received signal can be derived by multiplying this complex number by a complex oscillation at the original frequency, and then taking the real part of the result:

$$\begin{aligned} y_i(\tau_{i,} t) &= \Re\{h_i(t)\exp(-j\omega_c\tau_i)\exp(j\omega_c t)\} \\ &= \Re\{h_i(t)\exp(j\omega_c(t-\tau_i))\} \\ &= h_i(t)\cos(\omega_c(t-\tau_i)) \end{aligned} \tag{0.6}$$

where $y(\tau_i, t)$ is the signal received at time t from the group of rays with delay around τ_i.

At this point we can start to answer one of the two interesting questions: what is the power spectrum of this received signal? Remember that the transmitter is transmitting a constant frequency, so the transmitted power spectral density just consists of two delta functions:

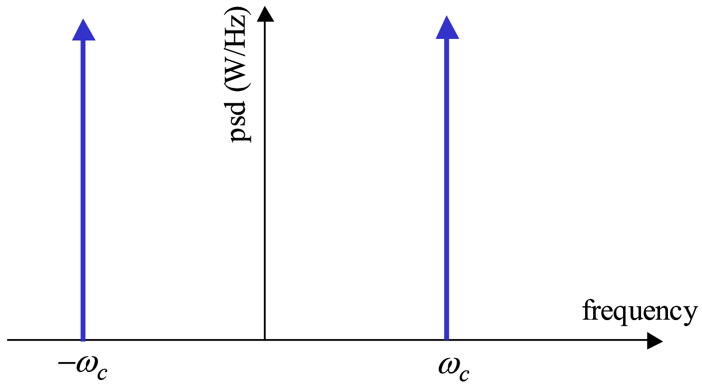

Figure 1-4 - Transmit Spectrum of a Single Frequency Signal

However the received signal has a varying phase and amplitude, which spreads the power over a range of frequencies. The received spectrum looks something more like this:

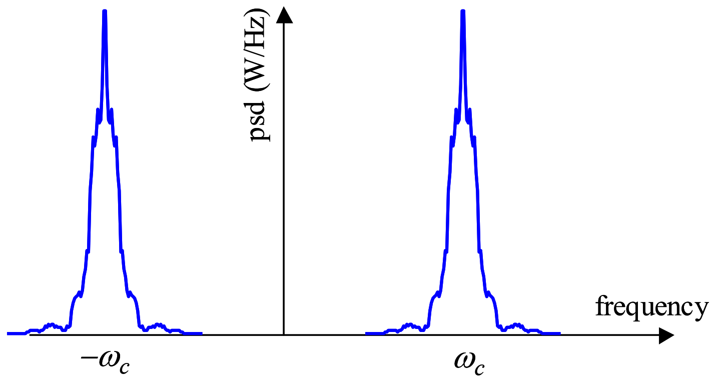

Figure 1-5 - Receive Spectrum of a Single Frequency Signal Through a Fading Channel

The received power spectral density is just the square of the magnitude of the Fourier transform of $y(\tau_i, t)$ with respect to time, and that Fourier transform is given by:

$$Y(\tau_i,\omega)=\int_{-\infty}^{\infty}\Re\left\{h_i(t)\exp(-j\omega_c\tau_i)\exp(j\omega_c t)\right\}\exp(-j\omega t)\,dt \tag{0.7}$$

and using the useful result that for the real part of a complex number z, $\Re\{z\}=\frac{z+z^*}{2}$:

$$\begin{aligned}Y(\tau_i,\omega)&=\frac{1}{2}\int_{-\infty}^{\infty}h_i(t)\exp(-j\omega_c\tau_i)\exp(-j(\omega-\omega_c)t)\,dt\\&\quad+\frac{1}{2}\int_{-\infty}^{\infty}h_i^*(t)\exp(+j\omega_c\tau_i)\exp(-j(\omega+\omega_c)t)\,dt\\&=\frac{\exp(-j\omega_c\tau_i)}{2}\int_{-\infty}^{\infty}h_i(t)\exp(-j(\omega-\omega_c)t)\,dt\\&\quad+\frac{\exp(j\omega_c\tau_i)}{2}\int_{-\infty}^{\infty}h_i^*(t)\exp(-j(\omega+\omega_c)t)\,dt\\&=\frac{\exp(-j\omega_c\tau_i)}{2}G_i(\omega-\omega_c)+\frac{\exp(j\omega_c\tau_i)}{2}G_i^*(-\omega-\omega_c)\end{aligned} \tag{0.8}$$

where[1] $G_i(\omega)$ is the Fourier transform of the amplitude and phase changes in the ray arriving with delay τ_i:

$$G_i(\omega)=\int_{-\infty}^{\infty}h_i(t)\exp(-j\omega t)\,dt \tag{0.9}$$

Take the square of the modulus of this to give the power spectral density, and we get[2]:

$$\begin{aligned}\left|Y(\tau_i,\omega)\right|^2&=\frac{1}{4}\left|G_i(\omega-\omega_c)\right|^2+\frac{1}{4}\left|G_i^*(-\omega-\omega_c)\right|^2\\&\quad+\frac{1}{4}\left|G_i(\omega-\omega_c)G_i^*(-\omega-\omega_c)\right|+\frac{1}{4}\left|G_i^*(\omega-\omega_c)G_i(-\omega-\omega_c)\right|\end{aligned} \tag{0.10}$$

For any radio signal of interest, all the power will occur either around the carrier frequency ω_c, or around $-\omega_c$, so there is no frequency ω for which $G_i(\omega_c-\omega_c)$ and $G_i(-\omega-\omega_c)$ both have a significant value: for any frequency one or the other (or both) will be negligible. So we can ignore the last two product terms in equation (0.10), and just write:

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[1] I'm using a 'G' for this function, rather than an 'H', since 'H' is usually used for the Fourier transform of the time-dependent impulse response with respect to delay, rather than time. $H(\omega, t)$ is the time-dependent frequency response for the channel at time t.

