
Bootstrapping the Battle of Britain



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Abstract

The Battle of Britain (1940) is the focus of much historical controversy. We show here how the statistical technique of *weighted bootstrapping* can be used to create a new quantitative basis to help address such controversies. Bootstrapping facilitates the exploration of alternative campaign possibilities with different tactics. This results in comparative probabilities of “victory” for the actual and various counterfactual campaigns, providing a quantified assessment of the likelihood of German achievement of air superiority, thereby facilitating invasion. We find this more likely had the Luftwaffe targeted airfields more heavily, and greatly more likely had Germany brought forward its air campaign.

1. Introduction

The Battle of Britain (1940) continues to generate fierce controversy amongst historians as its eightieth anniversary approaches. The conflict between critics and advocates of “Big Wings,” the massing of several squadrons or even wings in the air prior to interception and engagement, now has a considerable pedigree and

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has reached popular culture by many avenues, particularly the reputations of the personalities involved.¹ However, debates about the British conduct of the battle can distract attention from other fundamental questions. For example, some German veterans and historians deny that any such definitive conflict as a “Battle of Britain” occurred.² Whilst this is a minority view, victory and defeat are difficult to define with clarity even if we accept the existence of a discrete campaign. The German decision to postpone invasion of England to beyond 1940 might be an obvious yardstick, but some deny that invasion was ever German leader Adolf Hitler’s intention and this of course returns us to the question of whether or not there was ever a “battle” in the classical sense, rather than simply an active front.³

Nevertheless, there is no doubt that the German Luftwaffe was instructed to realize a definite goal of peace with the British Empire on German terms. Whether this was to be achieved through an invasion or political pressure caused by public susceptibility to “terror” bombing is perhaps beside the point. A German victory was intended and this would follow the definite achievement of air supremacy over at least southern England. The point at which German aerial success would change the political situation is impossible to quantify objectively, however. A decision by the British government to sue for peace would be an obvious measure—and did not occur. It can also be discounted as a realistic policy during the time frame of the battle, as the protracted and intense “Blitz” which followed also failed to achieve a moral collapse in the United Kingdom. Nor was an invasion attempted, although this remains the only realistic outcome to end the war quickly. Historians need to conceptualize, therefore, the Luftwaffe’s proximity to a success-enabling invasion if they are to possess a fully rounded understanding of historical developments between May and October 1940.

At this point the consideration of unrealized possibilities becomes unavoidable, and this is a difficult issue to discuss using academic historical techniques. The field has largely been left to popular writers willing to discuss the counterfactual possibility of invasion. These accounts require some sort of trigger for German action and fall broadly into two camps. One posits an invasion in September 1940 after German success in the prolonged air campaign, perhaps described in terms of

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1. For the literature, see Niall MacKay and Christopher Price, “Safety in Numbers: Ideas of Concentration in Royal Air Force Fighter Defence from Lanchester to the Battle of Britain,” *History* 96, no. 323 (2011): 304–25.

2. Stephen Bungay, *The Most Dangerous Enemy: A History of the Battle of Britain* (London: Aurum, 2000), 33; Len Deighton, *Fighter: The True Story of the Battle of Britain* (London: Jonathan Cape, 1977), 51.

3. Ian Kershaw, *Fateful choices: Ten decisions that changed the world, 1940–1941* (London: Penguin, 2008).

a retreat by Royal Air Force (RAF) Fighter Command north of the Thames into the 12 Group Area.⁴ This would enable an invasion attempt based on air supremacy over territory previously defended by 11 Group. Such a scenario possesses scholarly credentials as it most closely matches German planning and instructions to the Wehrmacht from Hitler. The second option, less frequently visited, favors an early German invasion, without a lengthy campaign to achieve air supremacy, to “bounce” the British defenses and exploit their temporary disarray in the aftermath of the evacuation of the British Expeditionary Force (BEF) from the French port of Dunkirk in late May.⁵ Intermediate options are not popular topics for speculative writing or academic historians, but they do fall within the scope of our analysis.⁶ How then might real data help in addressing the probability of a decisive German success and avoid historical analysis based on simple guesswork?

In recent years historians have demonstrated that quantitative analysis has a useful role in addressing circular arguments and apparently intractable issues. The purpose of this paper is to explain and advocate a quantitative analytical technique new to historical analysis, and to demonstrate its usefulness in addressing central issues in the Battle of Britain. The technique in statistics is called *bootstrapping*.⁷ The essential idea is to regard a set of historical data points as a sample from a single unknown distribution and then to resample, with replacement, from this distribution.

In this approach, the record of daily events and attrition in a military campaign constitutes all that we can know about the distribution of possible outcomes. From this daily record we can create alternative campaigns by resampling—choosing days at random, resulting in a set of days’ fighting, but in a different order, and perhaps with some days appearing more than once or not at all. Such resamples, all taken together, form a “bell curve” of possibilities for how the campaign might have turned out. The validity of this approach depends crucially on all days being, statistically speaking, alike. These conditions are rarely realized—for example, in the actual “Hundred Days” of the Waterloo Campaign, each day radically affected

4. Richard Cox, *Operation Sealion* (London: Thornton Cox, 1974); Egbert Kieser, *Hitler on the Doorstep: Operation “Sea Lion”: The German Plan to Invade Britain, 1940* (Annapolis, Md.: Naval Institute Press, 1997); Martin Marix Evans and Angus McGeoch, *Invasion! Operation Sea Lion, 1940* (London: Routledge, 2004); Robert Forczyk, *We March Against England: Operation Sea Lion, 1940–41* (London: Bloomsbury, 2016).

5. Cecil Scott Forester, “If Hitler had invaded England,” in *Gold from Crete* (London: Pan, 1971); Kenneth Macksey, *Invasion: The German Invasion of England, July 1940* (London: Greenhill, 1980); Charles Messenger, “The Battle of Britain 1940: Triumph of the Luftwaffe,” in *Third Reich Victorious: Alternative Histories of World War II*, ed. Peter G. Tsouras (London: Greenhill, 2002), 65–96.

6. The possibility of protracted indirect war on British seaborne trade and strategic interests in the Mediterranean lies outside the scope of this article. Forczyk, *We March Against England*.

7. Originally introduced by Bradley Efron in 1979, the bootstrap is described in B. Efron and Robert J. Tibshirani, *An Introduction to the Bootstrap* (Boca Raton: Chapman and Hall/CRC, 1994).

the next. However, this obvious caveat aside, close analysis of real data can be valuable in, for example, a geographically fixed campaign with a recurrent daily pattern of action such as the Battle of Britain.

Nonetheless, the historian's focus must remain principally on the real factors that influenced the campaign: variations between the days, the units involved, the conditions of engagement, the weather, and so on. More fundamental questions affecting the outcome include the intentions and assumptions of the protagonists, the quality of their decision-making, and the dilemmas they faced. The purpose of the present exercise is not to provide an alternative reality but rather to enable a more broad-based understanding of the historical outcome in all its complexity.

To address such questions we take a range of possible actions facing the protagonists, assign these decisions to particular days, and when we resample make such days more or less likely to be chosen depending on how the protagonists vary their decisions. This process is called *weighted bootstrapping*: we create alternative, counterfactual campaigns by varying the probabilities with which we choose individual days. As a historical technique, this wrings all that can be extracted from the known data with minimal assumptions and provides a standardized methodology for doing so. In any circumstance in which there is at least reasonable evidence that the underlying distribution is constant, the weighted bootstrap provides the best information that can be obtained from (nothing but) the data, and thus the best jumping-off point for the historian.

2. Bootstrapping a military campaign

The subtleties of mathematical statistics need not concern us here: in our historical context both the method and the result are simple and intuitive. Consider an extended military campaign, fought between Red and Blue, of 100 days, say, on each of which the underlying conditions—of equipment, terrain, etc.—are much the same. Suppose that we have data for numbers and casualties on each day, and that Red eventually “won” the campaign according to some quantitative measure. One view might be that these 100 days provide all that we can possibly know about the campaign, and that it is unproductive to consider alternatives. It can also truthfully be said, however, that these 100 days are merely a sample from the distribution of all possible days’ fighting, which might profitably be explored, if a methodology could be created for doing so.

Bootstrapping allows the two views to be somewhat reconciled. We acknowledge that all we know—all we *can* know—about the campaign is contained in those 100 days’ data. But we can create alternative campaigns by resampling, with replacement, from the original campaign. A single alternative campaign consists of 100 days each chosen at random from among the actual 100 days. In this alternative, some days, perhaps heavily favoring one side or the other, may be repeated, perhaps many times. Then, with the computing power available to modern researchers, we can create many such alternative campaigns—10,000, say. These are our best estimate, using the actual campaign data, of the distribution of all the ways the campaign might have turned out, based on the assumption that all

days' outcomes were equally likely. Next we can ask: according to our quantitative victory criterion, under which Red won the actual battle, what proportion of the alternative battles did Red win? If, say, 75 percent emerges, then this is our best estimate of the probability of Red's victory.

