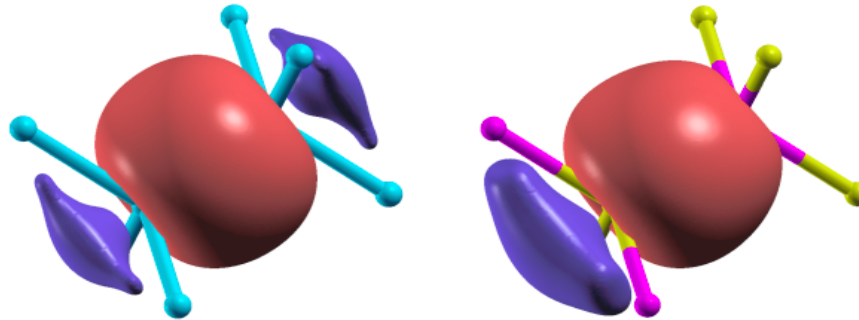


THE UNIVERSITY *of York*

Electron energy loss spectroscopy (EELS)



Phil Hasnip

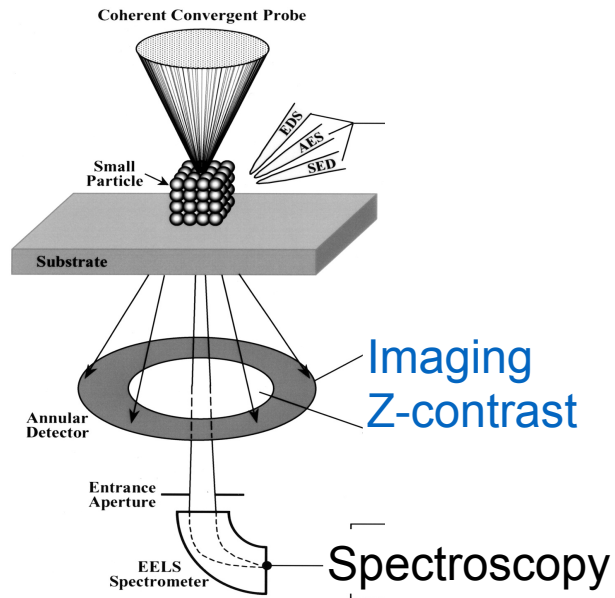
Condensed Matter Dynamics Group

Department of Physics,

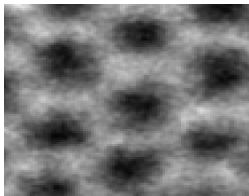
University of York, U.K.

<http://www-users.york.ac.uk/~pjh503>

Many slides courtesy of Jonathan Yates (University of Oxford)