[2] There's an easier way to get this result if you know that a multiplication of two signals in the time domain is equivalent to a convolution of two signals in the frequency domain. Convolving the Fourier transform of $h_i(t)$ with the Fourier transform of the cosine wave transmitted signal just involves making a copies of the Fourier transform of $h_i(t)$ centred around $-\omega_c$ and ω_c.

It's just like analogue amplitude modulation, except that in this case the amplitude modulation is not done intentionally by the transmitter, but by the channel itself.

$$\left|Y(\tau_i,\omega)\right|^2 = \frac{1}{4}\left|G_i(\omega-\omega_c)\right|^2 + \frac{1}{4}\left|G_i^*(-\omega-\omega_c)\right|^2 \tag{0.11}$$

Plotting $h_i(t)$, $S(\tau_i, \omega)$ and $|G(\tau_i, \omega - \omega_c)|^2$ for one value of delay τ_i gives:

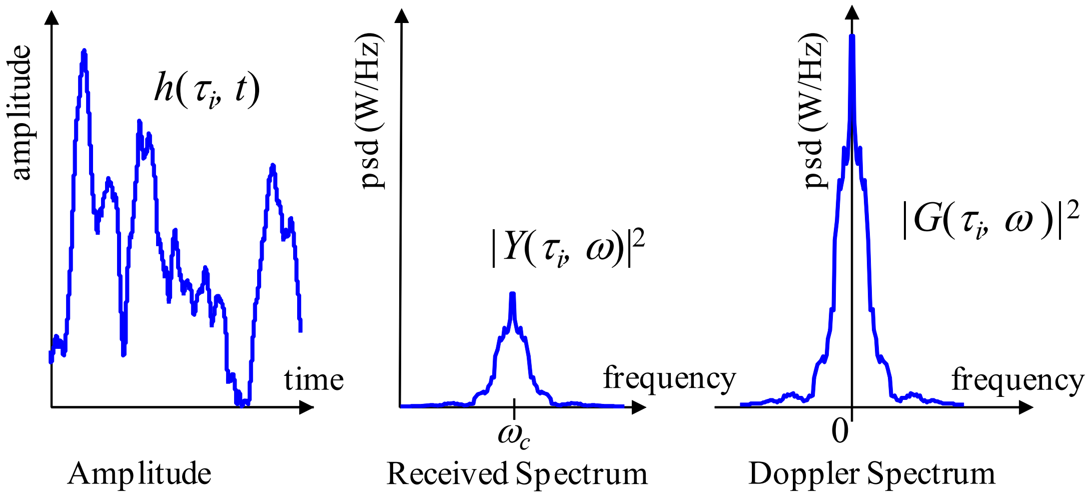

Figure 1-6 Fading and the Doppler Spectrum

The third plot in this figure shows the power spectral density of the complex representation of the received ray, which is proportional to the power spectral density of the real received signal (for positive frequencies) shifted down in frequency by the carrier frequency ω_c. This is commonly known as the Doppler spectrum. It's a measure of how much and transmitted power is spread out in frequency by the channel.

1.2.1 The Doppler Spread

The Doppler spectrum contains all the information we need about how fast the rays arriving with one particular delay are changing in amplitude and phase. However, just like the impulse response at a particular time, it's a graph. Just as it's useful to have a single parameter than gives a good indication of the range of delays over which an impulse arrives at the receiver (the delay spread), it's also useful to have a single parameter that gives a good indication of the range of frequencies over which a single-frequency transmission is spread when it arrives at the receiver. This one is known as the *rms Dopper spread*, or just the *Doppler spread*. It's defined in the same way as the delay spread: the Doppler spread is the standard deviation of the normalised Doppler spectrum.

We calculate it in a similar way, for example, with a continuous Doppler spectrum as shown above, the Doppler spread would be:

$$D = \sqrt{\frac{\int_{-\infty}^{\infty} P(\omega)(\omega-\bar{\omega})^2\, d\omega}{\int_{-\infty}^{\infty} P(\omega)\, d\omega}} \tag{0.12}$$

where D is the rms Doppler spread, $P(\omega)\, d\omega$ is the received power in the range of frequencies around ω, and $\bar{\omega}$ is the mean Doppler shift given by:

$$\bar{\omega} = \frac{\int_{-\infty}^{\infty} P(\omega)\omega\, d\omega}{\int_{-\infty}^{\infty} P(\omega)\, d\omega} \qquad (0.13)$$

1.2.2 Example of a Doppler Spectrum

Consider a man walking towards a wall. As long as he's not very far away from the wall, and if we can ignore reflections from all other obstacles (rather unrealistic, I know, but this is just a simple example), we only need to consider two rays: the direct ray from the transmitter, and the ray that bounces off the wall. Also, to make life even simpler, let's assume that both rays have the same power when they arrive at the receiver.