The bootstrap as described above appears to take us little further forward. It offers control over what the data are telling us about the way the campaign actually fought might have played out differently, but at this level of implementation it offers no genuine counterfactuals. Rather it principally enables us to turn the answers ("yes" or "no," "Red" or "Blue") to categorical questions about the campaign into probabilities.

Importantly, the bootstrap technique is *non-parametric*: no models are posited or underlying dynamics assumed; it relies on no assumptions about the nature of warfare in the campaign.⁸ In the bootstrap, there is nothing but the data, the record of what actually happened. The only artifice is our choice of a quantified threshold for victory: to the extent to which we differ about this, we will differ in the probabilities of victory we arrive at. So the Red victory probability will vary hugely according to our preconceptions about the narrowness of the margin and thereby the quantified victory criterion. The essence of our technique will be to *invert* this relationship: we begin with a victory probability, considering it as no more than an attempt to quantify, approximately, a historian's belief about the narrowness of victory, and use this probability to reverse engineer the threshold for victory. This threshold can then be used in various counterfactual scenarios to extract the *altered* victory probabilities associated with these scenarios.

The crucial step to perform such counterfactual history is non-uniform weighting, in which, when we resample, we choose different days with different probabilities. For example, suppose Blue used certain tactics for the first 70 days, and changed them utterly (to its advantage) for the last 30—and, as we know, lost the actual campaign. Now suppose we construct our alternative campaigns on the assumption that Blue instead changed its tactics much earlier—after about 40 days, say. We do this by selecting the 100 days of each alternative campaign as follows: each of the first 70 days of the actual campaign is chosen with probability not 1/100 but rather $1/100 \times 40/70$, and each of the last 30 days is chosen with probability $1/100 \times 60/30$. The sum of all the probabilities is still one, but now about 40 days will be fought with Blue's earlier, worse tactics, and about 60 with Blue's later, better tactics.

Suppose that Blue's proportion of victories in this set of alternative campaigns, using counterfactual tactics, is 65 percent, in contrast to its 25 percent victories using the actual (unweighted) campaign data. The difference—the turnaround, indeed—in these figures is a robust result. Our original Blue victory probability of only 25 percent might be thought a result of our preconceptions about the size of Red's victory margin—we have effectively calibrated it to them—but changes

8. For example, we make no use of Lanchester's scaling laws for attrition, stochastic duels, or any other assumptions about *how* casualties are produced or models for attrition. Alan R. Washburn and Moshe Kress, *Combat Modeling* (New York: Springer, 2009).

in those preconceptions will tend to move both of Blue's probabilities of victory, 25 percent in the real campaign and 65 percent in the counterfactual, in the same direction. The difference in these two figures is what is informative—it is much less a product of the historian's views than is either of the probabilities individually. This weighted bootstrap is no longer strictly non-parametric, for it introduces some ideas of our own in the form of the reweightings, but it is still free of any assumptions about the process of combat attrition. So, with no further assumptions, the historian who believed that Red won the actual campaign decisively, with a 75 percent chance of victory, should accept that if Blue had changed tactics earlier then it would have been the likely winner.

3. The Battle of Britain

The Battle of Britain, so evocative in British history, continues to generate academic debate. Historians, nevertheless, agree on a core narrative: the Luftwaffe fought an air campaign over the English Channel, southeastern England, and London in the summer and autumn of 1940 intended to achieve air supremacy through the neutralization of the RAF. Once the Luftwaffe switched in October to mainly night-bombing it was clear that German strategy had failed, at least for 1940: RAF Fighter Command remained in being, and was growing stronger.⁹

German air supremacy was consistently regarded, by both sides, as a necessary (albeit not necessarily sufficient) pre-condition for the invasion of England. Hitler was initially—at the time of the Dunkirk evacuation, at the end of May 1940—uncommitted to invasion. However, following meetings in late May and June, and resulting initial studies by the German high command, Hitler issued on 16 July an order to prepare for invasion, Operation Sea Lion. For the British, simultaneous with the defensive campaign, RAF Bomber Command and light forces of the Royal Navy conducted offensive campaigns against German craft in Channel ports whose obvious role would have been to form an invasion force. The Royal Navy would clearly have posed an existential threat to an invasion fleet.¹⁰ The idea that the air battle was a “myth” because of this recurs in an almost cyclical way, and journalists find it particularly tempting. Before Anthony Cumming's 2010 work in this vein appeared,¹¹ three eminent military historians were (in their view) ambushed by the journalist Brian James in an article for *History Today* and portrayed as agreeing with this perspective.¹² Their actual position, delivered in a riposte, was that invasion was thwarted by a holistic British defense involving

9. For an excellent recent summary, see Bungay, *Most Dangerous Enemy*. The 2015 seventy-fifth anniversary edition of *Royal Air Force Air Power Review* 18, no. 2 (Summer 2015) provides a useful collection of original RAF sources with commentary.

10. Duncan W. Grinnell-Milne, *The Silent Victory: September 1940* (London: Bodley Head, 1958).

11. Anthony J. Cumming, *The Royal Navy and the Battle of Britain* (Annapolis, Md.: Naval Institute Press, 2010).

12. Brian James, “Pie in the Sky?,” *History Today*, September 2006, 38.

all three services. One might say that this is an orthodox position and that no academic historian actively dismisses the importance of either the army or the navy in 1940. Such attempts constitute “a silly season story par excellence.”¹³ However, one would struggle to dismiss the Luftwaffe’s appearance over England in great strength as insignificant, or to argue that the obliteration of Fighter Command in the summer of 1940 would offer no implications for the course of the war. It is clear that Fighter Command won its defensive campaign, its “holding action,” and that this greatly reduced the probability of invasion.

A Fighter Command defeat would minimally have consisted of a forced withdrawal from forward-most airfields, then north of the Thames into 12 Group’s area, and finally effective abandonment of the skies south of the Thames. There was no obvious step-change for Fighter Command to make: rather its gradual failure would have appeared as first a steady, later an accelerating extension of RAF 11 Group commander Keith Park’s already parsimonious defensive approach. Given existing relative rates of attrition and replacement, the best German outcome would probably have been a brief time-window of air superiority.¹⁴ If this triggered invasion, a fresh air battle would have been fought, in parallel with a massive naval action.¹⁵

An invasion campaign is outside our scope: by the “Battle of Britain” we mean the first, purely aerial action. An analysis of the scaling of combat attrition and its relationship with Lanchester’s laws and ideas of concentration has been conducted in a series of works by the present authors,¹⁶ and in the first of these we used bootstrap techniques to verify that the Luftwaffe benefited more from large sortie numbers than did the RAF. Further, the Battle of Britain was fought over a relatively short period using largely similar aircraft. Various tests of the data show no obvious trends over time. It has been shown previously that the only

13. Christine Goulter, Andrew Gordon, and Gary Sheffield, “The Royal Navy did not win the ‘Battle of Britain,’” *Journal of the Royal United Services Institute* 151, no. 5 (2006): 66–67. This exchange is discussed in MacKay and Price, “Safety in Numbers,” 305.

14. Bungay, at one extreme, argues that the Luftwaffe “never came close” to their desired strategic result and that in terms of the important Sector stations, closing Biggin Hill for a few hours on one day was the extent of their success in damaging 11 Group’s infrastructure. Bungay, *Most Dangerous Enemy*, 368–69.

15. A fictionalized account of the 1974 Royal Military Academy wargame of Sea Lion is given by Cox, *Operation Sealion*. Details of German preparations are in Peter Schenk, *Invasion of England 1940: The Planning of Operation Sealion* (Berlin: Oberbaum, 1987; Engl. trans., London: Conway, 1990). A summary of the strategic planning is *German Plans for the Invasion of England: Operation Sealion, 1940*, Declassified Central Intelligence Agency report, published by Digital Publications 2017.

16. Ian R. Johnson and Niall J. MacKay, “Lanchester models and the Battle of Britain,” *Naval Research Logistics* 58 (2011): 210–22; MacKay and Price, “Safety in Numbers”; Niall J. MacKay, “Is air combat Lanchestrian?,” *Phalanx: The Bulletin of Military Operations Research* 44 (2011): 12–14; Ian Horwood, Niall MacKay, and Christopher Price, “Concentration and Asymmetry in Air Combat: Lessons for the defensive employment of air power,” *Royal Air Force Air Power Review* 17, no. 2 (2014): 68–91.

obvious change-point appears in a plot of (both sides') losses as a proportion of total sorties, which shows that the battle was more intense before 15 September 1940 than thereafter.¹⁷ However, there is no asymmetric (between the two sides) change in the data, and nothing to invalidate our assumption that the data can be considered independent and identically distributed.

