

Probabilistic modelling and verification using RoboChart and PRISM

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Outline

Background and motivations

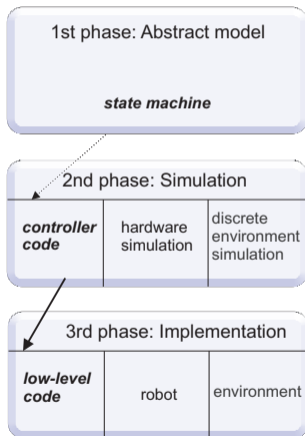
Probabilistic modelling and property language

Automated verification in RoboTool

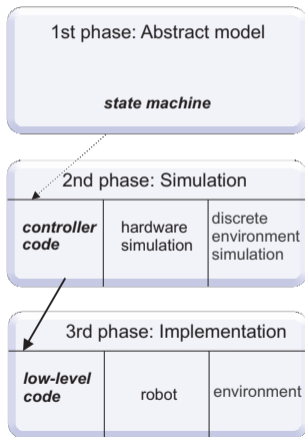
Probabilistic semantics in PRISM

Conclusion

Current practice and problems



Current practice and problems



- ▶ No **models**, or models without **precise syntax** or formal **semantics**,
- ▶ Time and uncertainty: discussed **informally**,
- ▶ No **tool** support,
- ▶ **Loose** connections of artefacts,
- ▶ Trial-and-error,
- ▶ No **assurance**.

RoboStar

- ▶ RoboStar framework: modern **modelling** and **verification** technologies, **software engineering** of robotics

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- ▶ Vision: **model** centred, **mathematical** semantics



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- ▶ A **component** model (platform independent + parallel composition of state machines)
- ▶ Previous work: modelling and verification of RoboChart with **time**

RoboChart

- ▶ Core notation of RoboStar, DSL for robotics, **state machines**
- ▶ A **component** model (platform independent + parallel composition of state machines)
- ▶ Previous work: modelling and verification of RoboChart with **time**
- ▶ but not **uncertainty**

Uncertainty in robotics

Unpredictable environment, sensors, actuators, model errors, and control algorithmic approximations (EKF SLAM, swarm robots).

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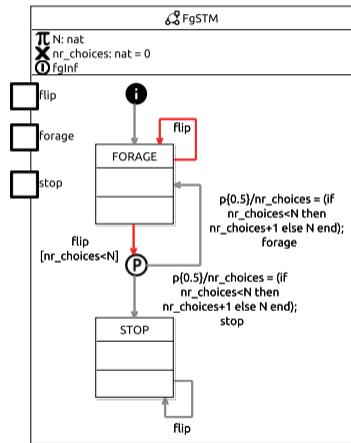
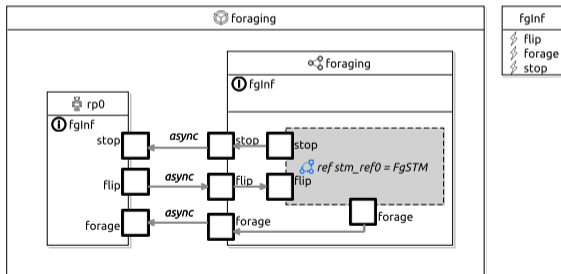
In RoboChart, we use **probabilism** to model uncertainty.

Novel contributions

- ▶ Extension of RoboChart with **probabilistic junctions**,
- ▶ RoboChart's probabilistic **semantics**: given in PRISM,
- ▶ A **metamodel** for PRISM,
- ▶ A probabilistic **property** language,
- ▶ Implementation in RoboTool for **automated** verification,

Probabilistic Modelling

A simple foraging robot with a randomising device, every time step (**flip**), limited number of choices (**N**)



Probabilistic Property Language

PRISM's property language (PCTL*) enriched with RoboChart elements

► Computation Tree Logic (CTL)

```
prob property P_deadlock_free:  
  not Exists [ Finally deadlock ]  
  with constant N from set {2 to 20 by step 2}
```

Deadlock freedom for various values of **N**

Probabilistic Property Language

PRISM's property language (PCTL*) enriched with RoboChart elements

- ▶ Computation Tree Logic (CTL)
- ▶ Linear Temporal Logic (LTL)

```
prob property P_1:  
  Forall [Globally ( Finally (fd==2) and (Next (fd==0)))]
```

For all paths, always eventually **fd** is 2 and **fd** is 0 immediately afterwards.