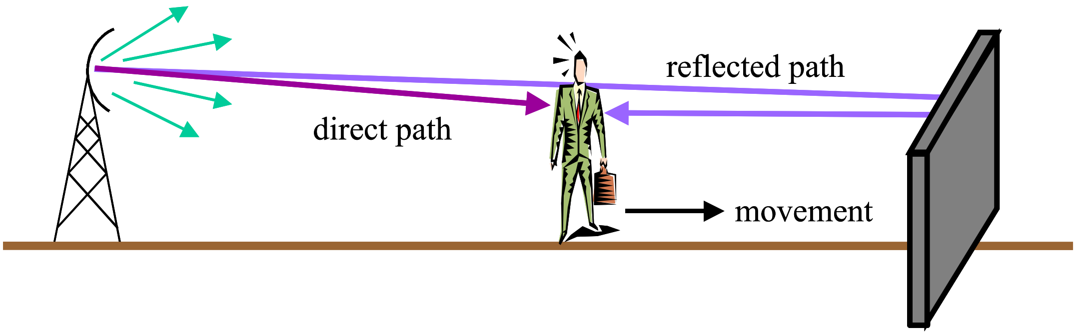

Figure 1-7 A Man Walking Towards a Wall

If the man is walking towards the wall, then the reflected ray will have a positive Doppler shift[3] of u/λ, and the direct ray will have a negative Doppler shift of $-u/\lambda$, where u is the speed of the man's walking, and λ is the wavelength. That gives a Doppler spectrum of:

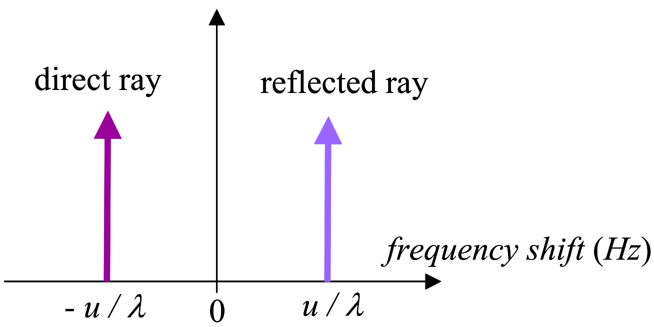

Figure 1-8 Doppler Spectrum of Power Received by Man Walking Towards a Wall

and the Doppler spread in this case is particularly easy to calculate, since we can use the fact that there are only two rays, and the mean value of the Doppler shift is zero (since the rays have the same power and opposite frequency shifts), so the formula simplifies to[4]:

[3] See the chapter on "Propagation for Mobile Radio" if you're unsure about the Doppler shift.

$$D = \sqrt{\frac{\sum_i P_i(\omega - 0)^2}{\sum_i P_i}} = \sqrt{\frac{P_i(\omega - 0)^2 + P_i(-\omega - 0)^2}{2P_i}} = \sqrt{\frac{\omega^2 + \omega^2}{2}} = \omega \qquad (0.14)$$

which means that here, the rms Doppler spread is just the frequency offset of either ray = u/λ Hz. For some more complex examples, see the problems.

1.2.3 The Classic Doppler Spectrum

There's one very common Doppler spectrum that follows directly from the multipath models we've been using, with lots of small scattering objects uniformly spread around the receiver. In this case, the probability distribution of the angle of arrival of the energy is uniform: power is equally likely to arrive at the receiver from any direction.

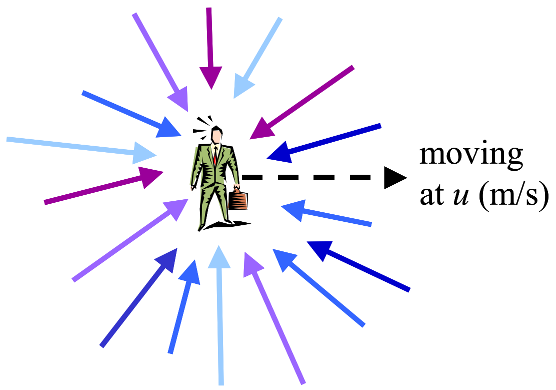

Figure 1-9 Uniform Angle of Arrival Probability

With a uniform distribution of scatterers, the probability that a ray arrives at angle between θ and $\theta + \delta\theta$ is $\delta\theta / 2\pi$. The Doppler shift of rays arriving between these two angles is somewhere between:

$$\begin{aligned} \xi &= \frac{u\cos(\theta)}{\lambda} \\ \xi + d\xi &= \frac{u\cos(\theta + d\theta)}{\lambda} \end{aligned} \qquad (0.15)$$

and therefore we can work out the probability that any given ray will arrive with a given Doppler shift. It's not quite as straightforward a calculation as we might wish, since there are two angles that give the same Doppler shift (θ and $-\theta$) and to work out the total probability of getting any particular Doppler shift, we need to add up the probabilities that the angle of arrival is between θ and $\theta + \delta\theta$ and between $-\theta$ and $-\theta - \delta\theta$. Fortunately, since the angle of arrival has a uniform probability density in this case, these probabilities are the same: $\delta\theta / 2\pi$.

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[4] Note that this formula uses the symbol ω which usually means an angular frequency in terms of rad/s is assumed. You can certainly use this formula like this, and it will produce an output rms Doppler spread in terms of rad/s also. However, the formula doesn't care what units the frequency shift is expressed in, provided they are all consistent. I could equally well use Doppler shifts in Hz, and the formula would produce an output Doppler spread also in Hz.