Our task is to apply bootstrap techniques to consider variations of Luftwaffe tactics, for it is clear that the Luftwaffe had no clear idea of how to achieve air supremacy.¹⁸ Early in the campaign bombers concentrated on Channel shipping. Later their targets were airfields and aircraft factories. Famously, on 6 September they altered their main target to London. This has been seen by many authors—among them contemporary actors,¹⁹ airpower theorists,²⁰ and historians²¹—as a crucial error, although from the contemporary Luftwaffe perspective it reflected a dilemma. Should the bombers destroy the RAF in production and on the ground, or were they merely bait to draw RAF fighters for the escorting Luftwaffe fighters to destroy? And what if the RAF declined action? Ultimately London was the target which the RAF must defend, a deceptive echo of German First World War naval attacks on east coast English towns designed to bring out the Royal Navy.²²

It has been argued that the most important component of the RAF's victory was its information system, aided by a strategic defensive over home territory.²³ The counterpoint of this is that the Luftwaffe, on the offensive over enemy territory, had very poor information, and thus little sense of which tactic was proving effective. A part of the British narrative, most famously argued in the eponymous work by Derek Dempster and Derek Wood, is that the victory was by a narrow margin.²⁴ We have added our voice elsewhere to the chorus of claims that the margin was not so narrow, based on high British fighter production and the large number of fighters stationed elsewhere in the United Kingdom and not directly

17. Johnson and MacKay, "Lanchester models and the Battle of Britain," Figure 1.

18. Bungay, *Most Dangerous Enemy*, chap. 30.

19. Carl Spaatz, "Strategic Air Power: Fulfillment of a Concept," *Foreign Affairs* 24, no. 3 (1946): 385–96; Peter Townsend, *Duel of Eagles* (London: Cassell, 1970); W. S. Churchill, *Their Finest Hour*, vol. 2 of *The Second World War*, 2nd ed. (London: Cassell, 1950).

20. John A. Warden, *The Air Campaign: Planning for Combat* (London: Brassey's, 1989), 103.

21. For example, Kenneth Macksey, *Military Errors of World War Two* (London: Arms & Armour, 1987), 45.

22. A discussion of German attacks on U.K. coastal towns in 1914 is given by Norman Friedman, *Fighting the Great War at Sea: Strategy, Tactics and Technology* (Annapolis, Md.: Naval Institute Press, 2014).

23. Sue Holwell and Peter Checkland, "An information system won the war," *IEEE Proceedings—Software* 145, no. 4 (1998): 95–99.

24. Derek D. Dempster and Derek Wood, *The Narrow Margin: The Battle of Britain and the rise of air power 1930–1940* (London: Hutchinson, 1961).

employed in the battle.²⁵ The strongest constraint was probably pilot training, for although newly trained pilot numbers were matching losses for most of the battle, RAF Fighter Command's leader Air Chief Marshal Hugh Dowding was strongly of the view that neither a novice monoplane fighter pilot, nor an experienced pilot of other types, was worth a combat-experienced fast monoplane fighter pilot.²⁶

In our exercise, weighted bootstraps allow us to model alternative campaigns in which the Luftwaffe prolongs or contracts the different phases of the battle and varies its targets. To avoid the switch to London is an obvious counterfactual. A more subtle possibility is to alter the belief of Luftwaffe leader Reichsmarschall Hermann Goering—analogueous to that of Air Vice-Marshal Trafford Leigh-Mallory, on the RAF side—that a fighter force must be destroyed in the air.²⁷ It is a common opinion that had the Luftwaffe prolonged its attacks on airfields, and indeed on any targets south of London, it would have had a much greater chance of victory.²⁸ Here we can quantify this, with weighted resamples in which the Luftwaffe chooses its targets in different proportions. The third and perhaps the deepest counterfactual is to make Hitler strongly in favor of invasion from the outset. In fictionalized accounts this leads to an invasion within a month of Dunkirk. More realistically, we allow for planning and preparation by all three arms, including the achievement of air superiority prior to invasion—the Luftwaffe was only able to make the necessary bases in France available during June. So our counterfactual brings forward the air campaign by three weeks.

In all of our cases, a governing factor is the necessity for neap tides, which are optimal for both military and seafaring reasons: to give slow strings of barges freedom of movement, to reduce tidal races over Channel sandbanks, and to permit an ebb tide to assist the barges down-Channel and allow beaching just before dawn with some moonlight.²⁹ This results in a fortnightly cycle, with invasion possible for three days either side of the quarter-moons on 26 August, 8 September, 24 September, and 8 October. Thus an initial decision for invasion needs to be made during one of the weeks beginning 13 August, 25 August, 11 September, or 25 September. To allow for necessary preparations for invasion—in reality the first feasible date was mid-September—the window beginning 13 August is included only in those counterfactuals in which the drive for invasion is brought forward by three weeks.

25. MacKay and Price, "Safety in Numbers"; Bungay, *Most Dangerous Enemy*; Deighton, *Fighter*.

26. Hugh Dowding, "Battle of Britain despatch," reprinted in *Royal Air Force Air Power Review* 18, no. 2 (2015): items 181–93.

27. Townsend, *Duel of Eagles*.

28. See, for example, Warden, *The Air Campaign*, for whom this is a matter of airpower theory, and the correct concentration of force. That airfield attacks were a winning tactic for the Luftwaffe is, as noted above, contested by Bungay, *Most Dangerous Enemy*.

29. *German Plans for the Invasion of England*; Grinnell-Milne, *Silent Victory*, chap. 7: "Time and the Tides."

4. Methods

4.1. Data

In order to better understand the Battle of Britain, we gathered a variety of data, presented in Tables 1 and 2. [All tables appear at the end of this article.] Table 1 consists of British and German total airframe losses (Hurricanes, Spitfires, or otherwise; and fighters, bombers, or otherwise; respectively), British pilot casualties, and primary target type (C = docks, shipping, and coastal; R = reconnaissance merely; A = aerodromes; L = London, Kent, and Thames estuary). Table 2 additionally provides the number of British sorties and (British estimates of) German sorties as well as regional weather (R = rain, C = clear, O = overcast).³⁰

We also divide the campaign into four phases (P) in Table 1, following the official history.³¹ These are (P1): 10 July–7 August, principally of coastal attacks and armed reconnaissance; (P2): 8 August–18 August, of heavy attacks on mostly coastal targets; (P3): 24 August–6 September, of sustained attacks gradually concentrating on airfields; and finally (P4): 7 September–31 October, following the Luftwaffe's switch to London as its principal target. Note the five-day lull between (P2) and (P3) and denoted 0 in Table 1; we treat this lull as a separate phase in our scenarios. We do not include 25 September and 16 October when reweighting based on target; these days correspond respectively to an attack on Filton and to general German air sweeps.

4.2. Victory criterion

We now need a “victory” criterion, by which we mean a trigger for invasion. Within RAF and War Cabinet policy there is no one obvious change which might have occurred and would have constituted the defeat of Fighter Command; the need to retain a force capable of contesting invasion, combined with the ability to withdraw Fighter Command northwards, ensures this. Further, we find no evidence of a plan for such a single collective withdrawal, which would probably rather have been gradual, and not immediately obvious to the Germans, who sometimes erroneously assumed that unrelated airfields belonged to Fighter Command.³²

30. Airframe losses were compiled from Winston G. Ramsey, *The Battle of Britain: Then and Now*, 5th ed. (London: Battle of Britain prints international, 1989). British pilot casualties were also compiled from Ramsey, *Battle of Britain*, and were cross-checked with the RAF Battle of Britain Honour Roll [online: raf.mod.uk/campaign/battle-of-britain-75th/the-few/battle-of-britain-roll-of-honour/, accessed 19 September 2017] and with Kenneth G. Wynn, *Men of the Battle of Britain: A Biographical Dictionary of The Few* (Barnsley, U.K.: Frontline, 2015). Targets are from Hugh Dowding, *Enemy Air Offensive Against Great Britain, 1941–1947*, held as AIR 2/7771, The National Archives (TNA), Kew, United Kingdom. Weather is from the RAF Campaign Diaries [online: raf.mod.uk/history/campaign-diaries.cfm, accessed 10 April 2017]. Sortie numbers are from T. C. G. James, *The Battle of Britain*, vol. 2 of *Air Defence of Great Britain*, ed. S. Cox, Royal Air Force Official Histories, new ed. (New York: Routledge, 2000). German sortie numbers are not generally available before 1 August.