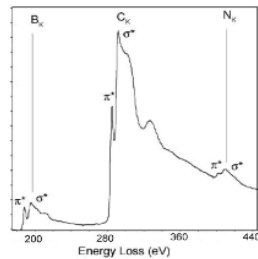
Imaging



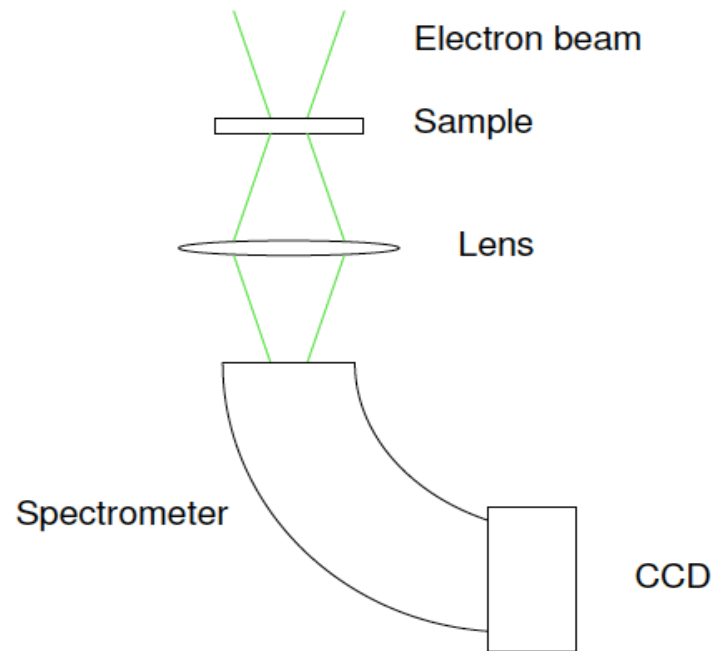
Diffraction



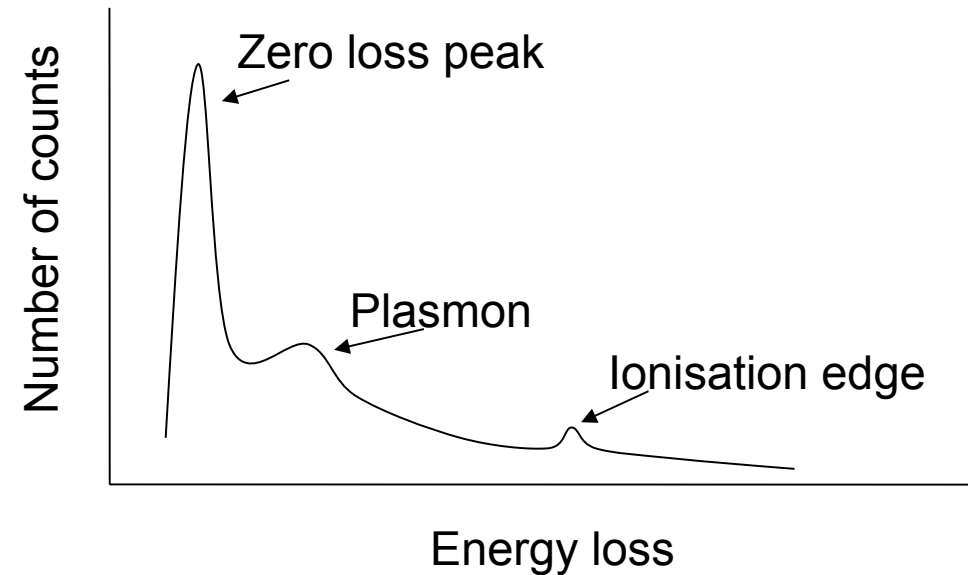
Spectroscopy



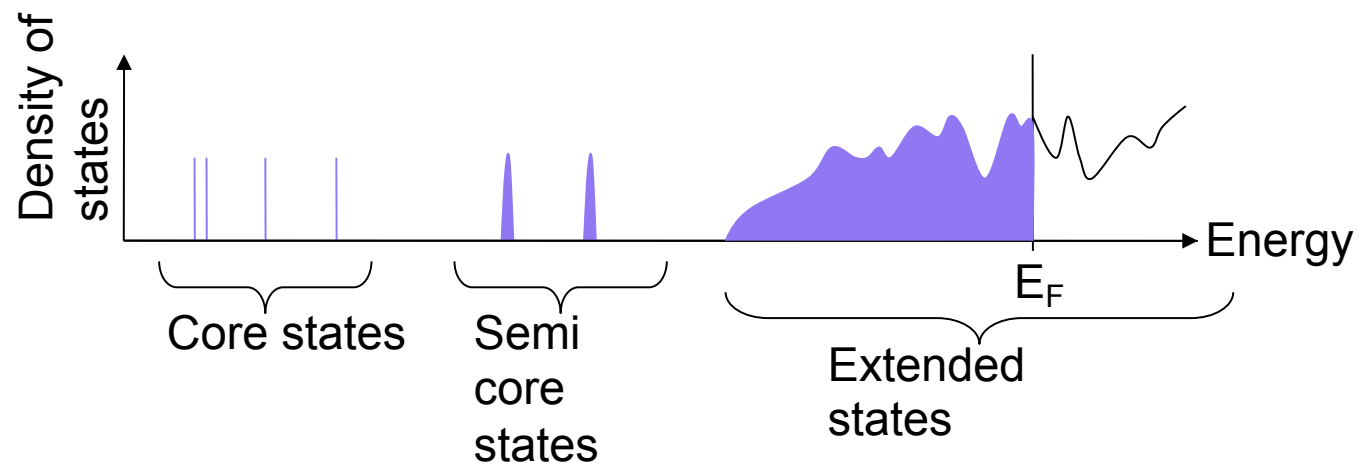
SuperSTEM II - Daresbury

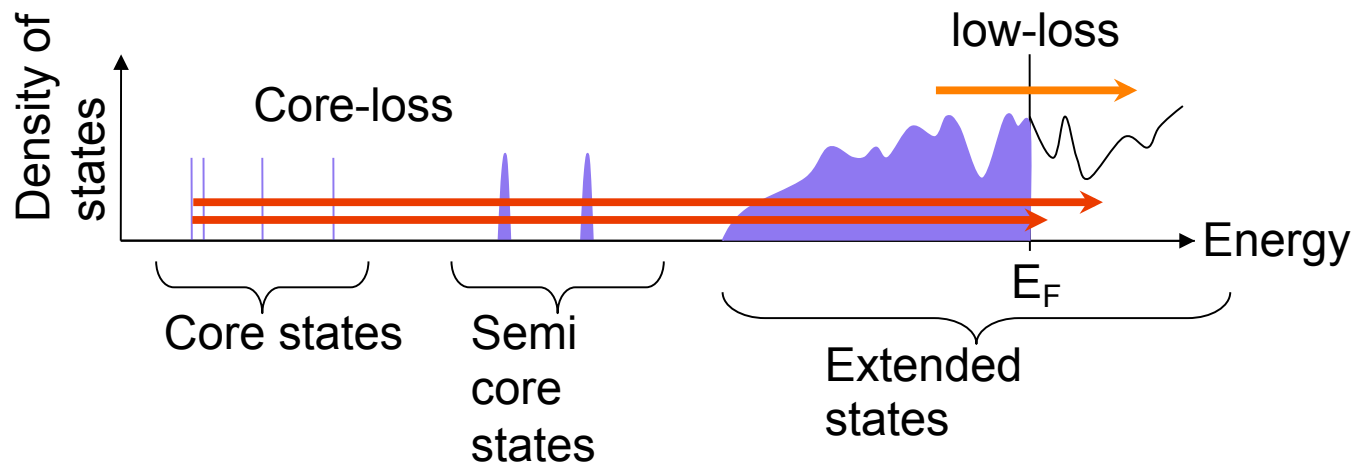


Plasmons $\sim 5\text{-}30\text{ eV}$
Single excitations $\sim 50\text{-}2000\text{ eV}$



- Phonon Scattering
- Plasmon Scattering
- Single electron excitation
- Direct Radiation losses





Fermi's Golden rule:

$$T_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle f | H | i \rangle|^2 \rho$$

The dielectric function $\varepsilon(\omega, \mathbf{q})$ tells us about the response of a material to an electric field

$$\mathbf{D}(\omega, \mathbf{q}) = \varepsilon(\omega, \mathbf{q}) \mathbf{E}(\omega, \mathbf{q})$$

$$\varepsilon(\omega, \mathbf{q}) = \varepsilon_1(\omega, \mathbf{q}) + i\varepsilon_2(\omega, \mathbf{q})$$

Loss function:

$$\text{Im} \left\{ \frac{-1}{\varepsilon(\omega, \mathbf{q})} \right\}$$

Kramers-Kronig:

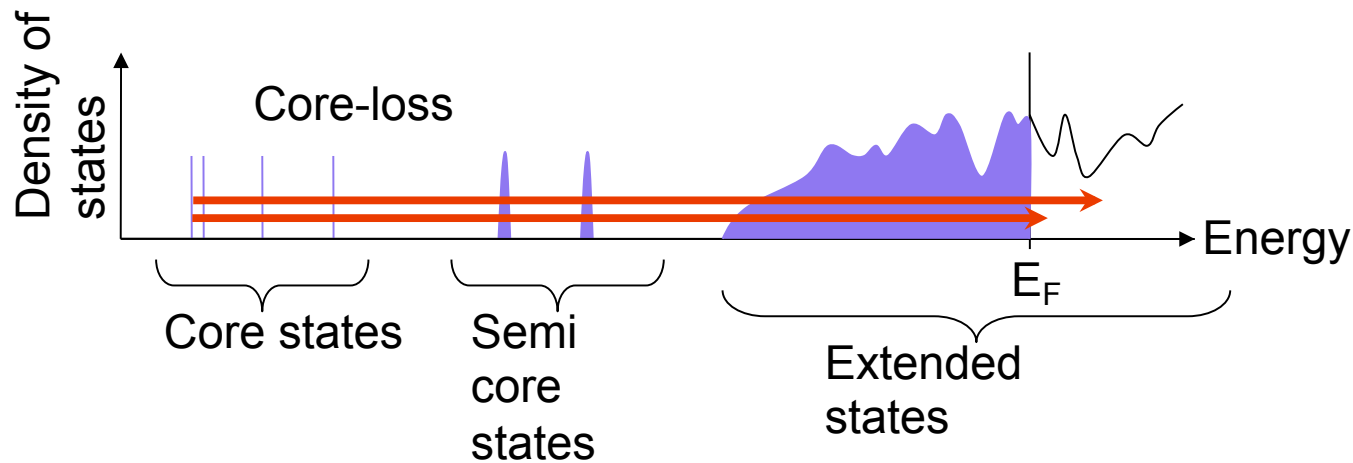
$$\varepsilon_1 \leftrightarrow \varepsilon_2$$

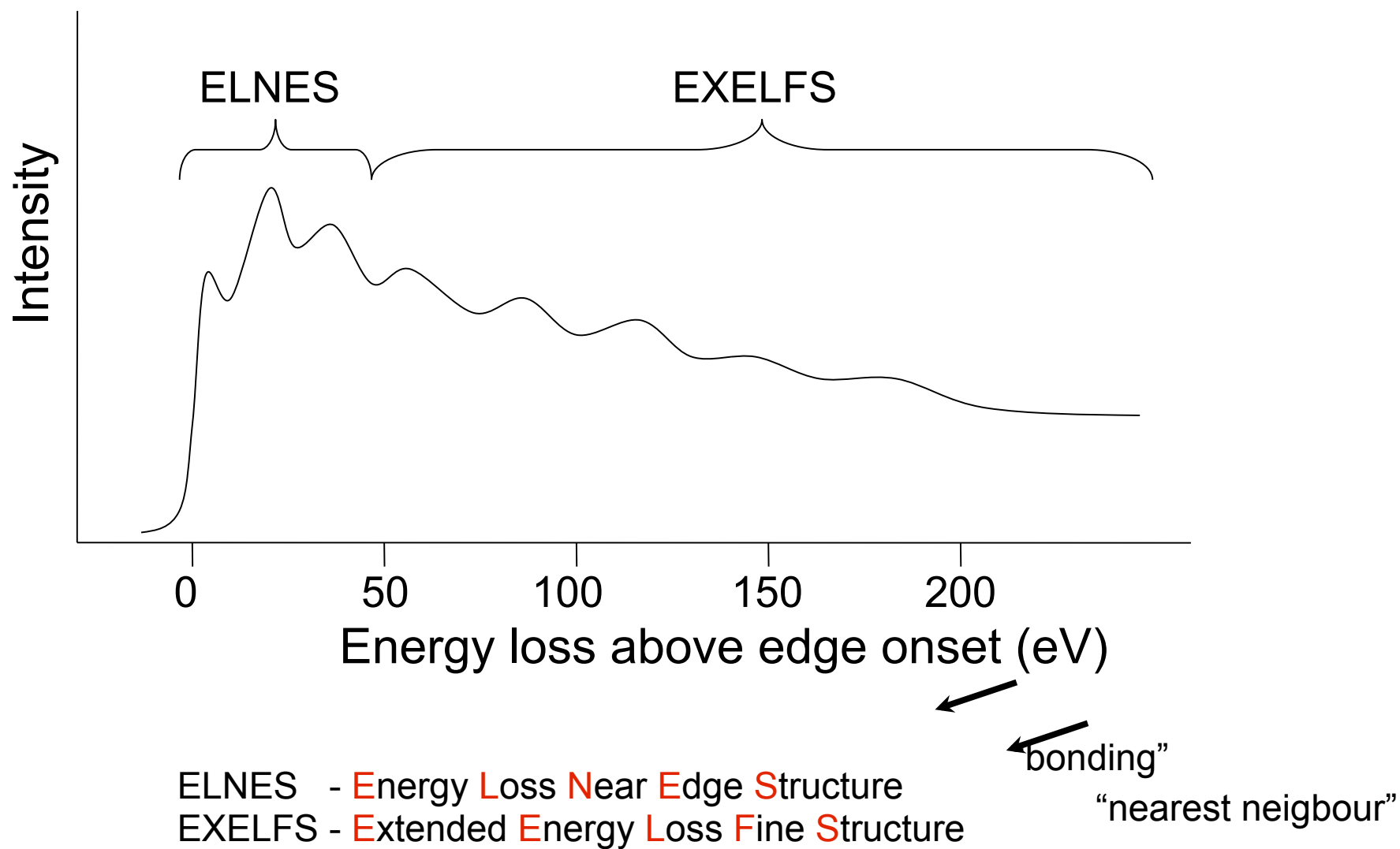
When an electron moves through a medium it creates a displacement field $\mathbf{D}$. The loss function gives the rate of energy absorption.

