## Probabilistic modelling and verification, and Animation in RoboChart

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## Probabilistic modelling: probabilistic junctions



ChooseUniformInf
$\mathbf{X}$ c: boolean
$\mathbf{X i}^{\text {i: nat }}$

ROBOSTAR

## Probabilistic verification: probabilistic property language



```
constants C1:
    ransacMOD::ransacRP::N set to 6,
    and ransacMOD::ransacRP::% set to 1/3
prob property P_deadlock_free:
    not Exists [ Finally deadlock]
    with constants C1
prob property P_goodfit:
    Prob=? of [Finally ransacMOD::ransacCTRL::stm_ref0
        is in ransacMOD::ransacCTRL::stm_ref0::goodFit]
prob property P_nr_of_choices:
    Reward {nrchoices} =? of [
        Reachable ransacMOD::ransacCTRL::stm_ref0 is in
            ransacMOD::ransacCTRL::stm_ref0::goodFit]
```


## Probabilistic verification: model checking with PRISM



## Result report

Assertion: P_deadlock_free

| Assertion | states: | transitions: | result: | checkTime: |
| :--- | :--- | :--- | :--- | :--- |
| P_deadlock_free | 3322 | 3742 | true | 0.004 seconds |

Assertion: P_nr_of_tries

| Assertion | states: | transitions: | result: | checkTime: |
| :--- | :--- | :--- | :--- | :--- |
| P_nr_of_tries | 3322 | 3742 | 1.6998561958204306 | 0.055 seconds |

Assertion: P_nr_of_choices

| Assertion | states: | transitions: | result: | checkTime: |
| :--- | :--- | :--- | :--- | :--- |
| P_nr_of_choices | 3322 | 3742 | 2.6998527952527036 | 0.092 seconds |

## Assertion: P_goodfit

| Assertion | states: | transitions: | result: | checkTime: |
| :--- | :--- | :--- | :--- | :--- |
| P_goodfit | 3322 | 3742 | 1.0 | 0.029 seconds |

## Large finite model or infinite model

## State space explosion

Previous example analyses $N=6$. However, if

- $N=100$ : construction (4s) + checking (0.002s);
- $N=10,000: 8 \mathrm{~s}+0.004 \mathrm{~s}$;
- $N=100,000: 830 \mathrm{~s}+0.011 \mathrm{~s}$;
- $N=1,000,000$ : not finished after several hours;

- $N=1, \ldots \ldots \ldots \ldots .$. ?


## Large finite model or infinite model

## Statistical model checking

- Approximate results (vs. exact)
- Monte Carlo simulations (executions)
- Analyse properties on simulations

Random walker $30 \times 30$ squares


## Large finite model or infinite model

## Theorem Proving (UTP, Isabelle/UTP)

For any $N \geq 1$,

$$
\begin{aligned}
& \left(\text { true } \vdash\binom{\left(\forall j \bullet j<(N-1) \Rightarrow\left(\operatorname{prob}^{\prime}(\mathbf{v}[j, \text { false } / i, c]=1 / N)\right)\right) \wedge}{\operatorname{prob} b^{\prime}(\mathbf{v}[(N-1), \text { true } / i, c])=1 / N}\right) \\
& \sqsubseteq \operatorname{ChooseUniform}(N)
\end{aligned}
$$



ChooseUniforminf $\mathbf{X}$ c: boolean $\mathbf{X i}_{\text {i: nat }}$

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\end{aligned}
$$

## Interpretation:



- If $j$ is between 0 and $(N-2), P(i=j)=1 / N$ and $c=$ false
- If $j$ is equal to $(N-1), P(i=j)=1 / N$ and

$$
c=\text { true }
$$

## Large finite model or infinite model

Theorem Proving (epistemic uncertainty)
Bayesian belief model: learn new facts

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Bayesian belief model: learn new facts Imperfect door sensor: 4 times more likely to be right than wrong

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init


Robot's belief

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Theorem Proving (epistemic uncertainty)
Bayesian belief model: learn new facts
Imperfect door sensor: 4 times more likely to be right than wrong
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init || sdoor


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(init || sdoor); mright
((init || sdoor) ; mright) || sdoor
(((init | sdoor); mright) || sdoor); mright
```



Robot's belief

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Theorem Proving (epistemic uncertainty)
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(((init | sdoor); mright) | sdoor); mright
((((init || sdoor) ; mright) | sdoor) ; mright) |
swall
```



Robot's belief

## Large finite model or infinite model

## Theorem Proving (epistemic uncertainty) <br> Bayesian belief model: learn new facts Imperfect door sensor: 4 times more likely to be right than wrong <br> init <br> init || sdoor <br> (init || sdoor) ; mright <br> ((init || sdoor) ; mright) || sdoor <br> (((init || sdoor) ; mright) || sdoor) ; mright ((((init || sdoor) ; mright) || sdoor) ; mright) || swall <br>  <br> Robot's belief

## Animation of RoboChart



```
Starting ITree animation...
Events: (1) RandomWalkCall (); (2) Gas (Din, []); ...;
[Choose: 1-22]: 1
Events: (1) Gas []; (2) Gas [(0,0)]; (3) Gas [(0,1)]; ...;
    (9) Gas [(0,0),(1,1)]; ...; (21) Gas [(1,1),(1,1)];
[Choose: 1-21]: 9
Events: (1) MoveCall (Q,Chemical_Angle_Front);
[Choose: 1-1]: 1
Events: (1) Flag Dout;
[Choose: 1-1]: 1
Terminated: ()
```


## Animation of RoboChart



```
[Choose: 1-22]: 1 RandomWalkCall ()
[Choose: 1-21]: 4 Gas (Din,[(1, 0)])
[Choose: 1-22]: 1 MoveCall (1,Chemical_Angle_Front)
[Choose: 1-24]: 2 Obstacle (Din,Location_Loc_right)
[Choose: 1-23]: 1 Odometer (Din,0)
[Choose: 1-22]: 1 MoveCall (1,Chemical_Angle_Left)
[Choose: 1-21]: }8\mathrm{ Gas (Din,[(0, 0), (1, 0)])
[Choose: 1-22]: 1 MoveCall (1,Chemical_Angle_Front)
[Choose: 1-24]: 1 Obstacle (Din, Location_Loc_left)
[Choose: 1-23]: 2 Odometer (Din,1)
[Choose: 1-23]: 1 Odometer (Din,0)
[Choose: 1-22]: 1 MoveCall (1,Chemical_Angle_Right)
[Choose: 1-21]: 4 Gas (Din,[(1, 0)])
[Choose: 1-22]: 1 MoveCall (1,Chemical_Angle_Front)
[Choose: 1-24]: 2 Obstacle (Din,Location_Loc_right)
[Choose: 1-23]: 1 Odometer (Din,0)
[Choose: 1-22]: 1 Stuck_timeout Din
[Choose: 1-22]: 1 ShortRandomWalkCall ()
```


## Thank you!

https://robostar.cs.york.ac.uk/