31. James, *Battle of Britain*.

32. Deighton, *Fighter*, 216; Bungay, *Most Dangerous Enemy*, 208, 221.

In terms of the constraints on Fighter Command, it was pilot supply that approached criticality. British monoplane fighter airframe production, running at about 400 per month,³³ would always have been sufficient to provide a modern aircraft for every available pilot. In contrast the supply of newly trained pilots was running at around 260 per month, supplemented by non-U.K. volunteers and refugees, and pilots re-allocated from other types. As noted above, Dowding considered a novice fighter pilot worth less than a combat-experienced one, and this is consistent with, for example, the 501 Squadron figures,³⁴ with 33 percent of novice pilots lost within a month compared with 22 percent of experienced ones. If we therefore assign a novice pilot a value of 67 percent/78 percent = 0.85 of a pilot lost, a reasonable estimate of the strength of Fighter Command, $BS(t)$, might therefore be $BS(t) := B_0 + sbt - BL$.

Here B_0 is the initial Fighter Command pilot strength, $B_0 = 1259$ on 6 July 1940 or $B_0 = 1094$ on 15 June, all assumed to be trained and experienced.³⁵ The new pilot contribution is $s = 0.85$, $b = 11$ is the average daily complement of new pilots, and BL is the total number of British pilots lost, so that $BS(t) = B_0 + sbt - BL$ is the total Fighter Command pilot strength at time t . Pilot losses BL are computed using data from a number of sources, given in Table 1: essentially they include all pilots killed, seriously wounded, or missing in action, and exclude the slightly wounded, who typically returned to action within a few days.

We now need to decide how the values of $BS(t)$ might be used to trigger invasion. Let T (for “T-Tag”) be the planned date of invasion, which we recall must lie within three days either side of the quarter-moon Q . Recall further that an initial decision for invasion needs to be taken on $T - 10$. For invasion to be triggered, the Luftwaffe must appear to have been gaining air superiority during (let us say) the five days before this, and for invasion not to be cancelled or postponed the same must apply for five days after the initial decision—beyond this, too much effort would have been put into preparations such as the sowing of minefields for poor air combat reports to cause cancellation. So we choose some critical value BS_c according to the procedure outlined in the next paragraph, and say that invasion is triggered if $BS < BS_c$ throughout the period from $T - 15$ to $T - 5$ for any T between $Q - 3$ and $Q + 3$. Thus the final date for which we simulate air combat is 6 October.

What critical threshold value BS_c of BS would constitute the defeat of Fighter Command? As noted earlier, the crux of our method is not to attempt to answer this directly, but rather to calibrate it to prior beliefs using bootstrap methods. Imagine three historians of differing views. One of them believes that the British margin of victory was nil—that the battle was won on a coin toss—and thus that the Germans had a 50.0 percent probability of victory. A second believes that the British had a modest margin of victory, that it would have taken a moderate

33. Dempster and Wood, *Narrow Margin*, 104.

34. Bungay, *Most Dangerous Enemy*, 373.

35. Dempster and Wood, *Narrow Margin*, Appendix 11.

amount of deviation from the expected (average) result for the Germans to win, and thus that the British probability of victory was 84.0 percent, corresponding to one so-called “standard deviation” σ from the expected (average) value in a normal distribution (a “bell curve”). A third believes that a German victory was very unlikely, and would have taken double such a deviation from the average (a “ 2σ event”), and thus (on a bell curve) that the British probability of victory was 97.7 percent. We then run a simple bootstrap on the Battle of Britain as actually fought, which results in a bell curve of outcomes centred on the actual outcome, and choose the three values of BS_c which generate the three historians’ British victory probabilities specified above.

We then use these three values of BS_c in our counterfactual scenarios, resulting for each scenario in three new probabilities. These are robust to small changes in the form of the victory criterion, since this merely mediates between the figures of interest, which are each historian’s belief (expressed as a victory probability estimate) about the actual battle, and the belief which it would then be rational for them to assign, on the basis only of the evidence from the actual fighting, to each counterfactual scenario.³⁶

4.3. Counterfactual scenarios

In the most radical counterfactual (CF) fiction the Luftwaffe’s initial hopes of swift achievement of (at least) air superiority are realized, followed by an early invasion.³⁷ We cannot and do not pursue such ideas: rather our counterfactuals are air campaigns which depart from the actual campaign in their dates or targeting but which are built up using data from it. Instead we consider five counterfactuals that can be well-posed in terms of our bootstrapping method. These are summarized in Table 3.

CF1: What if the switch to bombing London had not occurred?

That the Luftwaffe switch to bombing London was an error is a standard argument, as noted earlier. To capture it here we simply extend P3 to 6 October, eliminating P4 entirely.

CF2: What if Hitler had been fundamentally in favor of invasion from the outset?

In this case we assume that planning would be brought forward: German navy commander Grand Admiral Erich Raeder’s visit to Hitler on 21 May would, in its effects, have taken the place of that of 20 June; air campaign planning would have been initiated much earlier than the actual 30 June.³⁸ We take the net result as

36. That is, to the extent to which they believe that additional days’ fighting would have followed the pattern of actual days’ results. Since the bootstrap uses only the actual days of fighting, what it cannot do is to include any of the unlikely things that one could argue might have happened, but did not. There are no “black swans” in bootstrapped counterfactuals. Nassim Nicholas Taleb, *The Black Swan: The impact of the highly improbable* (New York: Random House, 2007).

37. Forester, *Gold from Crete*; Macksey, *Invasion*; Messenger, *Battle of Britain*.

38. Macksey, *Invasion*, 13.

bringing forward the air campaign by three weeks—as much as seems reasonable given the Luftwaffe’s need to make the Channel-littoral airbases operational. Thus we bring forward P1 to 16 June–17 July, and spread P2 and P3 proportionally over 18 July–6 September, with P4 thereafter. Since the battle begins early, this also gives time for the Germans to take advantage of the 26 August neap tides.

CF3 combines CF1 (no fourth phase) and CF2 (early onset):

We take CF2, but with no switch to London: bring forward P1 to 16 June–17 July, and spread proportionally P2 and P3 over 18 July–6 October.

For our next counterfactual we switch from contracting or prolonging phases to alterations of targeting. In the actual battle the numbers of days for the principal target types were $(A, C, L, R) = (16, 47, 36, 13)$.

CF4: What if Goering and his staff had believed that Fighter Command could be more easily destroyed on the ground than in the air?

Peter Townsend notes the belief of both Goering and staff officer Paul Deichmann that Fighter Command would be more easily destroyed in the air than on the ground (paralleling the beliefs of Big Wing advocate Leigh-Mallory in the RAF).³⁹ Indeed, Townsend records Deichmann’s view that the Luftwaffe should not destroy radar stations, whose work would simply bring the RAF’s fighters to the Luftwaffe’s, facilitating their destruction.⁴⁰ Thus for this counterfactual we take an eighty-nine-day battle terminating on 6 October, with R unchanged, L untargeted, and A exceeding C , with $(A, C, L, R) = (43, 33, 0, 13)$.

CF5 combines CF2 (an early start) with CF4 (targeting of Fighter Command on the ground):

We take $(A, C, L, R) = (54, 42, 0, 17)$. to combine a commitment for invasion with a firm belief in the destruction of Fighter Command on the ground as a prerequisite.

In addition to these well-posed counterfactuals, we will also look at the problems caused by trying to determine the impact of the weather on the Battle of Britain. We use our attempt to create counterfactual weather for the battle as a cautionary tale.

5. Results

5.1. Unweighted bootstrapping: the Battle of Britain

Before tackling the counterfactuals, we first need to apply the unweighted bootstrap, creating many new samples (henceforth we call these resamples “trials”) of the Battle of Britain from the actual days’ fighting. We begin with the results of a bootstrap with 100,000 trials. Compared to a standard 10,000 trials this will allow us to better fit a normal distribution and thereby obtain suitable critical values BS_c . We then compare the effects of bootstrapping to the battle as actually fought with a standard 10,000 trials. With these baselines in mind, we can then proceed to address the genuine counterfactuals.

39. Townsend, *Duel of Eagles*, 325. Townsend himself is quite clear: “Goering made a crucial error. For Fighter Command was more vulnerable on the ground than in the air” (p. 379).

40. Townsend, *Duel of Eagles*, 360.

As discussed earlier, to begin exploring the counterfactuals we must first obtain the threshold values which match various prior beliefs as to the probability of the Luftwaffe's obtaining air superiority. To do so, we will impose a normal distribution on the results of a large number of trials without any reweighting.⁴¹ The results of such a calibration run are shown in Figure 1. [All figures appear at the end of this article.] We observe immediately that the normal distribution provides a good fit. Thus we can take the expected value (the average or mean) of the normal distribution as the 50.0 percent threshold (median). The standard deviation and second standard deviation to the left of the average then agree well with our desired 84.0 percent and 97.7 percent probabilities of British victory. The corresponding thresholds are 1,437.5, 1,383, and 1,328.5 pilots respectively. To summarize: the historian who believes that the German invasion decision was evenly balanced would use a threshold pilot strength of 1,437.5 in our victory criterion, while the historians who believe in moderate and large British margins of victory would use 1,383 and 1,328.5 respectively.⁴²

Next, we compare our now-calibrated bootstrap to the battle as actually fought. We use the same number of trials as for our counterfactuals: 10,000. Owing to the lower number of trials, we should expect more variation, meaning that the results will be less precise and may deviate from our ideal values. The results of this run can be seen in Figures 2 and 3. Figure 2 is equivalent to Figure 1, but now for the smaller number of trials. Figure 3 shows plots of the day-to-day number of British pilots, with the actual number of pilots (by day) superimposed.⁴³

5.2. *Weighted bootstrapping: operational counterfactuals*

CF1: What if the switch to bombing London had not occurred?