At high energies (i.e. transitions from core-states) the loss function is given by the imaginary part of the dielectric function.

$$\text{Im} \left\{ \frac{-1}{\varepsilon(\mathbf{q}, \omega)} \right\} = \varepsilon_2(\mathbf{q}, \omega)$$

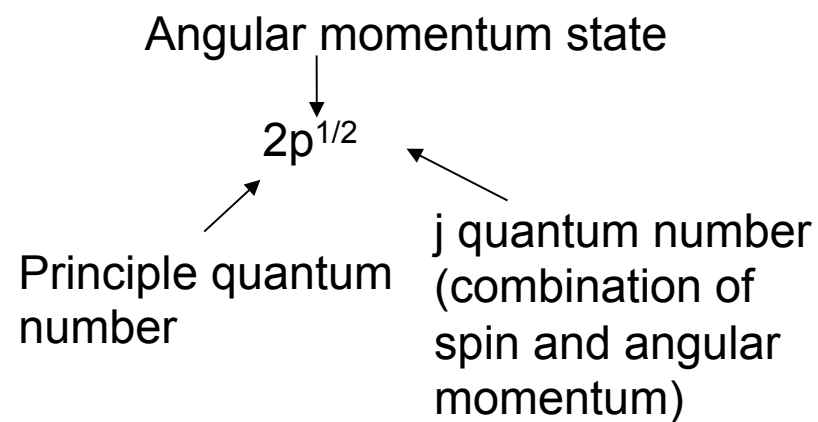
$$\varepsilon_2(\mathbf{q}, E) = \frac{4\pi e^2}{\Omega q^2} \sum_{n,\mathbf{k}} |\langle \psi_{\mathbf{k}}^n | e^{i\mathbf{q}\cdot\mathbf{r}} | c \rangle|^2 \delta(E_{\mathbf{k}}^c - E_{1s} - E)$$





Core electron	Edge
---------------	------

$1s^{1/2}$	K
$2s^{1/2}$	L_1
$2p^{1/2}$	L_2
$2p^{3/2}$	L_3
$3s^{1/2}$	M_1
$3p^{1/2}$	M_2
$3p^{3/2}$	M_3
$3d^{3/2}$	M_4
$3d^{5/2}$	M_5

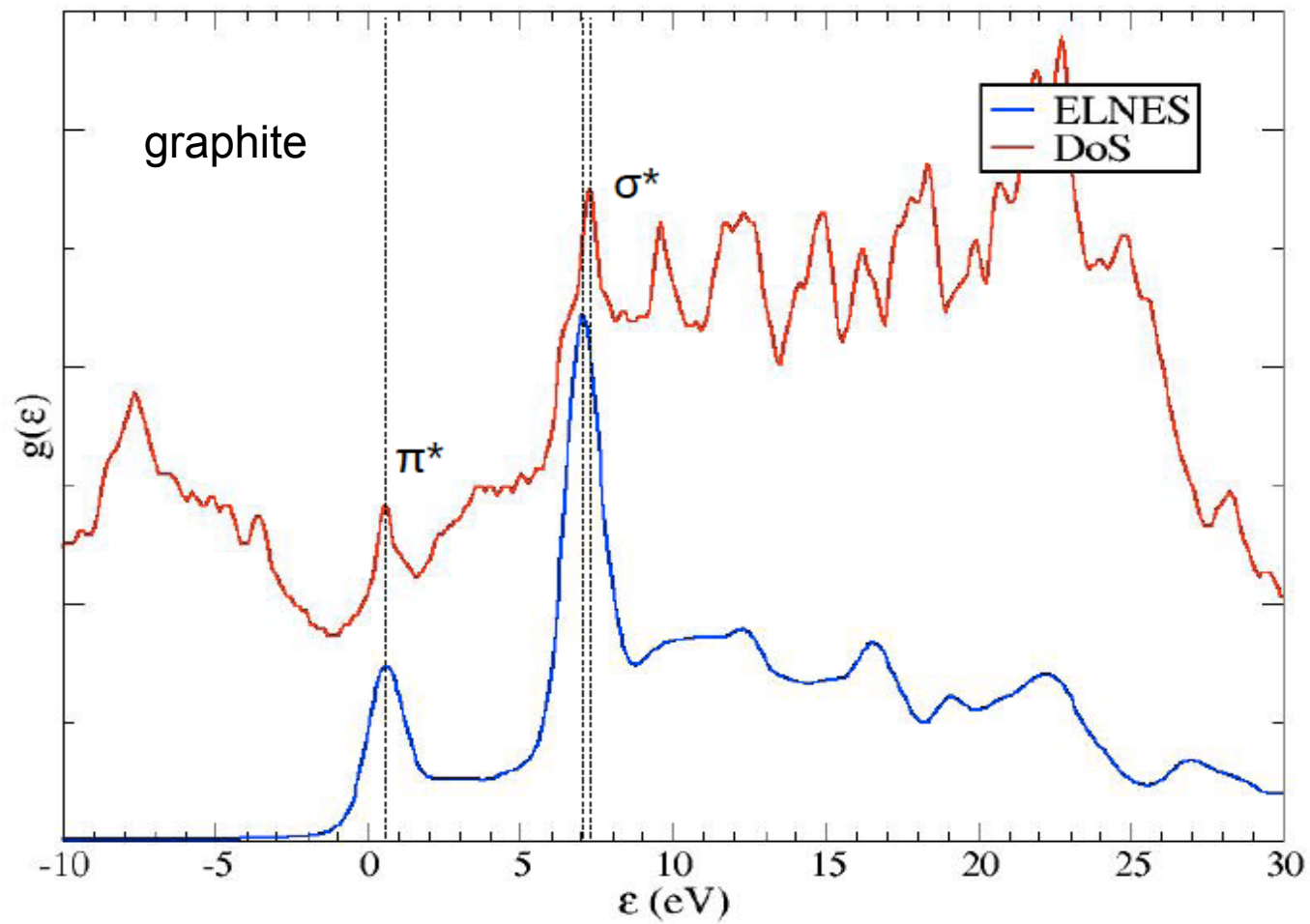


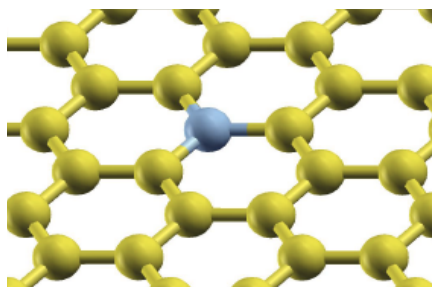
$$\langle f | e^{i\mathbf{q}\cdot\mathbf{r}} | i \rangle = \langle f | i \rangle - i \langle f | \mathbf{q}\cdot\mathbf{r} | i \rangle - \frac{1}{2} \langle f | (\mathbf{q}\cdot\mathbf{r})^2 | i \rangle - \frac{i}{6} \langle f | (\mathbf{q}\cdot\mathbf{r})^3 | i \rangle + \frac{1}{24} \langle f | (\mathbf{q}\cdot\mathbf{r})^4 | i \rangle + \dots$$

Dipole term

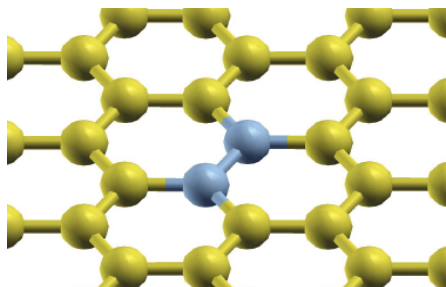
$$\langle f | x | 1s \rangle \approx \langle f | P_x \rangle$$

Core level spectra can be thought of as angular momentum projected density of states