Considering only the positive angles, the probability that the Doppler shift is between ξ and $\xi + d\xi$ is[5]:

$$p(\xi)d\xi = -p(\theta)d\theta \tag{0.16}$$

but since the angle of arrival is equally likely to be negative, the probability of getting this Doppler shift is exactly twice this:

$$p(\xi)d\xi = -2p(\theta)d\theta \tag{0.17}$$

and therefore:

$$\begin{aligned} p(\xi) &= -2p(\theta)\frac{d\theta}{d\xi} = -\frac{1}{\pi}\left(\frac{d\xi}{d\theta}\right)^{-1} = \frac{1}{\pi}\left(\frac{u}{\lambda}\sin(\theta)\right)^{-1} \\ &= \frac{1}{\pi}\frac{\lambda}{u}\frac{1}{\sqrt{1-\cos^2(\theta)}} \\ &= \frac{1}{\pi}\frac{\lambda}{u}\frac{1}{\sqrt{1-\left(\frac{\lambda\xi}{u}\right)^2}} = \frac{\lambda}{\pi}\frac{1}{\sqrt{u^2-(\lambda\xi)^2}} \end{aligned} \tag{0.18}$$

This probability density function looks like this:

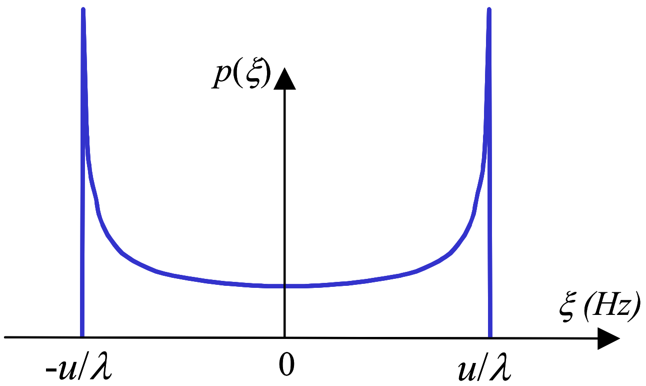

Figure 1-10 The 'Classic' Doppler Spectrum

Note that you are more likely to have power arriving at the extreme values of the possible frequency spread than in the centre. If you have no other information about the angles of arrival of the rays in a multipath case, this is the Doppler spectrum to use.

The rms Doppler spread in this case is calculated by integrating over the possible range of frequency shifts, and this gives the answer:

$$D = \frac{u}{\sqrt{2}\lambda} \tag{0.19}$$

[5] Note the factor of minus one in this equation. This is here because as the angle increases, the Doppler shift decreases, so a small change $d\theta$ in the angle of arrival corresponds to a negative change $-d\xi$ in the Doppler shift.

(for the derivation of this, see the problems).

1.3 Coherence Time and Coherence Distance

Just as the plot of the frequency response of the channel allowed us to define a coherence bandwidth (the range of frequencies over which the response of the channel does not significantly change), the plot of the Doppler spectrum allows us to define a coherence time (the period of time over which the channel does not change) and usually a coherence distance (the distance the receiver can move without experiencing a different channel).

The calculation is similar to the one used to derive the coherence bandwidth. The expected value of the correlation between the impulse responses with the same delay but at two different times could be expressed in terms of the inverse Fourier transform of the impulse responses:

$$\mathrm{E}\left\{h^*\left(\tau_i,t_1\right)h\left(\tau_i,t_2\right)\right\}=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{E}\left\{G_i^*\left(\omega_1\right)G_i\left(\omega_2\right)\right\}\exp\left(j\left(\omega_2 t_2-\omega_1 t_1\right)\right)d\omega_1 d\omega_2 \quad (0.20)$$

We can't proceed in quite the same way as we did when calculating the coherence between two frequencies, since the values of $G_i(\omega)$ (the Fourier transform of the complex impulse responses of the channels for a delay τ_i) are not independent for different frequencies: if the two frequencies are close together, the values of $G(\omega_1)$ and $G(\omega_2)$ are likely to be similar too.

However, if we're using the 'classic' Doppler spectrum scenario from section 1.2.3, we can work out the integral more directly for this case. Consider just one ray, arriving from an angle θ to the direction of movement of the receiver. A single frequency transmitted by the transmitter will result in a waveform at the receiver of:

$$x_i\left(t\right)=a_i\cos\left(\omega_c t+\phi_i\right) \quad (0.21)$$

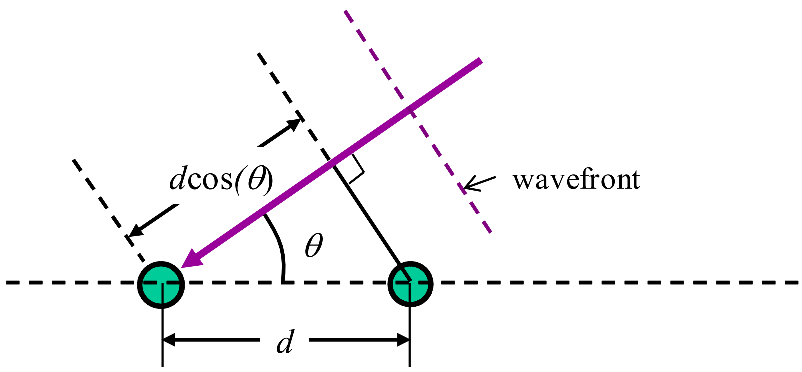

Figure 1-11 Two Receivers and the Phase Difference from an Incoming Ray

A receiver a distance d away would receive this signal a short time $d\cos(\theta)/c$ earlier where c is the speed of light, in other words a signal given by:

$$y_i\left(t\right)=a_i\cos\left(\omega_c\left(t-\frac{d\cos\left(\theta\right)}{c}\right)+\phi_i\right) \quad (0.22)$$