The results of our first counterfactual, where the Luftwaffe does not switch target to London (that is, enter into phase 4), are shown in Figure 4. It is immediately clear that the probability of British victory has significantly decreased. If one believed that the British won the real battle with probability 50 percent, the

41. To be clear: we are not imposing a normal distribution on the number of pilots evaluated on a single day, nor on one neap cycle's possible launch dates. Instead, we are imposing a normal distribution on the lowest number of pilots during any of the three possible neap windows. In this way, we ensure that the probabilities correspond to whether or not Germany launches an invasion at all during a given trial, and prevent overcounting trials where the number of pilots remains low multiple times.

42. Note that fractional values are possible due to our assigning less value to new pilots.

43. A box-and-whisker plot shows the spread of data by way of five values—the median, quartiles, and 5th and 95th centiles—supplemented by outliers. The middle value is the median, above and below which 50 percent of the data lie. The lower (respectively, upper) edges of the boxes correspond to the 1st (resp. 3rd) quartiles, which divide the data into the lowest 25 percent (resp. 75 percent) and highest 75 percent (resp. 25 percent). The distance between the 1st and 3rd quartiles is then used to compute the locations of the whiskers, beyond which all points are considered outliers. As we should expect, the number of pilots from day to day in the actual battle matches well with the trends of the bootstrap, due to ordering the days of our trials in order of phase.

implied threshold now yields a British victory probability of just 9.1 percent. Our lowest threshold, which had given the British a victory probability of 97.7 percent in the real battle, now corresponds to 63.7 percent. This reinforces the common narrative that the switch to targeting London was a mistake.

CF2: What if Hitler had been fundamentally in favor of invasion from the outset?

Our second counterfactual is grimmer still for the British: an eager Hitler pushes for an earlier beginning to the campaign, catching Fighter Command with approximately 165 fewer pilots initially available. The constant threat allows the Luftwaffe to engage earlier RAF pilots who in the real battle would have had more training and combat experience. Additionally, it gives the German forces access to the earliest possible invasion date on 24 August. Figure 5 shows the damage this does to the probability of British victory: the 50 percent victory possibility has now become 0.3 percent, and the most optimistic 97.7 percent threshold is now just 18.0 percent.

CF3 combines CF1 (no fourth phase) and CF2 (early onset):

As one might now expect, combining an early attack with no switch to London decreases further the viability of the British defense. This counterfactual also helps remind us of the prospect of diminishing returns: British chances are not utterly destroyed by the combined changes. The probability of British victory is simply reduced further, as seen in Figure 6: the 50 percent victory probability is now 0.1 percent (CF2: 0.3 percent), while the 97.7 percent is now 8.3 percent (CF2: 18.0 percent).

CF4: What if Goering and his staff had believed that Fighter Command could be more easily destroyed on the ground than in the air?

Instead of a change in the phasing of the battle, we now investigate the effects of not attacking London at all. We have already mentioned the German belief that it was easiest to destroy the RAF in the air, lured up by bombers, especially over London. The effects of this belief are brought out by our fourth counterfactual, in which the Luftwaffe focuses much more on the airfields. Results are shown in Figure 7. The results of CF4 are a more drastic variation of CF1 (no fourth phase), but are inevitably closely aligned with it. The 50 percent victory probability is reduced to 1.1 percent (CF1: 9.1 percent) and 97.7 percent to 33.6 percent (CF1: 63.7 percent). Not only should the Germans not have made their early September switch, but they paid dearly by choosing to attack London at all.

CF5 combines CF2 (an early start) with CF4 (targeting of Fighter Command on the ground):

We now come to our last counterfactual: if Hitler had been eager for invasion, giving the Luftwaffe an early start, and if the Luftwaffe had been dedicated to targets associated with destroying the RAF on the ground. This is our most negative counterfactual for Britain, as shown in Figure 8: if one believes that the probability of British victory in the battle as actually fought was 50 percent, or even 84 percent, then this alternative yields fewer than 1 in 10,000 victories for Britain. If one holds that the probability of British victory was 97.7 percent, this

situation yields just 0.4 percent for the British. However much one might not believe in a “narrow margin” of British victory in the battle as actually fought, the British aerial victory, it seems, depended very strongly on poor German choices. We summarize the results of all our counterfactuals in Table 4.

5.3. A cautionary tale: counterfactual weather

Finally, we briefly discuss problems with bootstrapping. Natural objections to the use of bootstrap techniques include changes in the way the data are distributed or hidden underlying common factors. Weather exhibits precisely these problems, and is illustrative of some of the difficulties.

Suppose one wanted to investigate the impact of the weather on the Battle of Britain—to imagine that it might have been more or less rainy or cloudy in 1940 than is typical for July to October. There are many problems in doing so. First: most available quantitative data are very coarse, or qualitative in nature, or insufficiently geographically precise. The second and more fundamental problem is that, in order to use the bootstrap, we must assume that the data are drawn from some unchanging distribution. Weather is not so here. Above all, since weather is time-dependent, by altering it we may inadvertently alter the proportions of other variables. For example, the phases of the battle are also time-dependent, and simple statistical tests show that weather and phase are related. In general, if the weather were unusually sunny during one phase but cloudy and rainy during another, then clearly it would be impossible to alter phasing without also altering the weather, and vice versa. If we were not aware of this fact, we might falsely ascribe our results to the influence of weather.

For illustrative purposes only, we proceed with an ill-posed bootstrap in which we alter the weather from the actual 1940 weather to more typical (median) weather by changing the proportion of clear days.⁴⁴ This leads to small increases in the probabilities of British victory: our 50 percent victory probability increases to 58 percent and our 97.7 percent increases to 98.5 percent. However, this is probably due almost entirely to the correlation between weather and phase. More important for us is that there is no indication from the procedure that something has gone wrong. The bootstrap is an unthinking tool that does exactly what is asked of it, leaving the user to make sure that assumptions of independence are justified.

6. Conclusions

We used the Battle of Britain data as a testbed for the bootstrapping technique in history. Our results match the standard historiographical view: the Germans

44. We used the 1929–79 record of sunlight hours at the Meteorological Office’s Oxford weather station [online: www.metoffice.gov.uk/pub/data/weather/uk/climate/stationdata/oxforddata.txt, accessed 24 July 2017] to give us typical ranges of sunlight during July to October, including 1940, and applied this to the more qualitative Meteorological Office 1940 Daily Weather Reports [online: digital.nmla.metoffice.gov.uk/archive/sdb%3AdeliverableUnit%7Ceda9f47f-c326-4991-ada4-a3e4c8bd11d9/, accessed 22 July 2017].

were most successful when attacking airfields in August and early September, and they blundered when they changed target to London. A major problem with this retrospective assessment, however, is that it did not occur to the Germans. Indeed, Wood and Dempster argue in their classic account that the Luftwaffe assumed that the British retreat north of the Thames had already occurred, hindering their necessary objective of destroying Fighter Command.⁴⁵ Thus, regardless of Hitler's motives for bombing London, the Luftwaffe considered it strategically essential.

As we noted above, the bootstrap technique relies on the assumption that, had the earlier attacks been extended, combat attrition would have continued in the same way. Whether the Germans would really have obtained continuing good results is a question beyond our methods. We do not and cannot know British tactical and strategic responses to a counterfactually improved German performance, and we revert to our central observation above: that the bootstrap can tell us only the possibilities which result from the battle as actually fought.

However, we can certainly nuance the view that the Battle of Britain was a narrow victory. For the battle as actually fought it may have been so, but the sheer scale of the reversal of probabilities when we vary German targeting policy or prolong the earlier phases of the battle suggests that the British victory in the air campaign was sensitively dependent on German strategy and Luftwaffe tactics, and could easily have been reversed, even were RAF pilots and tactics improving rapidly. Among such possibilities the switch to London (CF1) may not have been the most significant of the German errors: it only results in a movement of just over one standard deviation σ , a switch from a balanced campaign to one moderately favoring the Germans, or from moderately favoring the British to balance. Bringing the campaign forward by three weeks (CF2) has a much greater, almost 3σ effect: from significantly favoring one side to moderately favoring the other (or vice versa). Yet this would have required a fundamentally more aggressive approach towards Britain, with an early strategic understanding of the necessity of militarily defeating Britain.⁴⁶ Almost as significant—somewhere between 2σ and 3σ —would have been an understanding that Fighter Command should be defeated at least in part on the ground. The narrow margin, it seems, was not in the battle actually fought, but in German failure to pursue an optimal strategy.