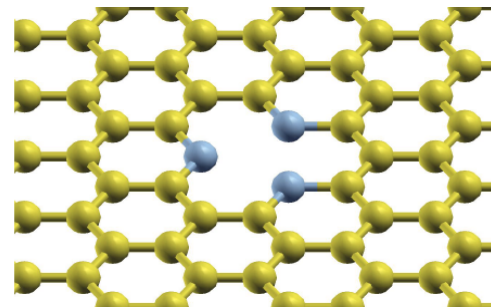




Substitutional

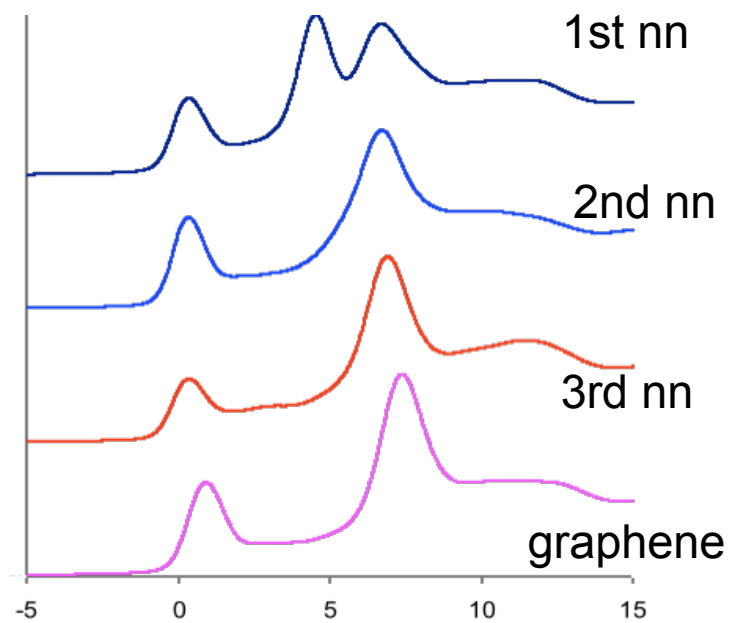


bi-nitrogen

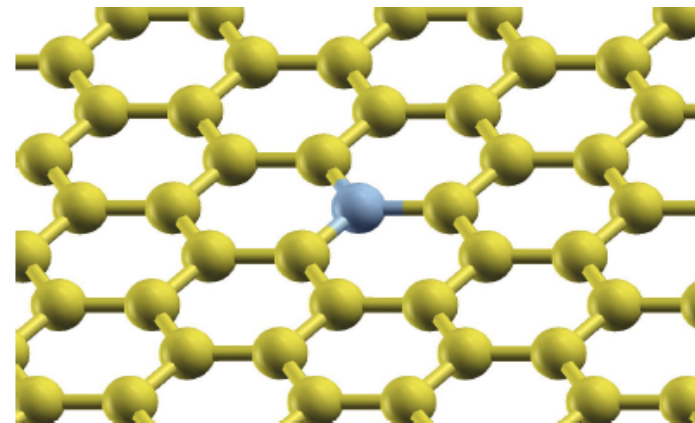


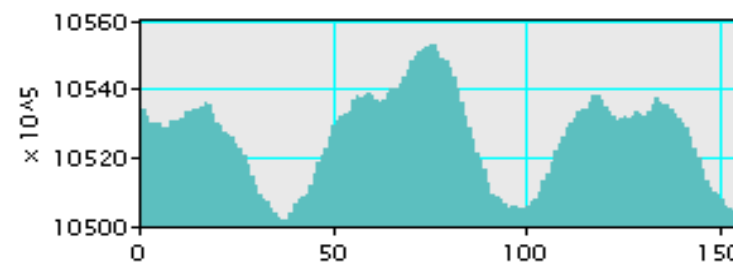
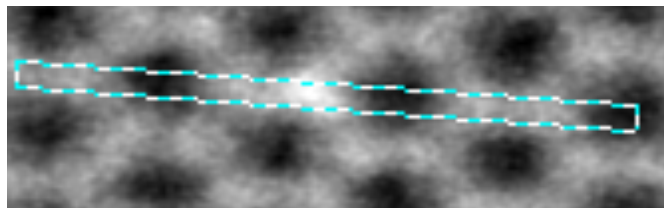
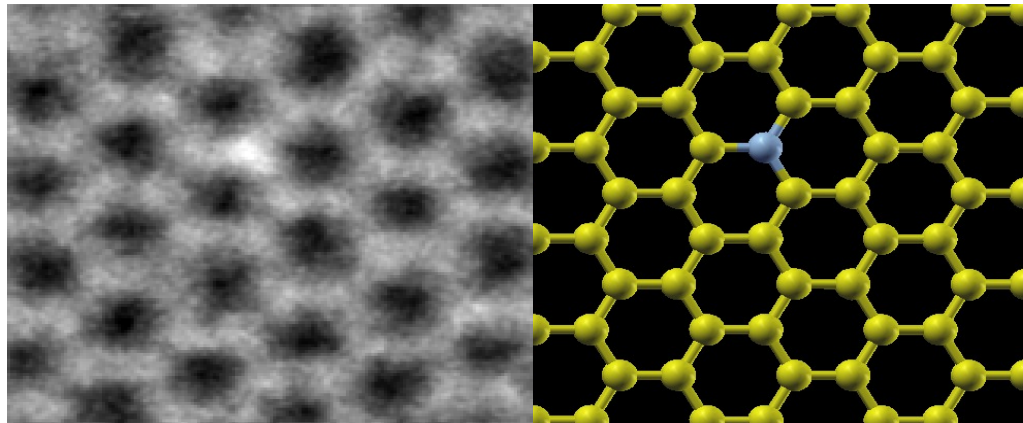
tri-pyridinic

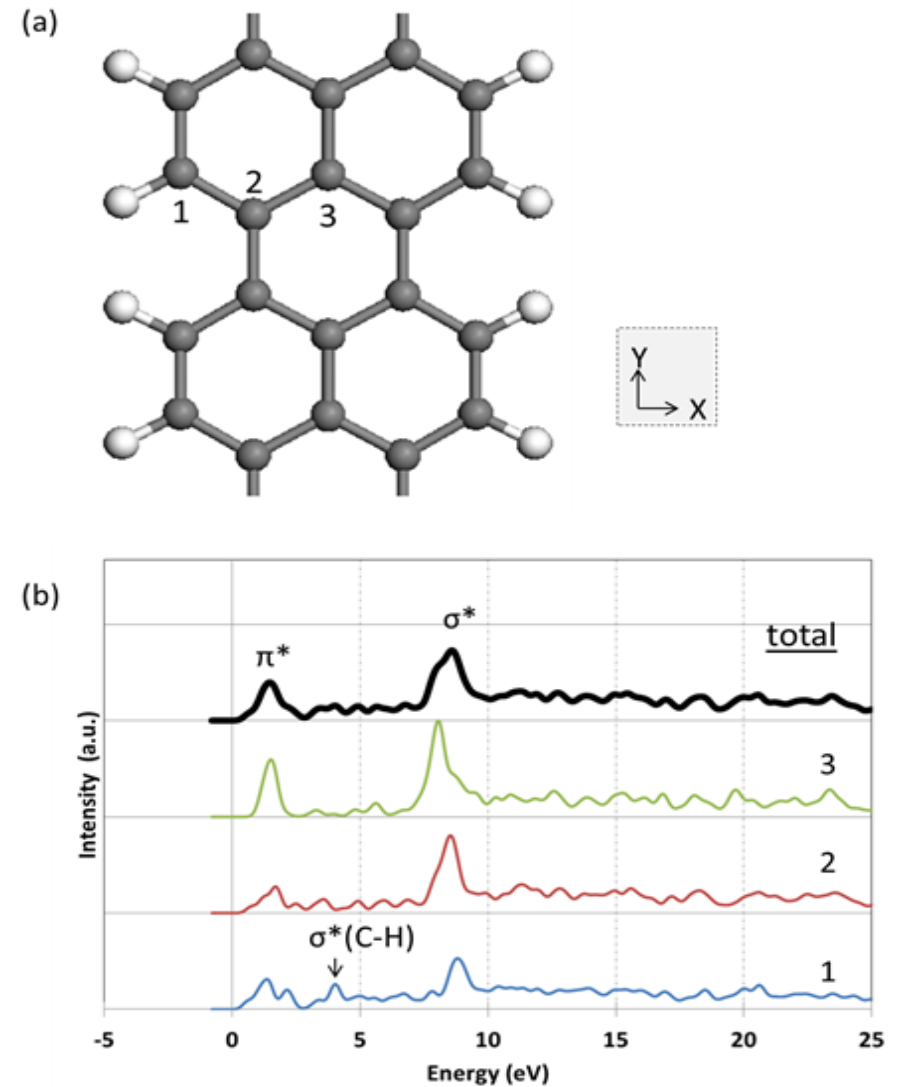
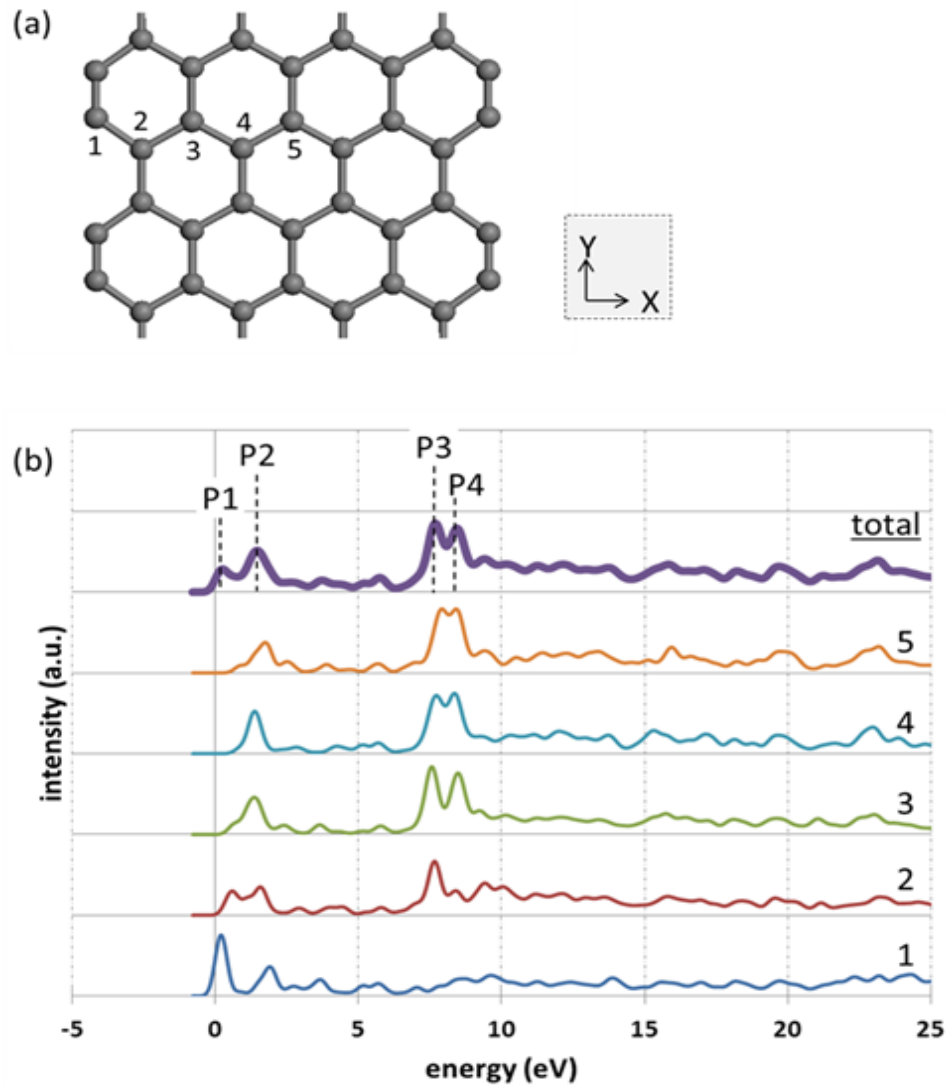
... and many others. Growth conditions, curvature etc. affects defect stability

