Probabilistic modelling and verification using RoboChart and PRISM

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Outline

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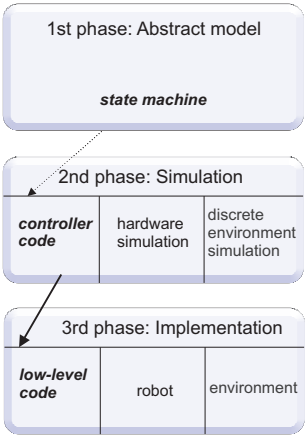
Current practice and problems



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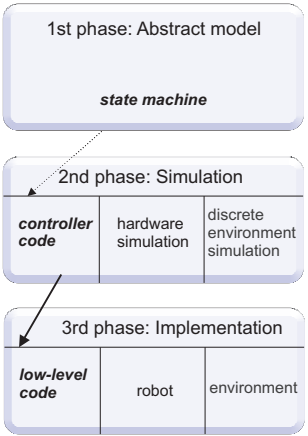
Current practice and problems



- No models, or models without precise syntax or formal semantics,
- Time and uncertainty: discussed informally,
- No tool support,
- Loose connections of artefacts,
- Trial-and-error,
- No assurance.

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RoboStar

- RoboStar framework: modern modelling and verification technologies, software engineering of robotics

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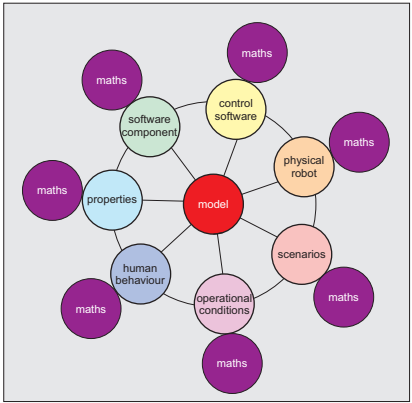
RoboStar

- RoboStar framework: modern modelling and verification technologies, software engineering of robotics
- Vision: model centred, mathematical semantics



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RoboChart

- Core notation of RoboStar, DSL for robotics, state machines

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- A component model (platform independent + parallel composition of state machines)

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- Previous work: modelling and verification of RoboChart with time

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- A component model (platform independent + parallel composition of state machines)
- Previous work: modelling and verification of RoboChart with time
- but not uncertainty

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Uncertainty in robotics

Unpredictable environment, sensors, actuators, model errors, and control algorithmic approximations (EKF SLAM, swarm robots).

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> *"Managing uncertainty is possibly the most important step towards robust real-world robot systems."*
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> — *Sebastian Thrun et al.*

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In RoboChart, we use probabilism to model uncertainty.

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Novel contributions

- Extension of RoboChart with probabilistic junctions,
- RoboChart's probabilistic semantics: given in PRISM,
- A metamodel for PRISM,
- A probabilistic property language,
- Implementation in RoboTool for automated verification,

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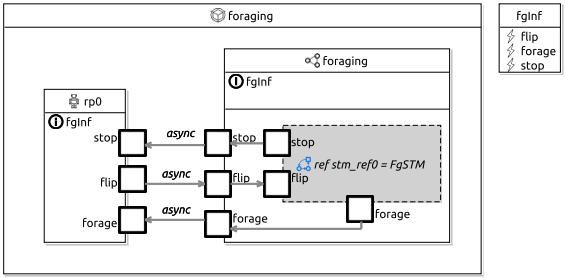
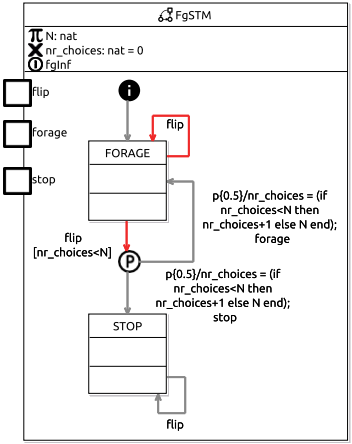
Probabilistic Modelling

A simple foraging robot with a randomising device, every time step (flip), limited number of choices (N)





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Probabilistic Property Language

PRISM's property language (PCTL*) enriched with RoboChart elements

- Computation Tree Logic (CTL)

```
prob property P_deadlock_free:
  not Exists [ Finally deadlock]
  with constant N from set {2 to 20 by step 2}
```

Deadlock freedom for various values of N

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Probabilistic Property Language

