1 What You Need To Know About… Basic Maths

Before we can start on any of the more interesting stuff, I thought I'd make sure that everyone has the basic mathematical background necessary to read and understand everything in the rest of the book. One of the goals of the book is to minimise the amount of mathematics required, but there's no getting away from it: a solid grounding in maths is essential to understanding the principles and techniques in modern communications engineering.

This isn't a maths textbook – I'm not going to try and teach anyone maths, but I thought it would be useful to summarise the maths you'll need to know for the rest of the book, so you can read over these first few chapters, and then feel confident that you are well equipped for the journey into the wonderful world of communications engineering. Or at least, you'll know what you're missing, and need to catch up on.

This first chapter of the "what you should know" section contains a brief summary of the basic essential maths knowledge most people do in school, and gives me a chance to introduce some of notation I'll be using the rest of the book. Ready? Then let's begin.

1.1 Variables, Functions and Signals

A variable is a quantity that we can't assign a fixed value to. So we call it something like x or y, and then try to write equations that let us work out the value of x or y. Most often, I'll write variables in italics[1]. For example, if I had a variable x, and I knew that $x + 3 = 9$, then I could work out that in this case $x = 6$. Sadly, most equations aren't that easy.

Functions are operations that transform one or more numbers or variables into other numbers or variables. Most of the functions we'll be meeting have a single output, but some of them take more than one input (note that an input to a function is often called an *argument*). For example, consider a function that takes one argument, and outputs double the input. I could write this as:

$$f(x) = 2x \tag{0.1}$$

where $f(x)$ is this function. When the variable $x = 3$, we just substitute all instances of x with the number 3, and we can calculate:

$$f(3) = 2 \times 3 = 6 \tag{0.2}$$

Another example: a function that takes two arguments, and outputs the square of difference between the two arguments:

$$g(x, y) = \left(x - y\right)^2 \tag{0.3}$$

so that if $x = 2$ and $y = 3$, we get:

$$g(2,3) = \left(2 - 3\right)^2 = \left(-1\right)^2 = 1 \tag{0.4}$$

[1] The exceptions are vector or matrix variables – which I'll write in bold. More about vectors and matrices later.

Notice that this particular function has the property that $g(x, y) = g(y, x)$. Functions with this property are said to be *commutative*. Most functions don't have this property.

A signal is a function of time only that outputs a single value[2]. For example, the voltage on a wire or the sound pressure in the air carrying a sound. They are usually written like $s(t)$ to show they are functions of time.

1.1.1 Symmetry and Periodicity

Some functions have particular properties of interest. For example, a function is said to be *even-symmetric* if for all values of x, $f(x) = f(–x)$, and *odd-symmetric* if for all value of x, $f(x) = –f(–x)$.

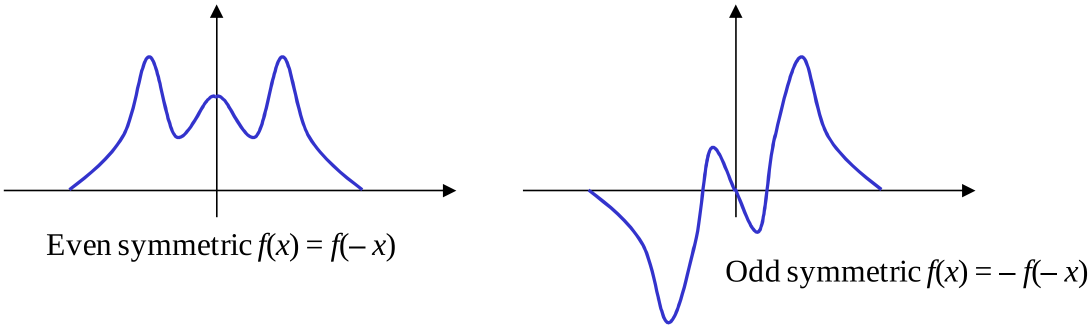

Figure 1-1 Even and Odd Functions

Notice that all odd-symmetric functions must be zero at the origin, in other words $f(0) = 0$.