The correlation between these two signals is:

$$E\left\{x_i(t)\,y_i(t)\right\} = E\left\{a_i^2\cos\left(\omega_c t + \phi_i\right)\cos\left(\omega_c\left(t - \frac{d\cos(\theta)}{c}\right) + \phi_i\right)\right\} \tag{0.23}$$

and using the well-known identity for the product of two cosines:

$$E\left\{x_i(t)\,y_i(t)\right\} = \frac{a_i^2}{2}E\left\{\cos\left(\omega_c\frac{d\cos(\theta)}{c}\right) + \cos\left(2\omega_c t - \omega_c\frac{d\cos(\theta)}{c}t + \phi_i\right)\right\} \tag{0.24}$$

The second term on the right-hand side is the mean value of a cosine wave over a totally random phase angle ϕ (which is equally likely to be anything), and is therefore zero. That just leaves:

$$E\left\{x_i(t)\,y_i(t)\right\} = \frac{a_i^2}{2}\cos\left(\omega_c\frac{d\cos(\theta)}{c}\right) = \frac{a_i^2}{2}\cos\left(\frac{2\pi d\cos(\theta)}{\lambda}\right) \tag{0.25}$$

where λ is the wavelength of the carrier frequency.

Of course there isn't just one ray being received, there's a huge number of them coming from all possible directions. Since they can be assumed to be arriving with entirely different and independent phases, the correlation between any two of these rays (the mean value of the product) will be zero, so the total correlation is just the sum of the correlations of each of the individual rays[6].

If the sum of all rays arriving at the first location is:

$$x(t) = \sum_i x_i(t) \tag{0.26}$$

and at the second location is:

$$y(t) = \sum_i y_i(t) \tag{0.27}$$

then the total correlation is:

$$E\left\{x(t)\,y(t)\right\} = \sum_i \frac{a_i^2}{2}\cos\left(\frac{2\pi d\cos(\theta)}{\lambda}\right) \tag{0.28}$$

[6] For example, consider the sum of two rays, $a(t) = \cos(\omega t + \alpha)$ and $b(t) = \cos(\omega t + \beta)$. The expectation value of the correlation between these rays is $E\{\cos(\omega t + \alpha)\cos(\omega t + \beta)\}$, where the average is taken over all possible values of α and β. This gives:

$$E\left\{\cos(\omega t + \alpha)\cos(\omega t + \beta)\right\} = \frac{1}{2}E\left\{\cos(2\omega t + \alpha + \beta)\right\} + \frac{1}{2}E\left\{\cos(\alpha - \beta)\right\}$$

and that's the sum of the average value of two cosines, averaged over all possible angles. Which is zero, because cosines have an average value of zero. In just the same way, the expectation value of the product of two different rays coming from two different angles is zero. That only leaves finite expectation values where the two rays being compared are coming from the same angle.

Since we are assuming that there are an infinite number of rays coming from all possible angles, and the probability distribution of them coming from any small range of angles $p(\theta)$ is $d\theta / 2\pi$, the summation can be replaced by an integral, and the correlation becomes:

$$E\{x(t)y(t)\} = \int_{-\pi}^{\pi} \cos\left(\frac{2\pi d \cos(\theta)}{\lambda}\right) p(\theta) d\theta = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos\left(\frac{2\pi d \cos(\theta)}{\lambda}\right) d\theta \qquad (0.29)$$

The solution to this integral is a Bessel function of the first kind with order zero:

$$E\{x(t)y(t)\} = J_0\left(\frac{2\pi d}{\lambda}\right) \qquad (0.30)$$

which looks like this:

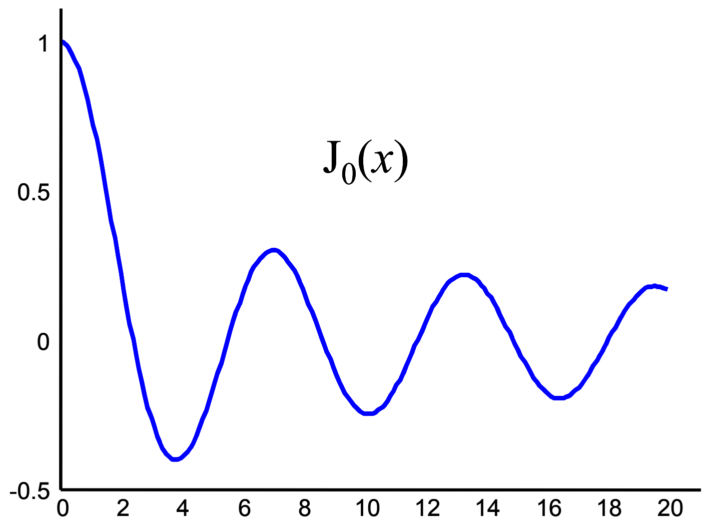

Figure 1-12 Bessel Function of the First Kind, Zeroeth Order

This function has the first zero (indicating completely independent fading) for a parameter of approximately 2.41. So, if we want to ensure that the fading between two points is uncorrelated, we need them to be separated by around:

$$d = \frac{2.41\lambda}{2\pi} = 0.384\lambda \qquad (0.31)$$

While the correlation between the fading at two points drops to zero after this distance, it then becomes negative, and reaches –0.4 at a separation of 0.61λ (indicating that if the signal in one place is strong, the signal a distance 0.61λ away is also likely to be fairly large, but with the opposite phase). Conversely, if the signal at one point is in a deep fade, the signal a distance 0.61λ away is more likely than not to also be in a deep fade.