That our analysis stresses the importance of an earlier beginning to the German campaign promotes a broader consideration of the topic. An Anglo-centric appreciation of the Battle of Britain tends implicitly to assume that the battle of France ended effectively with the Dunkirk evacuation,⁴⁷ liberating significant Luftwaffe assets to prepare for an early Sea Lion.⁴⁸ In fact, the second

45. Dempster and Wood, *Narrow Margin*, 212.

46. As Macksey notes, "The basic reason for Germany's failure to invade Britain in 1940 is . . . the lack of any preconceived will or intention to do so." Macksey, *Military Errors*.

47. Martin S. Alexander, "After Dunkirk: The French Army's Performance against Case Red, 25 May to 25 June 1940," *War in History* 14, no. 2 (2007): 219–64.

48. Macksey, *Invasion*.

phase of the German campaign in France, Fall Rot, was a hard-fought contest which occupied most of June and included a “second BEF” of 200,000 men. This German assault was initially repelled and, as with all “Blitzkrieg” campaigns, ultimate success depended on the whole-hearted participation of the Luftwaffe.⁴⁹ The initiation of a costly air and land campaign against the United Kingdom with an unbeaten and increasingly competent French military in the field and finite German resources stretched to the limit would constitute a very bold initiative. Despite Hitler’s reputation as a gambler, he was keenly aware of real possibilities of failure at this stage, and indeed his 24 May “halt order” to German armored units approaching Dunkirk is often attributed to this well-founded anxiety. Heinz Guderian, the famously thrusting tank commander, was unpleasantly surprised by the disappearance of Hitler’s previous boldness as his XIX Corps broke through towards the Channel coast, noting that it had “never occurred” to him that Hitler “would now be the one to be frightened by his own temerity and would order our advance to be stopped at once.”⁵⁰

Had Hitler expressed greater enthusiasm slightly later than the capture of Dunkirk and after the breaking of French resistance, as assumed in our bootstrap, an early air campaign against Britain would need still more counterfactual support: an assumption that the Luftwaffe had suffered less badly in conquering France. Stephen Bungay notes that the Luftwaffe lost 1,428 aircraft in the campaign, “about half its operational strength” with another 488 damaged. Clearly it was not only the RAF which needed to recuperate, and moreover the Germans had to rebuild their full force of fighters and bombers for the coming battle, whereas the RAF could focus on fighters alone.⁵¹ Similar problems apply to alternative targeting policies.⁵² Had the Luftwaffe attacked the RAF and its infrastructure on the ground more effectively and with better concentration than it did, then our analysis suggests results would have been better. However, this would have required a vastly improved Luftwaffe intelligence organization with a greatly developed conception of the nature and ranked importance of the many targets available for attack and of the results that could be achieved by bombing them.

In our analysis, therefore, we have used real data to conceptualize possible alternative outcomes to the Battle of Britain and in effect reinforced the most conservative position based on the actual outcome: that invasion never had much prospect of success based on air supremacy without serious counterfactual changes. In terms of the air campaign the best prospects for invasion would have required an earlier start, though no such opportunity realistically presented itself in wider strategic terms. Less promising would have been an invasion initiated at the end

49. Alexander, “After Dunkirk.”

50. Heinz Guderian, *Panzer Leader* (London: Penguin, 2009), 109.

51. Bungay, *Most Dangerous Enemy*, 104–5.

52. These are “cotenability problems” in the sense of Philip E. Tetlock and Aaron Belkin, eds., *Counterfactual thought experiments in world politics: Logical, methodological, and psychological perspectives* (Princeton, N.J.: Princeton University Press, 1996).

of September without a previous switch to attacking London, reversing actual German consensus in favor of the London attack. The prospects for the invasion itself would not have been significantly improved in this case from those which caused Hitler and the high command to decline it.

Beyond the Battle of Britain, a primary purpose of this paper is methodological innovation: to illustrate how weighted bootstrapping can provide a natural and intuitive tool for historians to investigate unrealized possibilities quantitatively and, by doing so, inform historical controversies and debates. The results can be presented, very simply, as the alteration in a historian's probability estimate for the possible compared to the actual, promoting fuller analysis of actual historical outcomes.

However, this technique requires not only a sufficient number of data points—of actual events each drawn from an implied underlying distribution of possibilities—but that this distribution be unchanging and the events be independent. The technique is justified only to the extent to which one day's events do not influence the next, and neither the sum of such influences nor other hidden trends cause the nature of the events to change over time. These conditions are almost unknown in the entirety of a land campaign, but at sea and in the air the position is less clear. Nevertheless, the bootstrap technique can provide a useful way of quantifying what can be said using nothing but the data: it has the virtues of being transparent—in the sense that the meaning of the reweighting of events is immediately clear—and free of any mathematical models. In a sense the bootstrap is the most transparent and conservative possible statistical technique to apply in a historical context: it accepts the data as all the events that are or can be known, and constructs alternative histories only as recombinations of these known events. But it thereby inevitably produces normal distributions (“bell curves”); it cannot access exceptional eventualities unseen in the actual data.

For future work, the main problem is the tension between the desire for a large number of data points, to better sample the underlying distribution, and the need for this distribution to be unvarying. A large number of data points will typically occur over either a long period of time or a large region of space, and either way there are likely to be important variations in the process which generates the data. Further, when the number of data points becomes too large, the bootstrap becomes less interesting precisely because all resamples look much the same—most of them will be close to the mean. One natural possibility potentially balancing these tensions would be an investigation of the Battle of the Atlantic with each data point corresponding to a single convoy, with hierarchical modelling which takes account of changing equipment and tactics. Possibilities exist to use bootstrapping at higher resolution or in different scenarios, perhaps in political or economic aspects of strategy. Either way, if assumptions regarding independence are valid, then bootstrapping can be a valuable historical tool.

Figures and Tables begin on the next page.

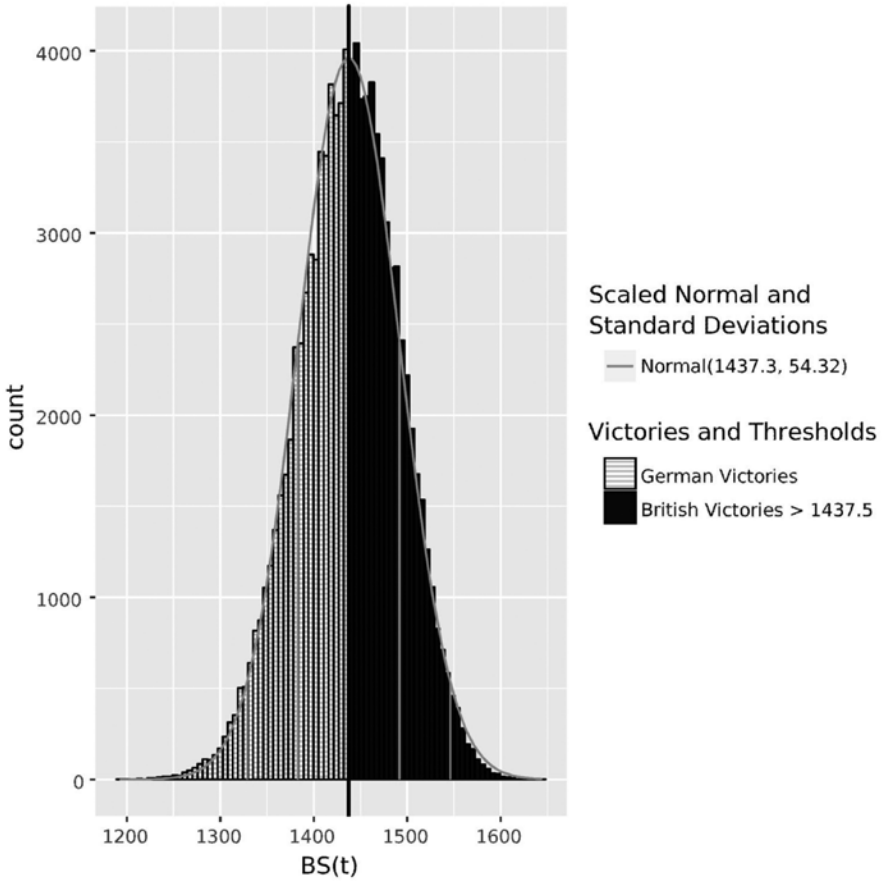


Figure 1: A calibration run in which 100,000 trials are run and then fitted with a normal distribution (superimposed grey curve). We use this run to inform our choices of thresholds, here shown as the leftmost two superimposed vertical grey lines, representing two and one standard deviations below the mean, and the solid black line dividing German victories from British victories, representing the mean.