Interestingly, any function of a single variable can be expressed as the sum of an even-symmetric function and an odd-symmetric function. It's done like this: let $o(x)$ be an odd-symmetric function, and $e(x)$ be an even symmetric function. Then, consider:

$$o(x) = \frac{f(x) - f(-x)}{2} \qquad \text{and} \qquad e(x) = \frac{f(x) + f(-x)}{2}$$

It's straightforward to prove that $o(x)$ is an odd-function, since:

$$o(-x) = \frac{f(-x) - f(x)}{2} = -\frac{f(x) - f(-x)}{2} = -o(x) \tag{0.5}$$

and even easier to prove that $e(x)$ is an even-symmetric function (I'll leave that one to you). It's also easy to show that:

$$o(x) + e(x) = \frac{f(x) - f(-x)}{2} + \frac{f(x) + f(-x)}{2} = f(x) \tag{0.6}$$

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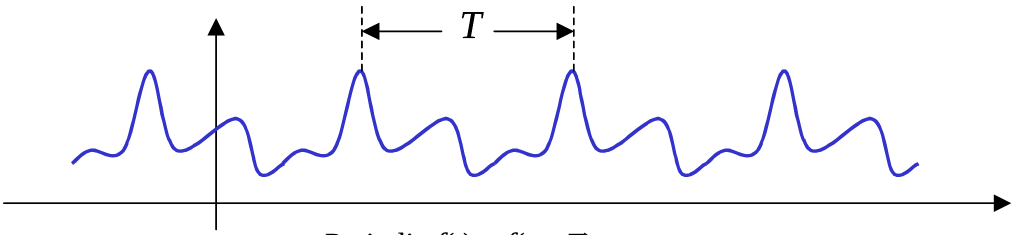
[2] Although the single value might be a vector with several components. There's more about vectors in later chapters, don't worry if that doesn't mean anything to you yet.

This turns out to be a very useful result.

Another interesting property of some functions is *periodicity*. A periodic function is one that obeys $f(t) = f(t + T)$ for all t, where T is known as the *period* of the function[3].



Periodic $f(t) = f(t + T)$

Note that if $f(t) = f(t + T)$, then this also implies that $f(t) = f(t + 2T)$, since:

$$f(t+2T) = f\big((t+T)+T\big) = f(t+T) = f(t) \tag{0.7}$$

and the same is true for $f(t) = f(t + 3T)$ and in fact that $f(t) = f(t + nT)$ for any integer value of n, however the period is taken to be the smallest possible positive offset for which the function repeats.

1.2 Logarithms

The logarithm is a particularly useful function of a single input, which provides a single output value. The only important point is that the input must be a positive number, since it's difficult to take the logarithm of a negative number[4]. The logarithm function is usually written as $\log_b(x)$, where b is the base of the logarithms.

The logarithm of a number x is the power to which a given number (known as the *base*) must be raised to give x. It's usually written as $\log_b(x)$, where b is the base of the logarithms. Therefore,

$$b^{\log_b(x)} = x \tag{0.8}$$

For example, the logarithm (base 10) of 1000 is 3, or in mathematical notation $\log_{10}(1000) = 3$, because $10^3 = 1000$.

Another example, the logarithm (base 2) of 64 is 6, or in mathematical notation $\log_2(64) = 6$, because $2^6 = 64$.

Logarithms are useful, since they allow the time-consuming process of multiplication to be replaced by the much simpler process of addition. This is achieved by noting the most useful property of logarithms: the sum of the logarithms of two numbers is the logarithm of the product of the two numbers. This is easy to prove:

[3] Most periodic functions of interest are functions of time, so I've used the notation $f(t)$ here instead of $f(x)$.

[4] It's not impossible to take the logarithm of a negative number, but the result is a complex number, not a real number. More about this in the chapter on complex numbers.

$$base^{\log(x)+\log(y)} = base^{\log(x)}\, base^{\log(y)} = xy = base^{\log(xy)} \tag{0.9}$$

A similar result is true for the subtraction of two logarithms: this provides a simple way to do the even more time-consuming process of dividing two numbers:

$$base^{\log(x)-\log(y)} = base^{\log(x)}\, base^{-\log(y)} = \frac{base^{\log(x)}}{base^{\log(y)}} = \frac{x}{y} = base^{\log(x/y)} \tag{0.10}$$

Of course, you have to find the logarithm of the numbers first, and then take the power of the base to the logarithm of the answer to get the final answer.