This suggests there is an optimum distance apart to put two antennas when using them to provide receive diversity. Ideally the two antennas should be spaced so that if one is in a deep fade the other has the best possible chance of not being in a fade, and that suggests a spacing of 0.384λ where the correlation in the fading is zero.

This is one definition of the coherence distance: the distance two antennas have to be apart to experience uncorrelated fading. Just as with coherence bandwidth there are a range of definitions of coherence distance, depending on just how uncorrelated the fading has to be.

For example: in practice, a correlation of less than 0.5 gives almost as good performance as completely uncorrelated fading, and that suggests a separation of $\lambda/4$ is good enough.

For a typical mobile phone operating with a carrier frequency of around 2 GHz, this suggests a minimum distance between two antennas of around 4 cm. Operate at 900 MHz, and the minimum separation increases to 8.3 cm.

This is one reason why MIMO techniques[7] haven't really caught on with mobile phones: the trend is to make mobile phones smaller and lighter, and there just isn't room for two antennas this far apart.

1.3.1 Coherence Time

If a channel has a coherence distance of d, then a user moving at u m/s will move through this distance in a time d/u, and hence have a coherence time of d/u. In the 'classic Doppler' case described above, this leads to a correlation between the channels received by the same (moving) antenna after a time τ of:

$$E\{x(t)y(t)\} = J_0\left(\frac{2\pi u \tau}{\lambda}\right) \qquad (0.32)$$

and hence a coherence time (assuming a definition that requires a correlation of zero between the fading at the two times) of:

$$t = \frac{d}{u} = \frac{2.41\lambda}{2\pi u} = \frac{0.384\lambda}{u} \qquad (0.33)$$

Compare this to the rms Doppler spread of this channel:

$$D = \frac{u}{\sqrt{2}\lambda} \qquad (0.34)$$

and it can be easily shown that:

$$t = \frac{0.384\lambda}{u} = \frac{0.384}{D\sqrt{2}} = \frac{0.27}{D} \qquad (0.35)$$

and the coherence time is inversely proportional to the rms Doppler spread.

The relationship between Doppler spectrum and coherence time is very similar to that between rms delay spread and coherence frequency, and similar rules-of-thumb can be used. While the constant of proportionality is a function of the correlation required and the scenario, it's generally true that the coherence time is inversely proportional to the Doppler spread for any channel.

[7] Multiple Input Multiple Output. You can increase the capacity of a wireless link by having more than one antenna at the transmitter and the receiver, provided they are sufficiently far apart that they don't just transmit or receive the same signal.

1.4 The Scattering Function

Back in section 1.2 we calculated the power spectral density of the i^{th} ray affected by a fading channel and found that it could be expressed as:

$$\left|Y(\tau_i,\omega)\right|^2 = \frac{1}{4}\left|G_i(\omega-\omega_c)\right|^2 + \frac{1}{4}\left|G_i^*(-\omega-\omega_c)\right|^2 \tag{0.36}$$

where $G_i(\omega)$ is the Fourier transform of the amplitude and phase of the arriving ray relative to the transmitter ray. The positive frequency components of this power spectral density can be determined from the power spectral density of the complex signal representing the amplitude and phase of this ray:

$$S_i(\omega) = \left|G_i(\omega)\right|^2 \tag{0.37}$$

This is just for one ray: the i^{th} ray arriving after a delay of τ_i. If we include all of the arriving rays with different delays, we can define a function of delay and frequency shift that represents the Doppler spectra of all of the arriving rays:

$$S(\omega,\tau) = \sum_i \left|G_i(\omega)\right|^2 \delta(\tau-\tau_i) \tag{0.38}$$

This is known as the scattering function. It's a very useful way to plot channels, since every ray arriving at the user is represented on a plot of the scattering function by a single peak. For example, consider that man walking towards the wall again.

The ray reflected from the wall will have a slightly higher frequency than the ray arriving directly from the transmitter, due to the Doppler shifts (the man is walking towards the wall, but away from the transmitter), however it will arrive slightly later (since it has further to travel). Plotting the scattering function for this channel would result in a plot looking like this:

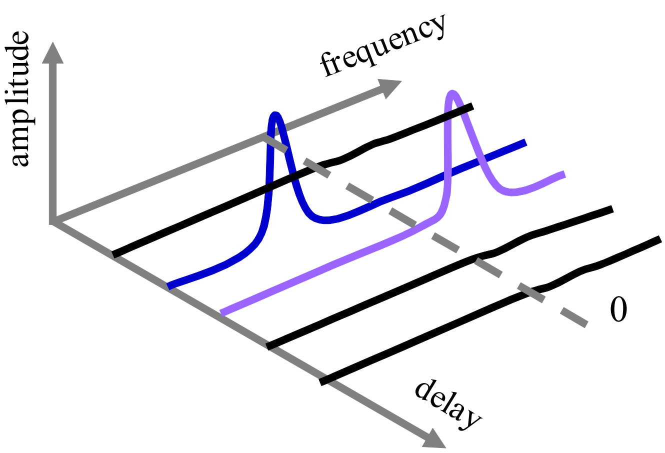

Figure 13 Scattering Function for Man and Wall

It's immediately obvious from this plot that the first ray to arrive has a negative Doppler shift, and the second ray to arrive has a positive Doppler shift.