Probabilistic Property Language

PRISM's property language (PCTL*) enriched with RoboChart elements

- ▶ Computation Tree Logic (CTL)
- ▶ Linear Temporal Logic (LTL)
- ▶ Probabilistic CTL (quantitative)

```
prob property P_min_terminate:  
  Prob min=? of [Finally FgSTM is in STOP]  
  with constant N from set {2 to 20 by step 2}
```

```
prob property P_max_terminate:  
  Prob max=? of [Finally FgSTM is in STOP]  
  with constant N from set {2 to 20 by step 2}
```

What's the minimum and maximum probabilities of **FgSTM** finally in state **STOP**?

Probabilistic Property Language

PRISM's property language (PCTL*) enriched with RoboChart elements

- ▶ Computation Tree Logic (CTL)
- ▶ Linear Temporal Logic (LTL)
- ▶ Probabilistic CTL (quantitative)
- ▶ Rewards/costs

```
rewards nr_of_forages =  
  [forage.out] true : 1;  
endrewards  
  
prob property R_max_stop:  
  Reward {nr_of_forages} max=? of [Reachable FgSTM is in  
    STOP]  
  with constant N set to 10
```

Each synchronisation on event **forage** costs 1, and what's the maximum expectation of the cost when **FgSTM** reaches **STOP**, considering **N** is 10.

Probabilistic Property Language

PRISM's property language (PCTL*) enriched with RoboChart elements

- ▶ Computation Tree Logic (CTL)
- ▶ Linear Temporal Logic (LTL)
- ▶ Probabilistic CTL (quantitative)
- ▶ Rewards/costs
- ▶ Simulations

```
prob property P_max_terminate_sim:  
  Prob max=? of [Finally FgSTM is in STOP]  
  using sim with CI at alpha=0.01, n=1000, and pathlen=10000  
  with constant N from set {2 to 20 by step 2}
```

What's the maximum probabilities of **FgSTM** finally in state **STOP** using statistic model checking with method CI (confidence interval)?

Automated verification in RoboTool

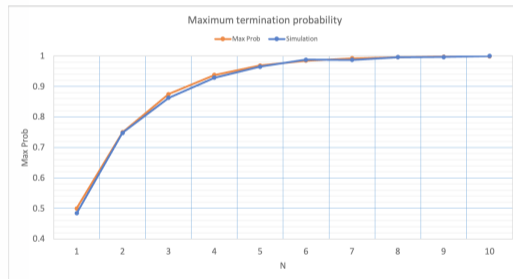
Results of probabilistic analysis of assertions in simulation.assertions using PRISM

Assertion: P_max_terminate

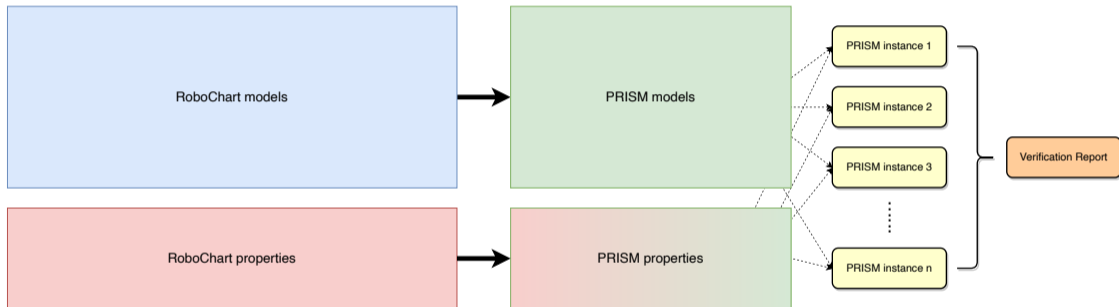
Assertion	Const	states:	transitions:	result:	checkTime:
P_max_terminate	foraging::foraging::stm_ref0::N=2	16	20	0.75	0.004 seconds
P_max_terminate	foraging::foraging::stm_ref0::N=4	30	38	0.9375	0.007 seconds
P_max_terminate	foraging::foraging::stm_ref0::N=6	44	56	0.984375	0.011 seconds
P_max_terminate	foraging::foraging::stm_ref0::N=8	58	74	0.99609375	0.008 seconds
P_max_terminate	foraging::foraging::stm_ref0::N=10	72	92	0.9990234375	0.012 seconds
P_max_terminate	foraging::foraging::stm_ref0::N=12	86	110	0.999755859375	0.013 seconds

Assertion: P_max_terminate_sim

Assertion	Const	states:	transitions:	result:	checkTime:
P_max_terminate_sim	foraging::foraging::stm_ref0::N=2			0.738	
P_max_terminate_sim	foraging::foraging::stm_ref0::N=4			0.947	
P_max_terminate_sim	foraging::foraging::stm_ref0::N=6			0.977	
P_max_terminate_sim	foraging::foraging::stm_ref0::N=8			0.995	
P_max_terminate_sim	foraging::foraging::stm_ref0::N=10			1.0	
P_max_terminate_sim	foraging::foraging::stm_ref0::N=12			1.0	