Logarithms have some more useful and important properties as well, for example if you know the logarithm base *A* of a number *x*, and you need the logarithm base *B*, then:

$$\log_A(x) = \frac{\log_B(x)}{\log_B(A)} \tag{0.11}$$

and if you know the logarithm base *A* of a number *x*, and you need the logarithm of x^a, then:

$$\log_A(x^a) = a\log_A(x) \tag{0.12}$$

because:

$$A^{\log_A(x^a)} = x^a = \left(x\right)^a = \left(A^{\log_A(x)}\right)^a = A^{a\log_A(x)} \tag{0.13}$$

1.3 Algebra

Algebra is an essential tool in engineering – in particular the ability to simplify expressions, and to express formulas in terms of whichever variables are of most interest. It's important to be very familiar with results such as:

$$\left(x^2 - y^2\right) = \left(x-y\right)\left(x+y\right) \tag{0.14}$$

and to recognise results such as:

$$\left(x^3 - y^3\right) = \left(x-y\right)\left(x^2 + xy + y^2\right) \tag{0.15}$$

which can be proved by multiplying out the right-hand-side:

$$\begin{aligned}\left(x-y\right)\left(x^2+xy+y^2\right) &= \left(x^3+x^2y+xy^2\right) - \left(x^2y+xy^2+y^3\right)\\ &= x^3+x^2y+xy^2-x^2y-xy^2-y^3\\ &= x^3-y^3\end{aligned} \tag{0.16}$$

Changing the variable in an expression is an essential skill – for example you'll need to be able to change the formula:

$$Q = \frac{M}{(n-1) + D + Bn} \tag{0.17}$$

into

$$n = \frac{M - Q(D-1)}{Q(B+1)} \tag{0.18}$$

It's done by doing the same operation to both sides of the equation, until the only term left on the left-hand side is the one you want, in this case *n*. Here, this is accomplished by multiplying both sides by $(n-1) + D + Bn$, then subtracting $Q(D-1)$, and finally by dividing by $Q(B+1)$.

1.3.1 Quadratic Equations

A particularly important result in algebra is the formula for quadratic equations[5]:

$$\begin{aligned} ax^2 + bx + c &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned} \tag{0.19}$$

In example problems, it's useful to be able to spot the factors of simple quadratic equations: for example, if given an equation of the form:

$$x^2 - x - 6 = 0 \tag{0.20}$$

with practice it's possible to solve this by inspection, by noting that:

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[5] This is derived by a process known as "completing the square", and goes like this:

$$ax^2 + bx + c = 0$$

$$ax^2 + bx + \frac{b^2}{4a} - \frac{b^2}{4a} + c = 0$$

$$\left(\sqrt{a}x + \frac{b}{2\sqrt{a}}\right)^2 = \frac{b^2}{4a} - c$$

$$\sqrt{a}x + \frac{b}{2\sqrt{a}} = \pm\sqrt{\frac{b^2}{4a} - c}$$

$$\sqrt{a}x = \frac{-b}{2\sqrt{a}} \pm \frac{\sqrt{b^2 - 4ac}}{2\sqrt{a}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 - x - 6 = (x-3)(x+2) \tag{0.21}$$

and hence that this is solved by $x = 3$ or –2. However, in the real world, the solutions are rarely that simple.

1.4 Problems

1) If $x^3 - 7x^2 + 6x = 0$, what are the possible values of x?

2) Given the formula:

$$3n = \frac{2}{n-1} + 4,$$

solve for all possible values of n. What are the largest and smallest possible values of n that satisfy this equation?

3) What is $\log_x(x^2)$?

4) What is the result of dividing $x^3 + y^3$ by $x + y$?

5) Two numbers when multiplied together give 31.25, but when added together give 15. What are the two numbers?

6) If $\log_x(125) = 3$, what is x?

7) If $\log_a(x) = 4$, and $\log_a(y)$ is 2, what is $\log_a(xy)$?

8) If $(1.05)^x = 2$, take the logarithm of both sides of this equation, and hence solve for x.