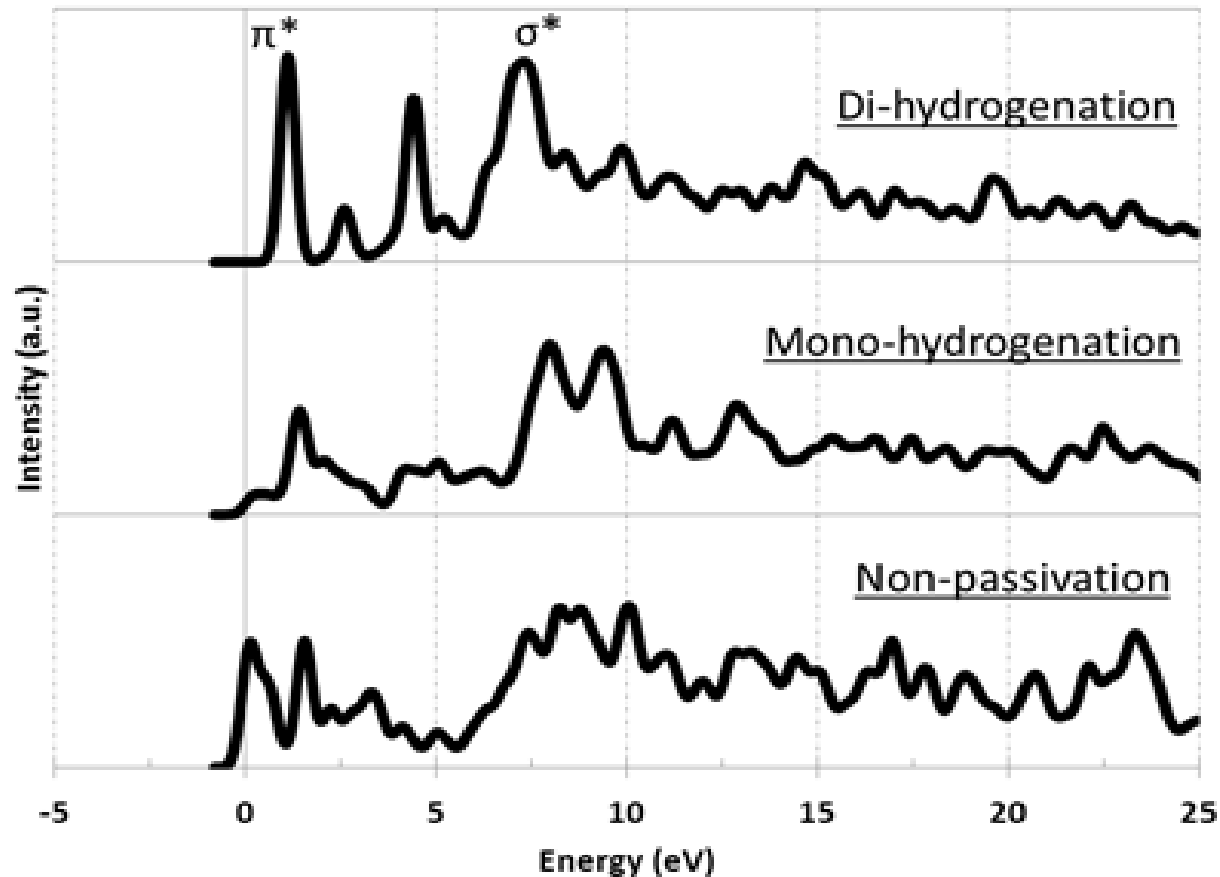


Carbon K-edge







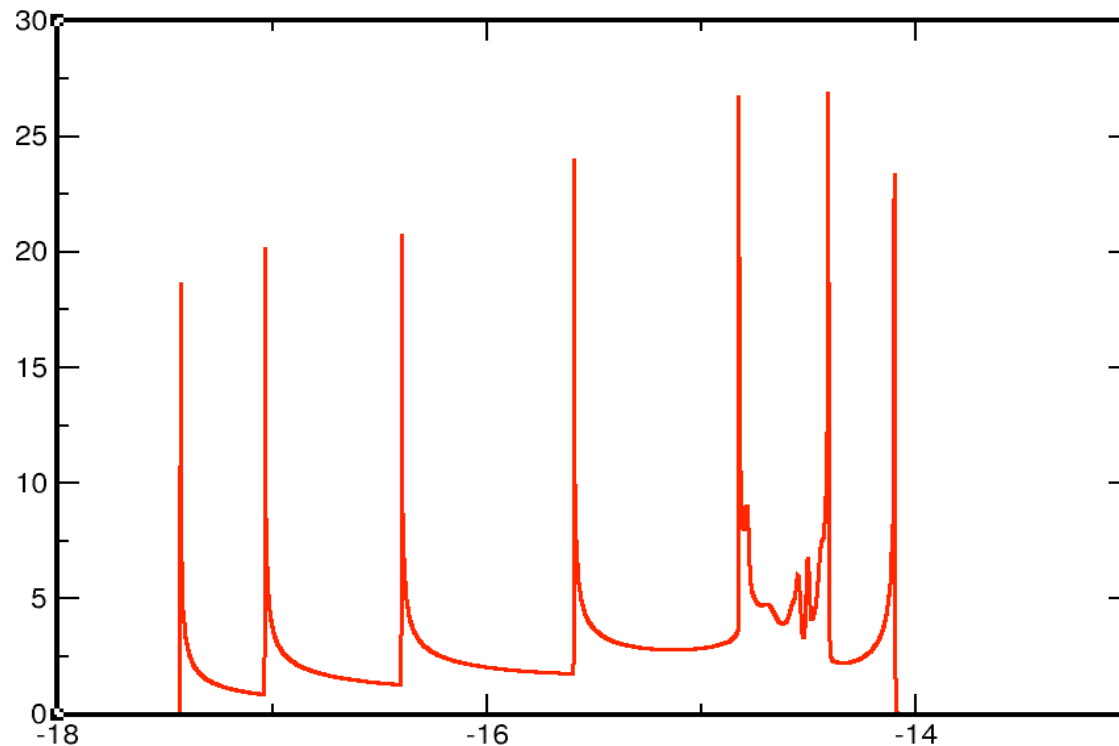


"Theoretical study of core-loss electron energy-loss spectroscopy at graphene nanoribbon edges",
Nobuyuki Fujita, Philip Hasnip, Matt Probert and Jun Yuan,
[J. Phys. Condens. Matter 27, 305301 \(2015\).](#)

Numerically Accurate Spectral Properties

www.optados.org*Andrew Morris (Cambridge), Rebecca Nicholls (Oxford), Chris Pickard (UCL), Jonathan Yates (Oxford)*