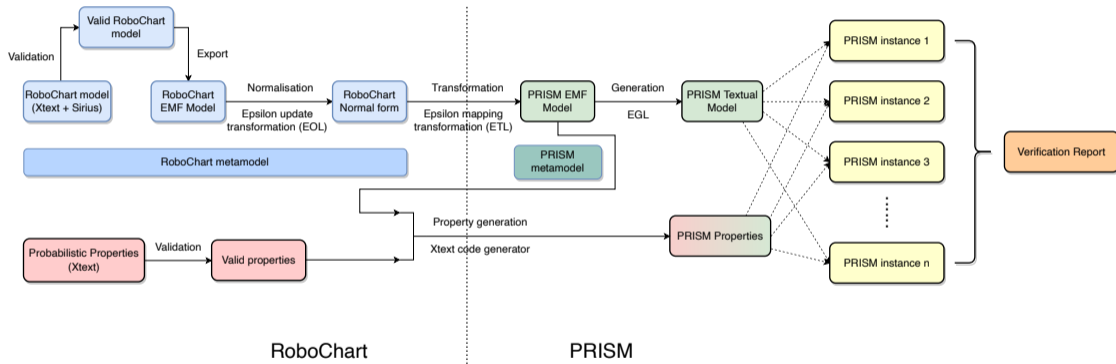


Automated verification in RoboTool - approach



Modelling techniques and **mathematical semantics**

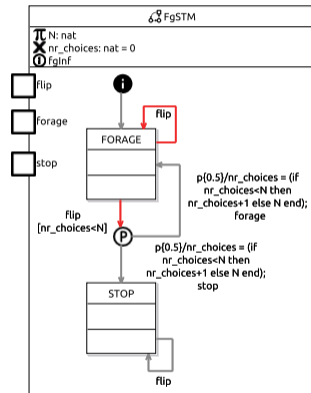
Automated verification in RoboTool - approach



Modelling techniques and **mathematical semantics**

Probabilistic Semantics

- ▶ Nondeterministic choice resolved at **states**,
- ▶ Transitions that **exit** states implicitly or explicitly **enter** probabilistic choices,
- ▶ Given as **DTMC** and **MDP**,
- ▶ Defined by a **formal translation** to PRISM: for verification.



Translation

Challenges

- ① Component model,
- ② State machines and composite states,
- ③ Transitions and actions,
- ④ Communication: input/output triggers and actions,
- ⑤ Operations, asynchronous communication, during actions, ...

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Solutions

two-stage translation and its **Formalisation**:

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Solutions

two-stage translation and its **Formalisation**:

- ▶ **normalisation** of RoboChart, and
- ▶ **transformation** to PRISM.

Formalisation

Rules: functions and Z notation.

$$\llbracket x : Tx, y : Ty, z : Tz, \dots \rrbracket_{\mathcal{S}} : T_1 \times T_2 \times \dots$$
$$(e_1, e_2, \dots)$$

where

$$e_1 \stackrel{\Delta}{=} \dots$$

$$e_2 \stackrel{\Delta}{=} \dots$$

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$$e_1 \triangleq \dots$$

$$e_2 \triangleq \dots$$

Rule 32. Probabilistic junction (transitions)

$$\left[\begin{array}{l} n : \text{ProbJunc}, cs : \text{NodeContainer}, stm : \text{StateMachineDef}, exit : \text{VarDecl}_{pr}, \\ scpname : \text{Name}, pconstrs : \mathbb{P} \text{ BoolExpr}_{pr}, stnumber : \text{int}, trnumber : \text{int} \\ : \text{int} \times \text{int} \times \mathbb{P} \text{ Constant} \times \mathbb{P} \text{ Command} = \end{array} \right]_{TV_{prob}}$$