The scattering function has all the information in the power delay profile and the Doppler spectrum in it. Add up all powers in all the rays arriving for each delay, and you get the power delay profile:

$$P(\tau) = \int_{-\infty}^{\infty} S(\omega,\tau)\,d\omega \qquad (0.39)$$

Equally, add up all the power in all the rays arriving with different delays but with the same Doppler shift, and you get the Doppler spectrum:

$$G(\omega) = \int_{0}^{\infty} S(\omega,\tau)\,d\tau \qquad (0.40)$$

1.4.1 Another Example of a Scattering Function

If you consider the scenario of Figure 1-1 with three groups of rays arriving at a moving receiver in a dense multipath environment, the scattering function would look like this:

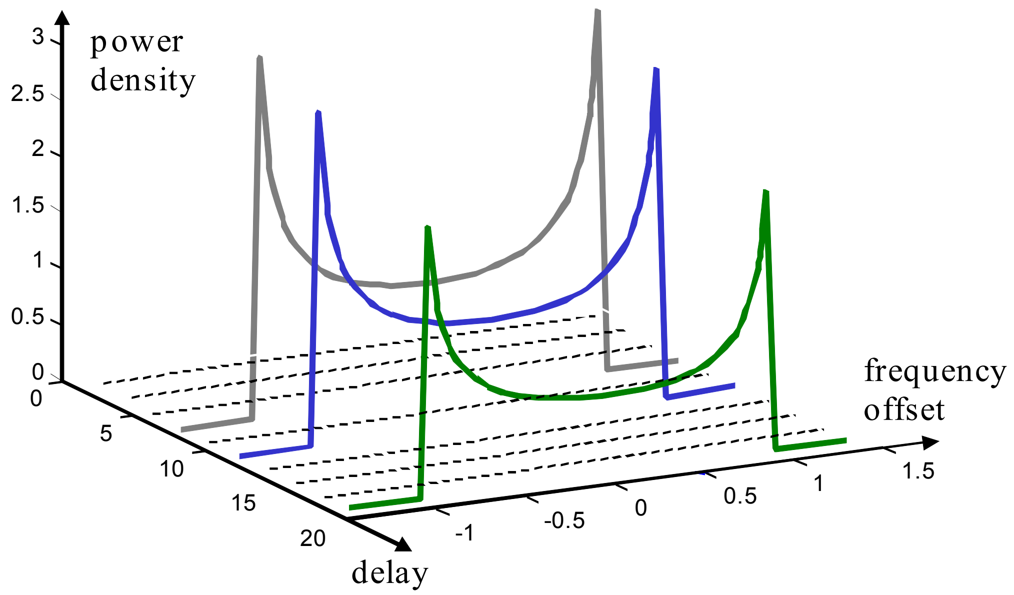

Figure 14 Scattering Function for User Moving in Environment from Figure 1-1

Each of the three rays has the classic Doppler spectrum, but no power is received at any delay times other than the three delays in the impulse response from Figure 1-2. The graphs is normalised to a maximum frequency deviation of one, the real frequency deviations will be a factor u / λ greater than this, where u is the velocity of the user, and λ is the wavelength.

For indoor environments and dense urban environments (for example in the middle of cities), there can be a huge number of rays arriving from all directions, and some line-of-sight paths that appear as discrete peaks in the scattering diagram.

1.5 Four Channel Types

Combining the results in the previous chapter with the results from this one, we can imagine four extreme examples of different channel, classified in terms of their delay spread and coherence time. (That's delay spread compared to the length of a symbol, and coherence time compared to the length of time it takes to do something about the fading.) All real channels will lie somewhere between these four extremes. I can illustrate the idea with a diagram:

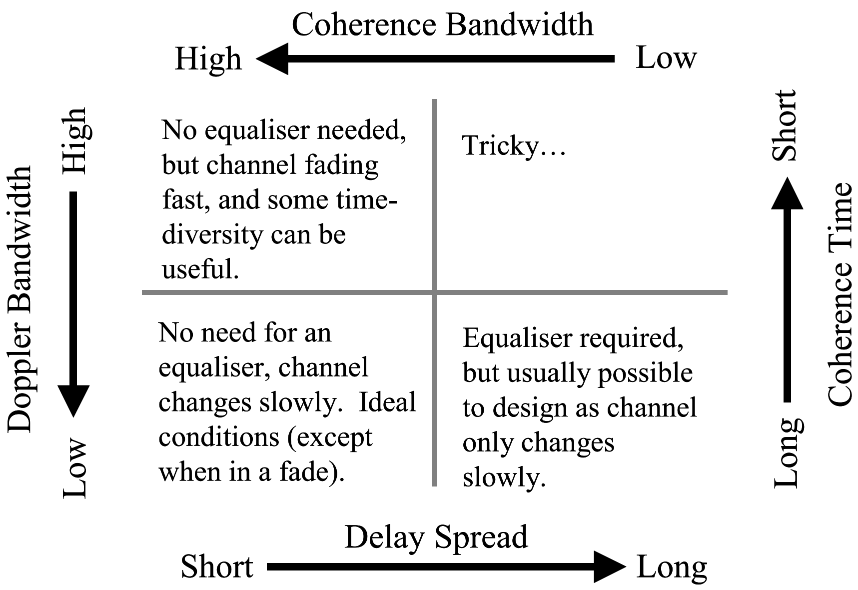

Figure 15 The Four Extreme Channel Types

A few notes about the four extreme types of channel (again, remember that real channels are likely to lie somewhere between these extremes).

1.5.1 Long Delay Spread and Long Coherence Time

In the frequency domain this implies a wideband channel (one with a coherence bandwidth narrower than the transmitted signal's spectrum), and a very stable channel that fades slowly.