DOS BN nanoribbon



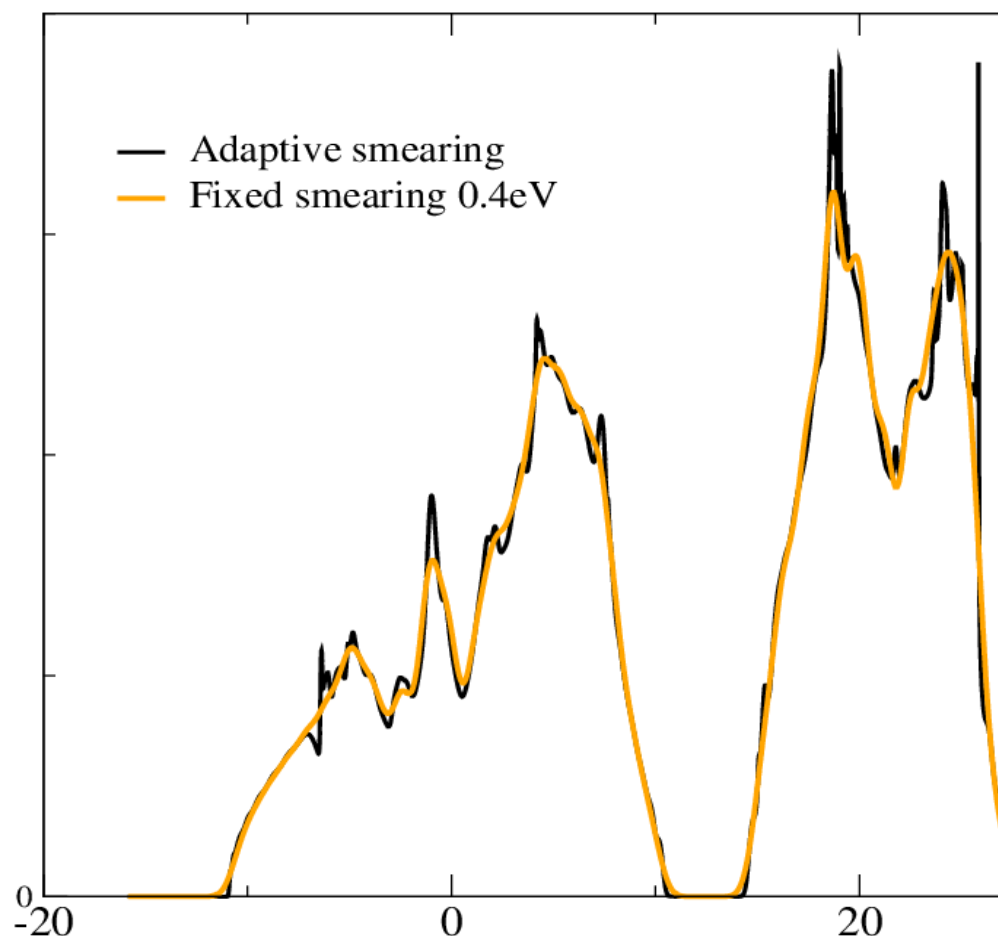
Adaptive broadening convolves each energy band with a Gaussian or Lorentzian whose width depends on the band-gradient (in reciprocal-space);

from J.Yates, X.Wang, D.Vanderbilt and I.Souza, Phys. Rev. B, 75, 195121 (2007)

www.optados.org

Andrew Morris, Rebecca Nicholls,
Chris Pickard, Jonathan Yates

- Density of states (DOS)
- Projected DOS
- Joint-density of states (JDOS)
- Core-loss
 - Absorption and Emission
 - momentum (q) decomposition
 - Instrument and lifetime broadening
- Low loss
 - Metals & insulators



Adaptive broadening convolves each energy band with a Gaussian or Lorentzian whose width depends on the band-gradient (in reciprocal-space);

from J.Yates, X.Wang, D.Vanderbilt and I.Souza, Phys. Rev. B, 75, 195121 (2007)

CASTEP

- computes core and valence wavefunctions
- forms matrix elements

$$\langle \psi_{\mathbf{k}}^n | \mathbf{r} | c \rangle$$

Must use on-the-fly pseudo-potentials

Keywords:

```
task : spectral  
spectral_task : coreloss
```

OPTADOS www.optados.org

- Reads matrix elements
 - Forms spectrum
 - Can add instrument and lifetime broadening effects
- more details in this afternoon's practical

Single-particle approximation

Beyond the “sudden” approximation

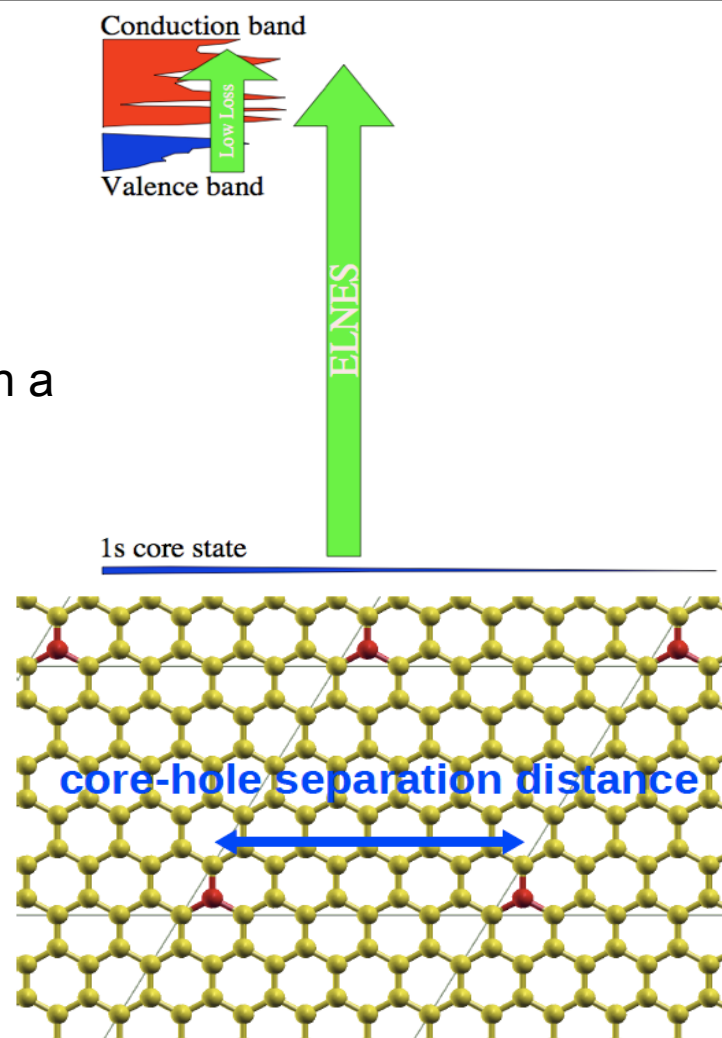
- 1) Core electron ejected
- 2) Lattice remains fixed
- 3) Final states relax

Initial core state from an atomic calculation

Final states from self consistent calculation with a single excited atom

Supercell approximation

- Core-hole atom breaks translational symmetry
- Create “supercell” of unit cell to reduce interaction between images
- Converge wrt supercell size