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```
prob property P_1:
  Forall  [Globally  ( Finally  (fd==2) and (Next (fd==0)))]
```

For all paths, always eventually fd is 2 and fd is 0 immediately afterwards.

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Probabilistic Property Language

PRISM's property language (PCTL*) enriched with RoboChart elements

- Computation Tree Logic (CTL)
- Linear Temporal Logic (LTL)
- Probabilistic CTL (quantitative)

```
prob property P_min_terminate:
  Prob min=? of [Finally FgSTM is in STOP]
  with constant N from set {2 to 20 by step 2}

prob property P_max_terminate:
  Prob max=? of [Finally FgSTM is in STOP]
  with constant N from set {2 to 20 by step 2}
```

What's the minimum and maximum probabilities of FgSTM finally in state STOP?

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- Computation Tree Logic (CTL)
- Linear Temporal Logic (LTL)
- Probabilistic CTL (quantitative)
- Rewards/costs

```
rewards nr_of_forages =
  [forage.out] true : 1;
endrewards

prob property R_max_stop:
  Reward {nr_of_forages} max=? of [Reachable FgSTM is in
      STOP]
 with constant N set to 10
```

Each synchronisation on event forage costs 1, and what's the maximum expectation of the cost when FgSTM reaches STOP, considering N is 10.

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Probabilistic Property Language

PRISM's property language (PCTL*) enriched with RoboChart elements

- Computation Tree Logic (CTL)
- Linear Temporal Logic (LTL)
- Probabilistic CTL (quantitative)
- Rewards/costs
- Simulations