(transret.1, transret.2, transret.3, {cmd} \cup transret.5)

where

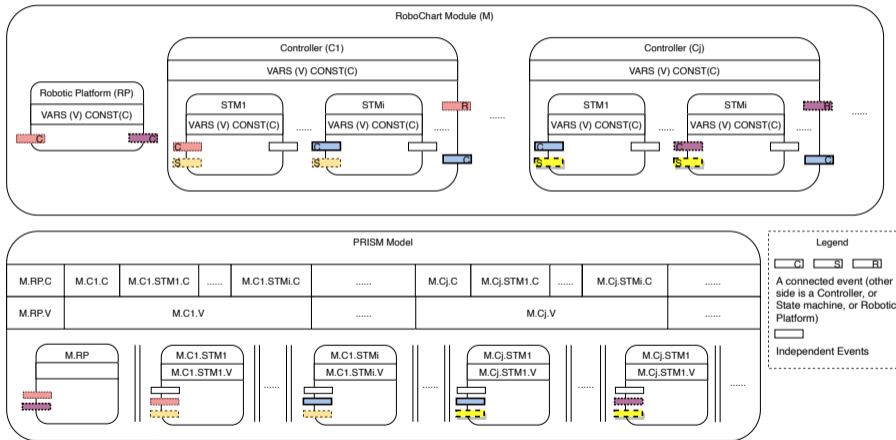
trans = {t : cs.transitions | t.source = n}

transret = \llbracket trans, n, stm, exit, scpname, pconstrs, stnumber, trnumber \rrbracket_{TS}

cmd = \llbracket (andExprs⁰(pconstrs) & (scpname = id(n))) \rightarrow transret.4;

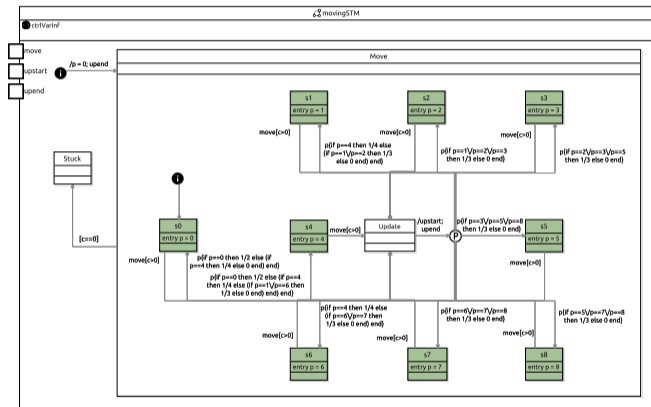
12 normalisation rules and 84 transformation rules (RoboChart reference manual)

Translation: ① component model



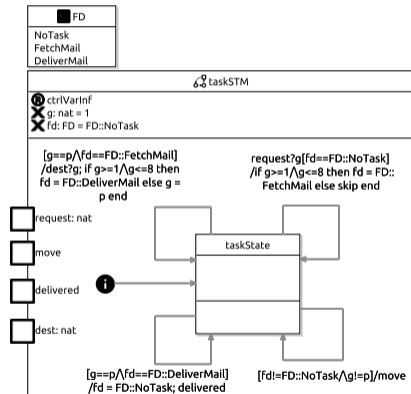
Translation: ② state machines and composite states

- ▶ Counter **scpc** for each machine and composite state,
- ▶ Conjunction of counters,
- ▶ **exit**: six stages,
 - ▶ None,
 - ▶ Requested,
 - ▶ Request substate,
 - ▶ Waiting,
 - ▶ Substate exited,
 - ▶ Exited.



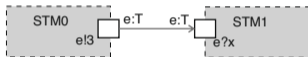
Translation: ③ transitions and actions

- ▶ Transition **lock** for each machine,
- ▶ One transition corresponding to multiple commands in PRISM,
- ▶ **lock** avoids interfere from other transitions,
- ▶ A transition taken, other transitions in the machine not to be taken till the transition is **completed**
- ▶ Suitable for composite states

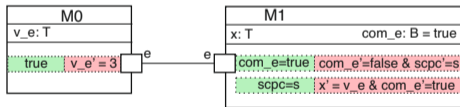


Translation: ④ input and output communication

- ▶ A variable (output): **value** for exchange,
- ▶ A boolean variable (input): exchange **finished**?

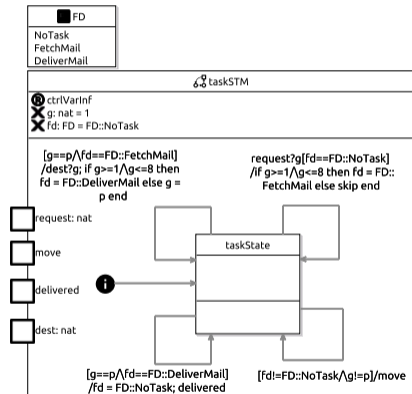


RoboChart state machines
with a connection for exchange



Communication in
PRISM modules

v_e : the variable to store the value for exchange
 com_e : a boolean variable denoting if exchange is complete or not



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- ▶ A probabilistic **property** language;
- ▶ **Automation** and **verification** support in RoboTool;
- ▶ Future work: time, during actions, rich types/expressions (data refinement), more case studies

Thank you!

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