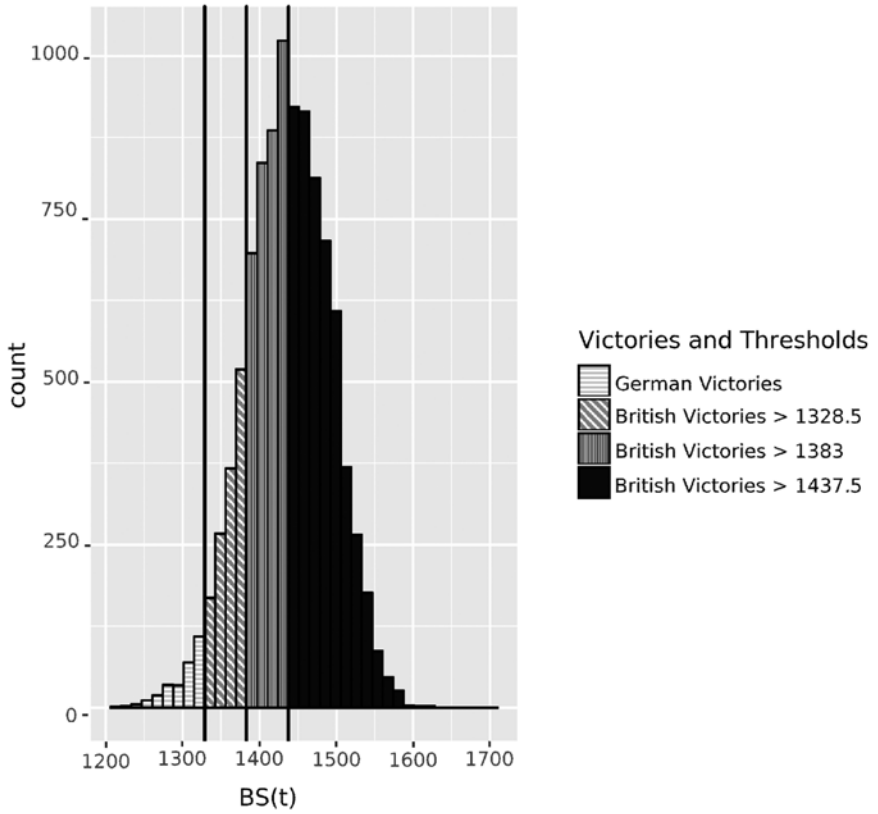


Figure 2: Bootstrapping the Battle of Britain with sampling in proportion to the phases as actually fought. The thresholds correspond to 97.2%, 83.9%, and 49.5% probabilities of British victories.



Figure 3: Bootstrapping the Battle of Britain with sampling in proportion to the phases as actually fought. The box-and-whisker plots show the day-to-day distributions of the number of pilots, the dashed vertical lines show boundaries between the phases of the battle as actually fought, and the solid curve is the number of pilots in the battle as actually fought.

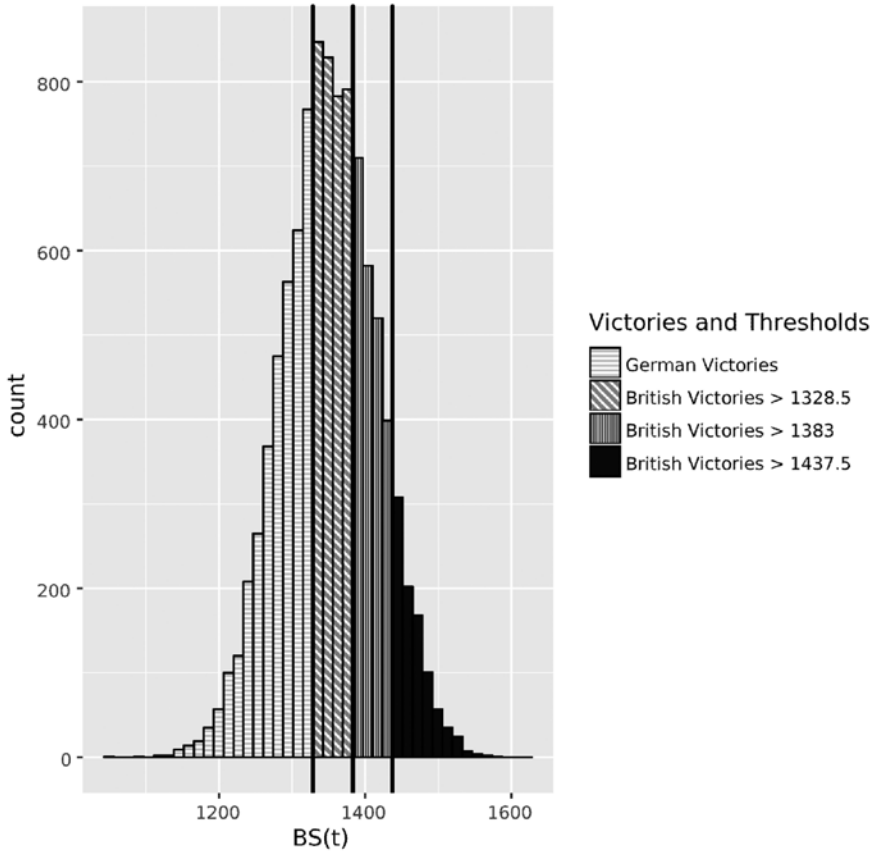


Figure 4: CF1. Bootstrapped results of the scenario where the Luftwaffe had not switched to targeting London by entering the fourth phase. Our thresholds now correspond to 63.7%, 31.2%, and 9.1% probabilities of British victory.

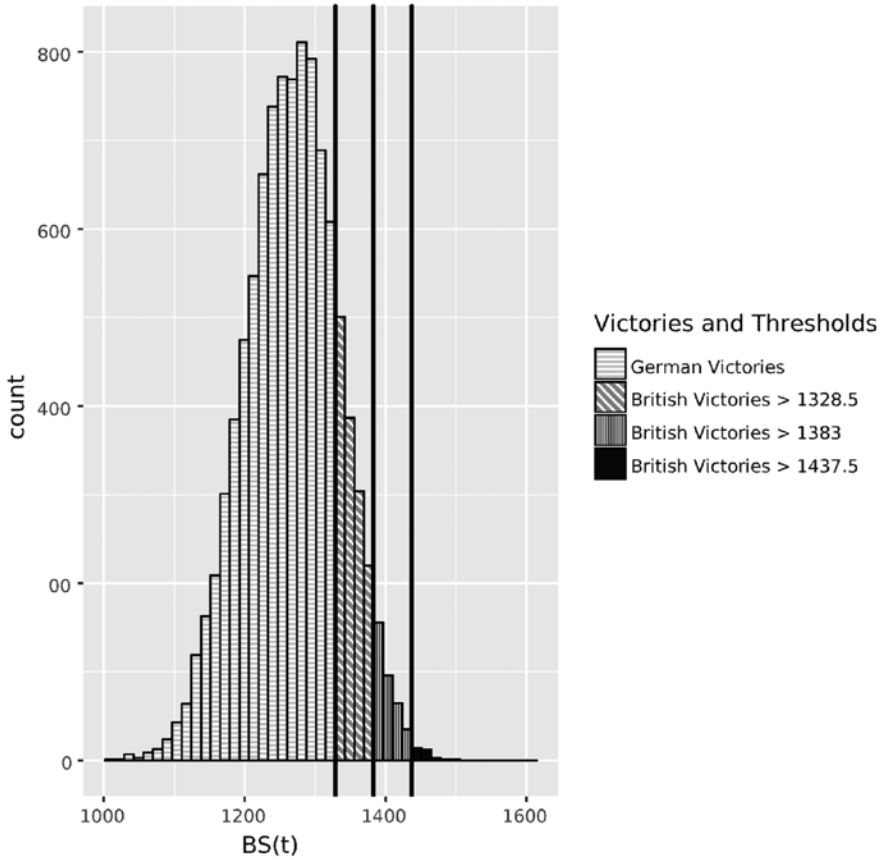


Figure 5: CF2. Bootstrapped results of the scenario where the Luftwaffe began their assault earlier. Our thresholds now correspond to 18.0%, 3.8%, and 0.3% probabilities of British victory.