```
prob property P_max_terminate_sim:
  Prob max=? of [Finally FgSTM is in STOP]
  using sim with CI at alpha=0.01, n=1000, and pathlen=10000
  with constant N from set {2 to 20 by step 2}
```

What's the maximum probabilities of FgSTM finally in state STOP using statistic model checking with method CI (confidence interval)?

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Automated verification in RoboTool

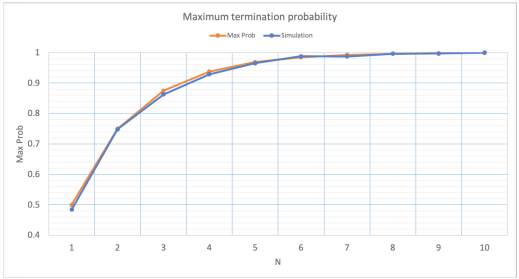
Results of probabilistic analysis of assertions in simulation.assertions using PRISM

Assertion: P_max_terminate

Assertion	Const	states:	transitions:	result:	checkTime:
P_max_terminate	foraging::foraging::stm_ref0::N=2	16	20	0.75	0.004 seconds
P_max_terminate	foraging::foraging::stm_ref0::N=4	30	38	0.9375	0.007 seconds
P_max_terminate	foraging::foraging::stm_ref0::N=6	44	56	0.984375	0.011 seconds
P_max_terminate	foraging::foraging::stm_ref0::N=8	58	74	0.99609375	0.008 seconds
P_max_terminate	foraging::foraging::stm_ref0::N=10	72	92	0.9990234375	0.012 seconds
P_max_terminate	foraging::foraging::stm_ref0::N=12	86	110	0.999755859375	0.013 seconds

Assertion: P_max_terminate_sim

Assertion	Const	states:	transitions:	result:	checkTime:
P_max_terminate_sim	foraging::foraging::stm_ref0::N=2			0.738	
P_max_terminate_sim	foraging::foraging::stm_ref0::N=4			0.947	
P_max_terminate_sim	foraging::foraging::stm_ref0::N=6			0.977	
P_max_terminate_sim	foraging::foraging::stm_ref0::N=8			0.995	
P_max_terminate_sim	foraging::foraging::stm_ref0::N=10			1.0	
P_max_terminate_sim	foraging::foraging::stm_ref0::N=12			1.0	



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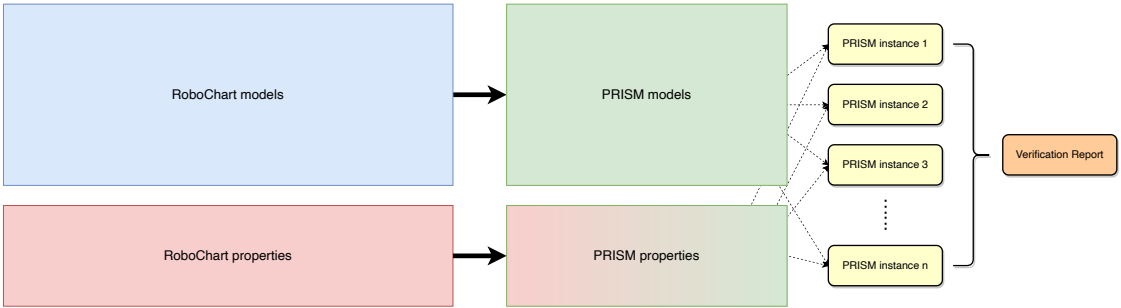
Automated verification in RoboTool - approach



Modelling techniques and mathematical semantics

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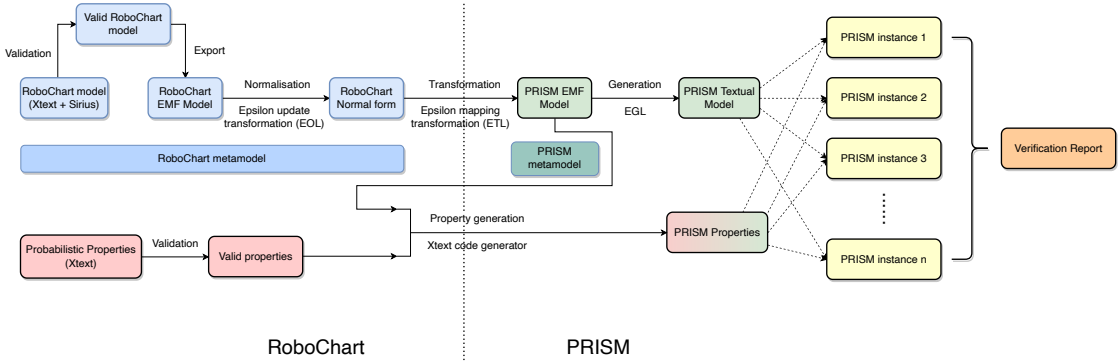

Modelling techniques and mathematical semantics

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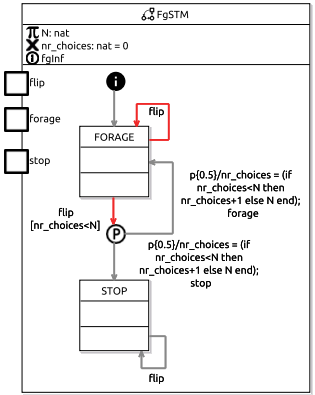
Probabilistic Semantics

- Nondeterministic choice resolved at states,
- Transitions that exit states implicitly or explicitly enter probabilistic choices,
- Given as DTMC and MDP,
- Defined by a formal translation to PRISM: for verification.



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Translation

Challenges

1. Component model,
2. State machines and composite states,
3. Transitions and actions,
4. Communication: input/output triggers and actions,
5. Operations, asynchronous communication, during actions, ...

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Solutions

two-stage translation and its Formalisation:

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Solutions

two-stage translation and its Formalisation:

- normalisation of RoboChart, and

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Solutions

two-stage translation and its Formalisation:

- normalisation of RoboChart, and
- transformation to PRISM.

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Formalisation

Rules: functions and Z notation.

$$[\![x : Tx, y : Ty, z : Tz, \cdots]\!]_{\mathcal{S}} : T_1 \times T_2 \times \cdots$$
$$(e_1, e_2, \cdots)$$

where

$$e_1 \triangleq \cdots$$
$$e_2 \triangleq \cdots$$

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Rule 32. Probabilistic junction (transitions)

$$\frac{[\![n : ProbJunc, cs : NodeContainer, stm : StateMachineDef, exit : VarDecl_{pr}, scpcname : Name, pcconstrs : \mathbb{P}\, BoolExpr_{pr}, stnumber : int, trnumber : int : int \times int \times \mathbb{P}\, Constant \times \mathbb{P}\, Command = (transret.1, transret.2, transret.3, \{cmd\} \cup transret.5)]\!]_{TN/prob}}{\begin{array}{l}\textbf{where} \\ trans = \{t : cs.transitions \mid t.source = n\} \\ transret = [\![trans, n, stm, exit, scpcname, pcconstrs, stnumber, trnumber]\!]_{TS/} \\ cmd = [i \mid (andExprs^{\wedge}(pcconstrs) \,\&\, (scpcname = id(n))) \rightarrow transret.4;\end{array}}$$

12 normalisation rules and 84 transformation rules (RoboChart reference manual)

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Conclusion

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- Extension of RoboChart with probabilistic junction;
- Probabilistic semantics of RoboChart in PRISM;
- Formalisation of semantics by two-stage translation;
- A probabilistic property language;
- Automation and verification support in RoboTool;
- Future work: time, during actions, rich types/expressions (data refinement), more case studies

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Thank you!

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