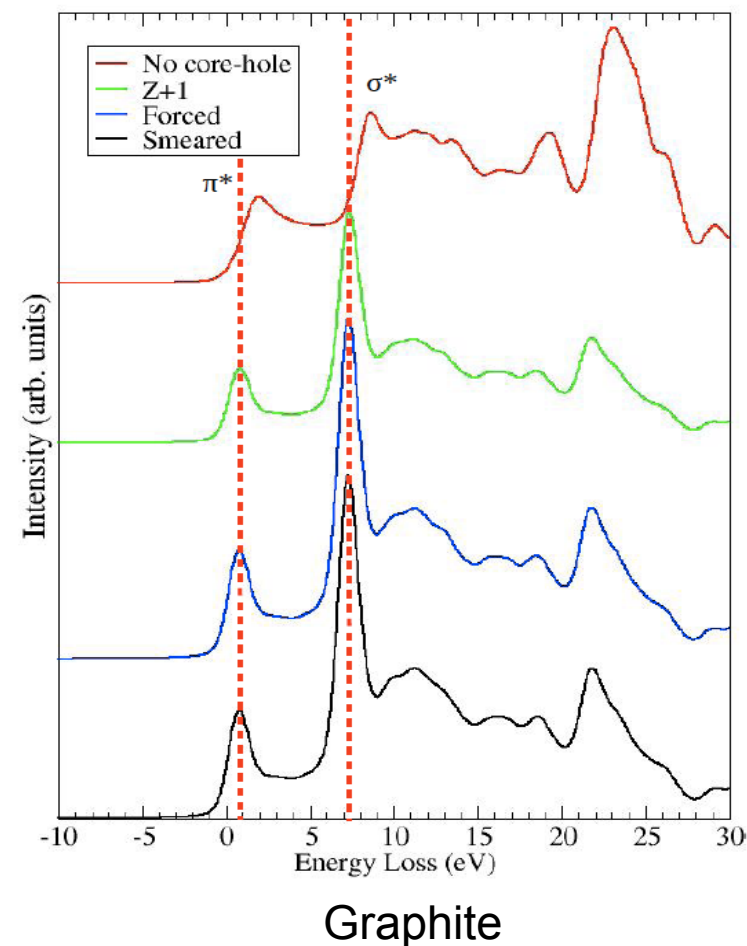


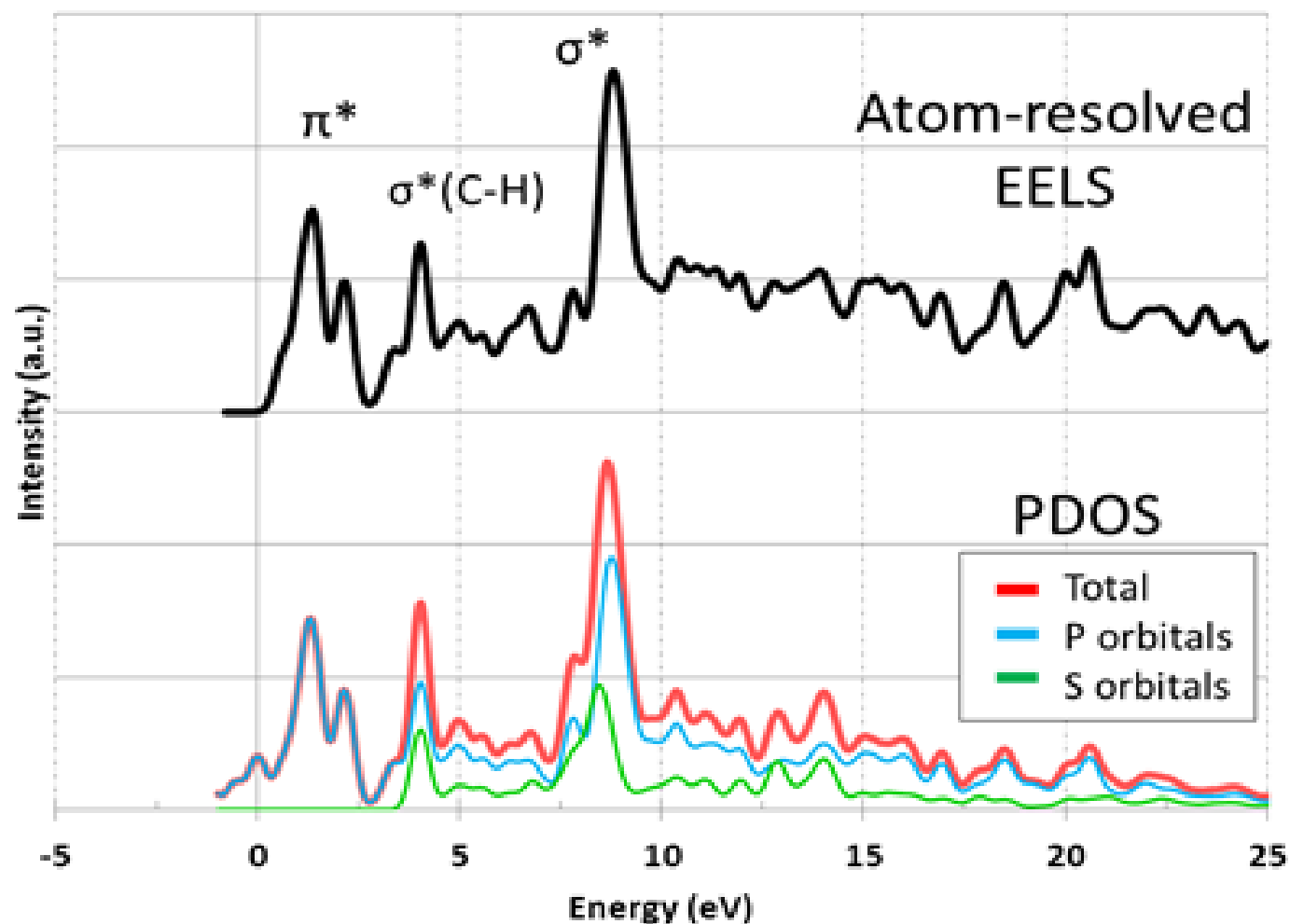
Methods to include core-hole

- Z+1 approximation (e.g. replace C with N)
- Constrained calculation (*only possible with AE code*)
- Excited-state pseudopotentials

CASTEP's on-the-fly pseudopotential generator can create excited-state pseudopotentials (see practical)

Effect of core-hole depends on the material



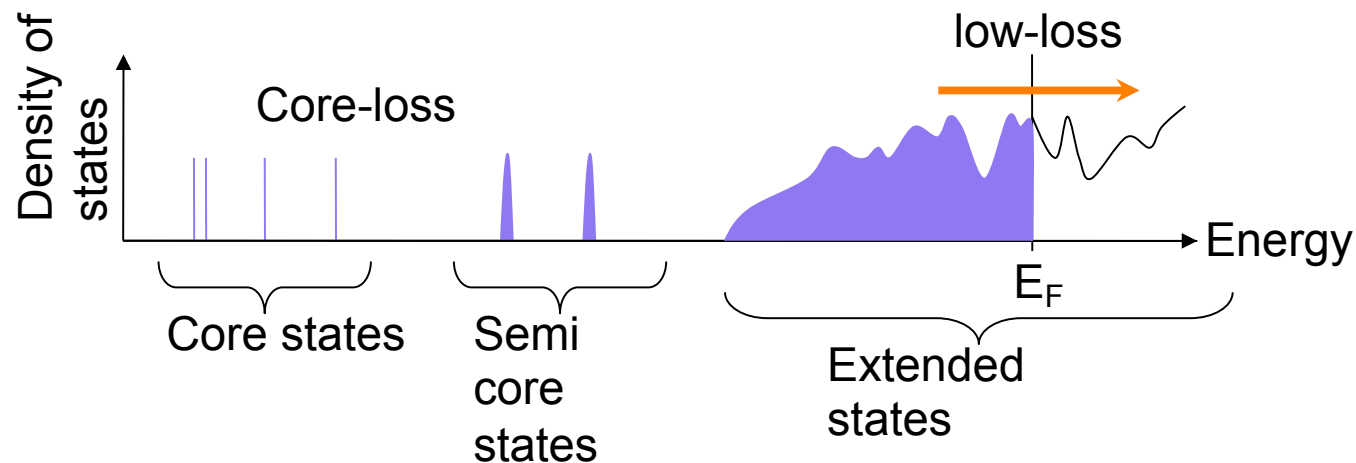


"Theoretical study of core-loss electron energy-loss spectroscopy at graphene nanoribbon edges",
Nobuyuki Fujita, Philip Hasnip, Matt Probert and Jun Yuan,
[J. Phys. Condens. Matter 27, 305301 \(2015\).](#)

$$\epsilon_2(E) = \frac{2\pi e^2}{\Omega\epsilon_0} \sum_{i,j,\mathbf{k}} | \langle \psi_{i\mathbf{k}} | \hat{\mathbf{u}} \cdot \mathbf{r} | \psi_{j\mathbf{k}} \rangle |^2 \delta(\epsilon_{i\mathbf{k}} - \epsilon_{j\mathbf{k}} - E)$$

The low-loss dielectric function is a weighted joint-density of states

$$jDOS(E) = \frac{1}{N_k} \sum_{i,j,\mathbf{k}} \delta(\epsilon_{i\mathbf{k}} - \epsilon_{j\mathbf{k}} - E)$$



CASTEP

- computes valence wavefunctions
 - forms optical matrix elements
- keywords

```
task : spectral  
spectral_task : optics
```

OPTADOS www.optados.org

- Reads matrix elements
- Forms
- Uses Kramers-Kronig to obtain
- Computes loss function, conductivity, refractive index etc
- Can add broadening effects

more details in this afternoon's practical

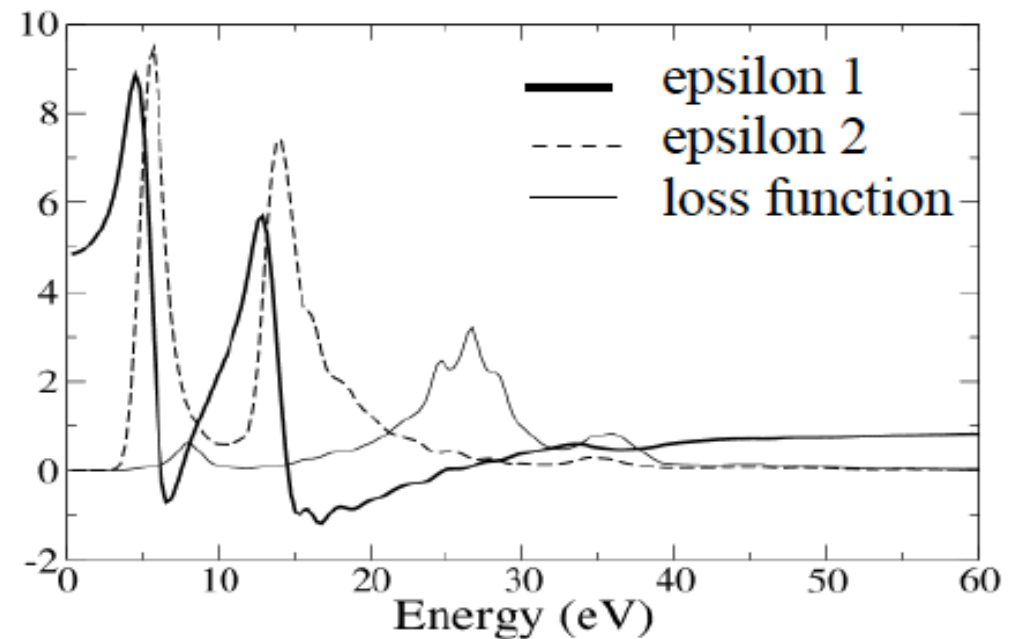
CASTEP $\epsilon_2(\omega) = \frac{2\pi e^2}{\Omega \epsilon_0} \sum_{i,j,k} | \langle \psi_{i\mathbf{k}} | \hat{\mathbf{u}} \cdot \mathbf{r} | \psi_{j\mathbf{k}} \rangle |^2 \delta(\epsilon_{i\mathbf{k}} - \epsilon_{j\mathbf{k}} - E)$