The wideband channel suggests a lot of distortion in the radio channel, which results in intersymbol interference. Provided there isn't too much of this, an equaliser can be used to improve the quality of the link, and since the fading is so slow, there should be plenty of time to characterise the channel and work out a good equaliser structure to use.

Fading is very slow, but since this is a wideband channel, not all of the frequencies in the transmitted spectrum will be faded at the same time. Some energy will get through, and there are a range of techniques that can be used to make sure that the data can be recovered from the energy that does make it to the receiver, including RAKE receivers and coded OFDM modulation[8].

1.5.2 Long Delay Spread and Short Coherence Time

In the frequency domain this implies a wideband channel (one with a coherence bandwidth narrower than the transmitted signal's spectrum), and an unstable channel that fades quickly.

[8] See the chapter on CDMA for more about RAKE receivers, and the chapter on OFDMA for more about the use of coded OFDM in mobile communications.

This is the nightmare channel. Lots of intersymbol interference, and yet a channel that fades so fast that there isn't time to characterise the channel before it changes. There are a couple of techniques that can be used: if the channel is fading very fast, then there is the possibility of using long symbol periods (reducing the problems with intersymbol interference caused by the long delay spread) and letting the fading average itself out. Some form of frequency division multiplexing can then allow reasonable data rates with long symbols, however to ensure no interference between the different sub-carriers, they may have to be spaced some distance apart[9].

1.5.3 Short Delay Spread and Long Coherence Time

In the frequency domain this implies a narrowband channel (one with a coherence bandwidth wider than the transmitted signal's spectrum), and a very stable channel that fades slowly.

Provided you're not in a fade, this is the perfect channel. Short delay spread so negligible intersymbol interference, and long coherence time so the channel remains stable for long periods. The only slight problem is what happens when a fade does (eventually) occur, as fades can last a very long time. The usual solutions are to employ some sort of *diversity*: providing two (or more) versions of the signal that are unlikely to fade at the same time[10].

There are several types of diversity, but the most likely to be useful in these channels are space diversity (providing two antennas at the receiver spaced by the coherence distance apart) and frequency diversity (transmitting the information on more than one frequency, the frequencies being greater than the coherence bandwidth apart). Space diversity is popular provided the receiver[11] is large enough to have two antennas sufficiently far apart, the problem with frequency diversity is that it requires more spectrum, and that's expensive. Then there's always the obvious: just wait until the received signal comes out of its fade. That might be fine for emails, but if you're trying to have a conversation with someone, it's possible you can't wait that long.

1.5.4 Short Delay Spread and Short Coherence Time

In the frequency domain this implies a narrowband channel (one with a coherence bandwidth wider than the transmitted signal's spectrum), and an unstable channel that fades slowly.

An equaliser wouldn't help much since there is no intersymbol interference, however there is that fast fading. If the fading is very fast (much more rapid than a symbol time) the receiver can just average it out, if not, then some symbols are likely to be lost. If it's slower, or occurs over similar time periods to a symbol, some sort of forward error correction coding[12] can be very effective. In both cases, some form of diversity is useful as well.

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[9] See the chapter on Frequency Division Multiplexing for more details about these ideas.

[10] See the chapter on Diversity for much more about the various forms of diversity.

[11] You can also get the advantages of space diversity by using two antennas at the transmitter, and only one at the receiver. For more details about how this works, look up the chapter on Space-Time Codes. It's really clever.

[12] See the chapter on Forward Error Control coding for more about FEC codes.

1.6 Problems

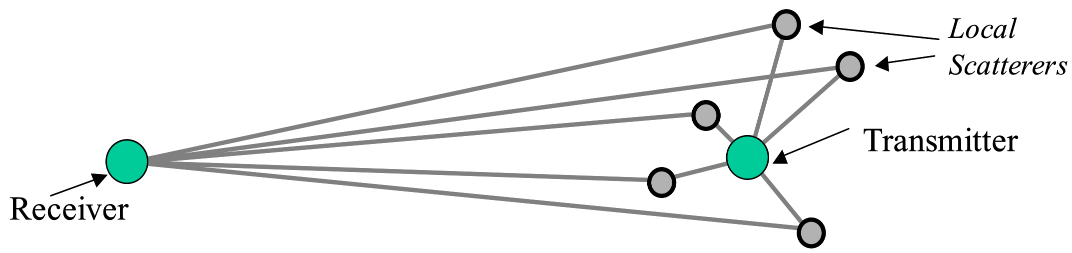
1) A mobile phone user receives three reflected rays – one from the north, one from the east and one from the west. If the frequency of the mobile phone signal is 420 MHz, and the user starts to move North-East with a speed of 3 m/s, what frequencies would he receive?

If all three rays arrive with the same power and after the same delay, sketch the scattering function for this channel, and determine the rms Doppler bandwidth.

2) Consider a mobile phone user receiving two rays at 900 MHz of equal power, one from the north, and one from the west. The user is moving south-west at 1 meter per second. How often do fades occur? If a fade is defined as being a received signal less than 10 dB below the median received signal, how long are these fades?

What is the Doppler spread of this link? What is the coherence time?

3) In the downlink, we can readily imagine that power is arriving at the receiver from all directions with equal probability. However, the same is not true of the uplink. Imagine the case shown in the figure below, where there are a large number of local scatterers around the mobile user, but the base station is on top of a tall building.



In this case, we could assume that the angle of arrival at the base station is equally distributed between $+\theta$ and $-\theta$ degrees only. What is the coherence distance at the base station now? How far apart should two antennas be placed at this base station to provide diversity gain?