Kramers Kronig
↓

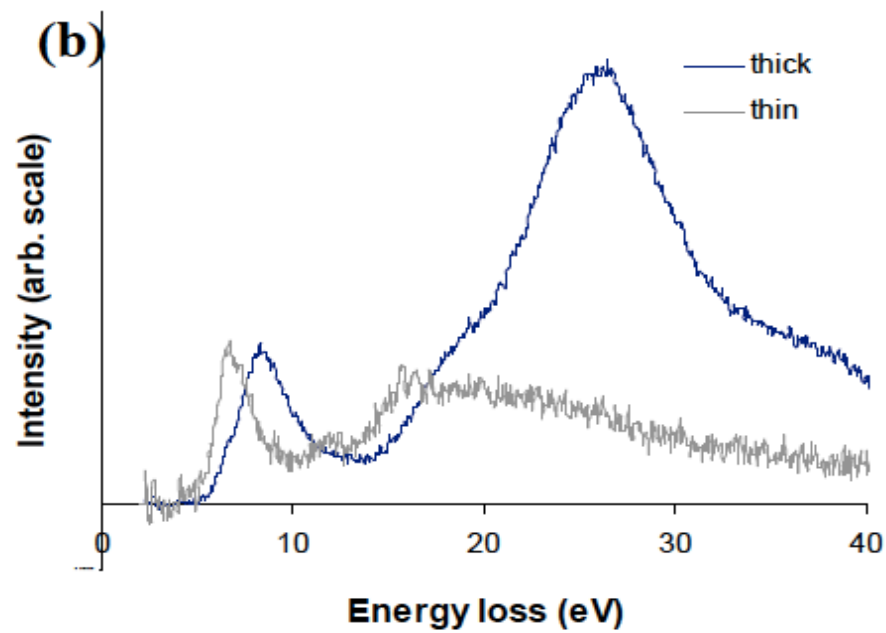
OPTADOS $\epsilon_1(\omega)$

↓

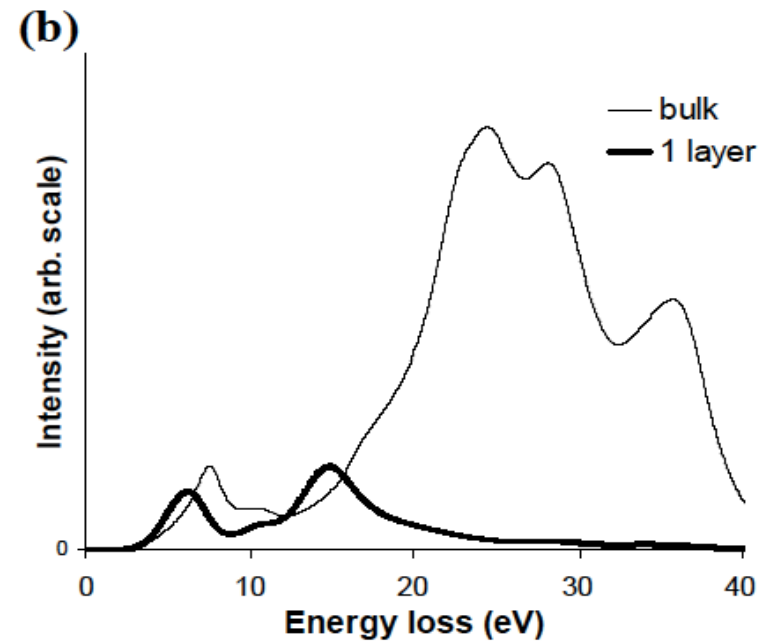
OPTADOS $Im \left\{ \frac{-1}{\epsilon(\omega)} \right\}$



Experiment



Calculation



Many-body perturbation theory

Start with DFT

Use GW to get corrected excited states

Use Bethe-Salpeter Equation (BSE) to include electron-hole interactions (exciton)

Time-Dependent DFT

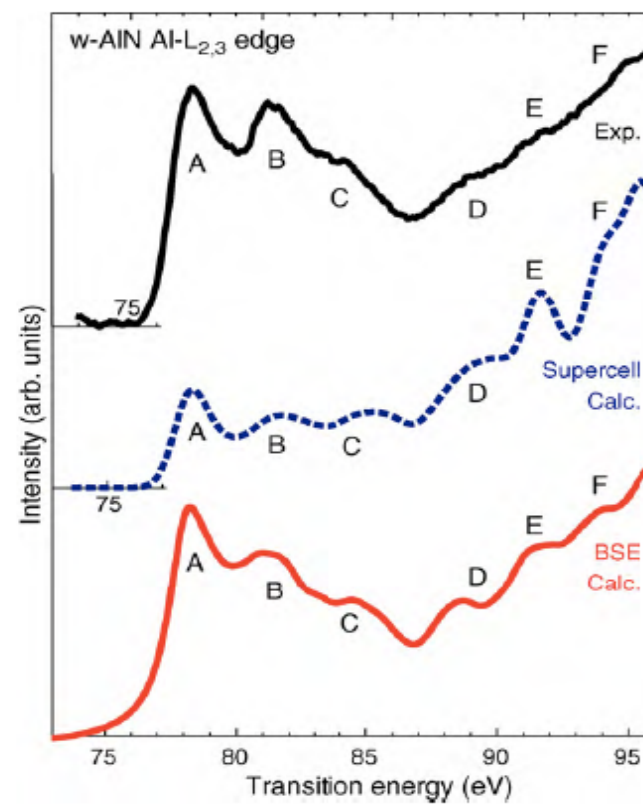
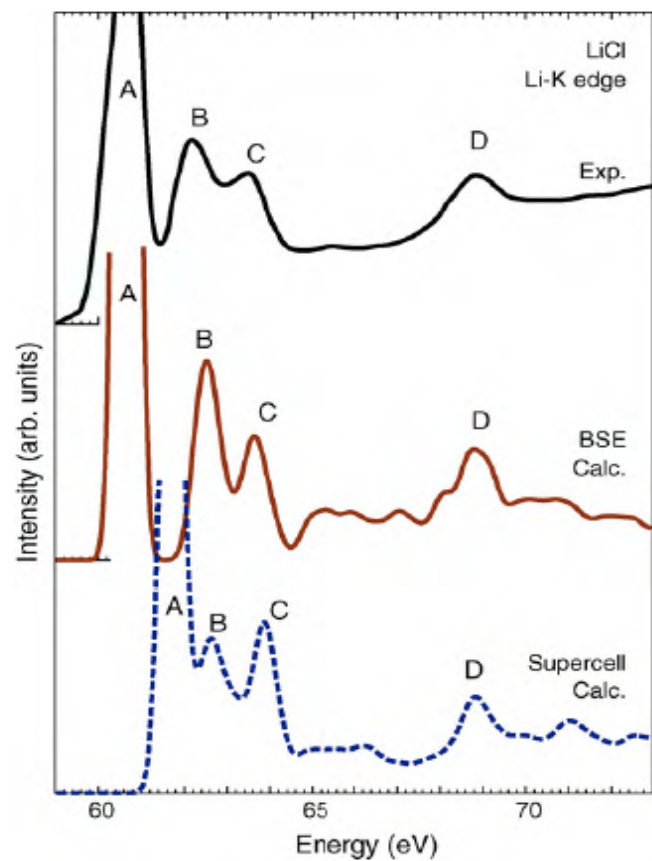
Exact solution of time-dependent QM

Need to devise time/frequency-dependent exchange-correlation kernel (see TDDFT lecture);
may be useful for low-loss EELS

Multiplet Effects

For edges such as the $L_{2,3}$ there can be strong overlap between the core and valence wavefunctions

Li-K edges of LiCl



T. Mizoguchi et al. / Micron 41 (2010) 695–709

Standard electronic structure tools can provide qualitative interpretation of core and low-loss EELS

Care must be taken in interpreting unoccupied states in DFT; recall Hohenberg-Kohn theorem is only for *ground state* properties.

Many-body approaches such as GW (see next lecture) provide a route to *quantitative* simulations, but at a v. high computational cost.

For core states with $l > 0$ there can be strong core-valence overlap, which is often not well captured; i.e. K-edge EELS simulated much more reliably than L- or M-edge EELS.

Can TD-DFT (see previous lecture) provide a useful balance of speed and accuracy?

OptaDOS manual

<http://www.optados.org>

Chris Pickard's PhD thesis:

<http://www.tcm.phy.cam.ac.uk/~cjp20/old/thesis.ps.gz>

Summary paper of Castep's spectroscopic capabilities:

["Electron and vibrational spectroscopies using DFT, plane waves and pseudopotentials: CASTEP implementation"](#)

V. Milman, K. Refson, S.J. Clark, C.J. Pickard, J.R. Yates, S.P. Gao,
P.J. Hasnip, M.I.J. Probert, A. Perlov, M.D. Segall

Journal of Molecular Structure: THEOCHEM 954 (1-3) 22-35 (2010)