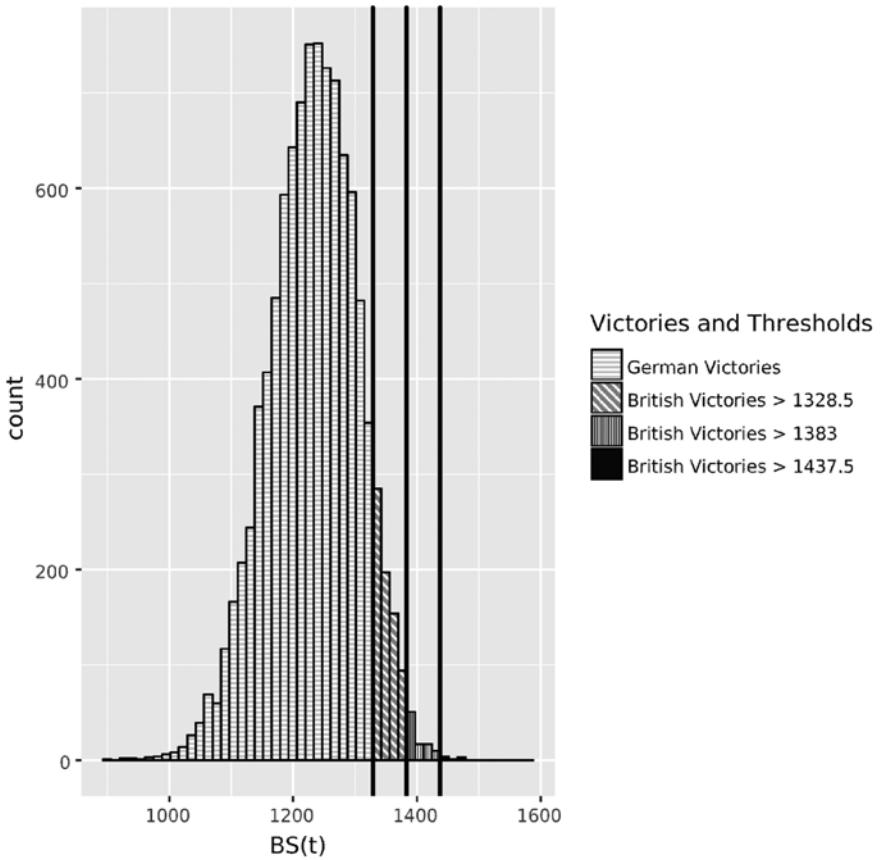


Figure 6: CF3. Bootstrapped results of the scenario where the Luftwaffe began their assault early and did not choose to switch to primarily targeting London by entering the fourth phase. Our thresholds now correspond to 8.3%, 1.0%, and 0.1% probabilities of British victory.

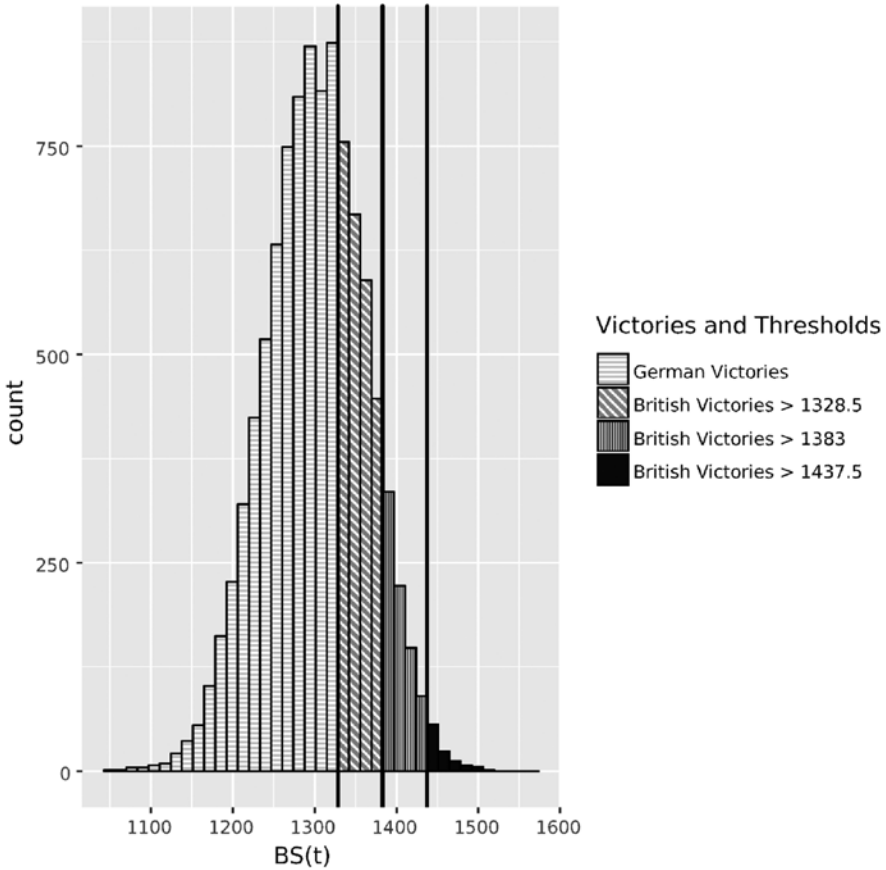


Figure 7: CF4. Bootstrapped results of the scenario where the Luftwaffe neglected to attack London entirely and had instead focused on the airfields. Our thresholds now correspond to 33.6%, 9.0%, and 1.1% probabilities of British victory.

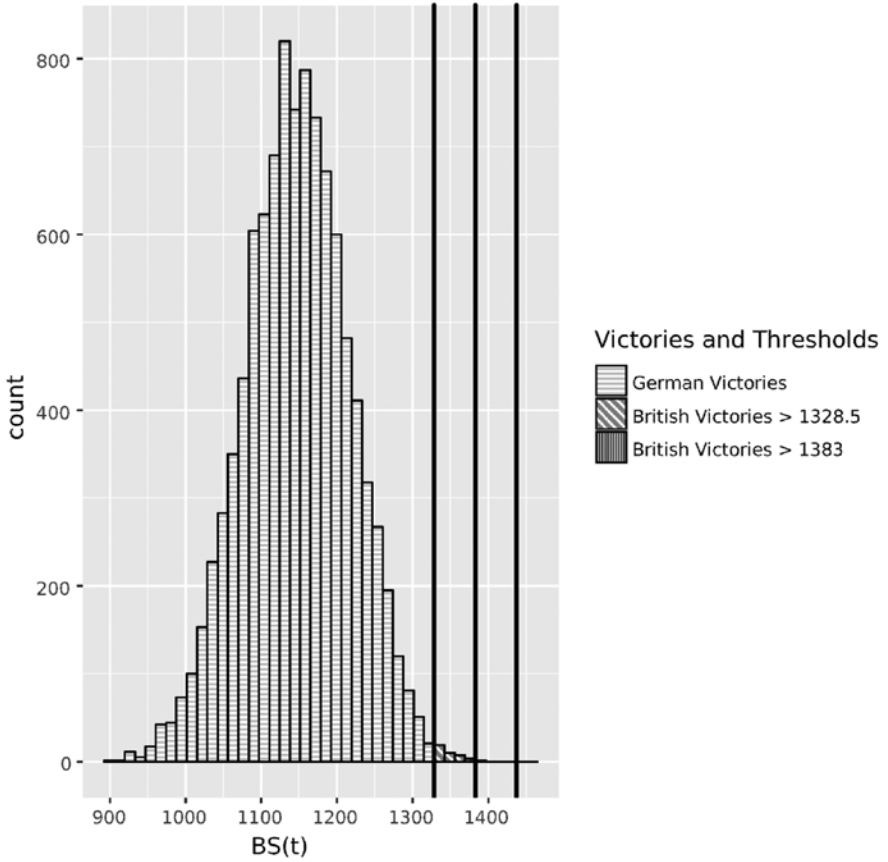


Figure 8: CF5. Bootstrapped results of the scenario where the Luftwaffe began their assault and did not attack London, instead focusing on the airfields. Our thresholds now correspond to 0.4%, 0.01%, and almost-nil probabilities of British victory.

**Table 1: Data used in Counterfactuals
Combat Loss and Target Data**

Day	Date	British Airframe Losses	German Airframe Losses	British Pilots Lost	British Pilots Wounded	British Pilots Slightly Wounded	Phase	Primary Target
1	7/10/1940	2	11	2	0	0	1	C
2	7/11/1940	6	17	3	1	0	1	C
3	7/12/1940	5	9	4	0	1	1	C
4	7/13/1940	6	6	5	0	1	1	C
5	7/14/1940	1	3	1	0	0	1	C
6	7/15/1940	2	5	0	1	0	1	C
7	7/16/1940	1	4	1	0	0	1	C
8	7/17/1940	1	4	1	1	0	1	C
9	7/18/1940	5	6	4	0	0	1	C
10	7/19/1940	10	5	4	4	0	1	C
11	7/20/1940	9	12	6	0	2	1	C
12	7/21/1940	2	12	1	0	0	1	C
13	7/22/1940	2	4	1	0	0	1	C
14	7/23/1940	2	5	0	1	0	1	C
15	7/24/1940	5	15	3	0	1	1	C
16	7/25/1940	9	19	7	2	2	1	C
17	7/26/1940	1	5	1	0	0	1	C
18	7/27/1940	2	5	2	0	0	1	C
19	7/28/1940	6	11	1	4	1	1	C
20	7/29/1940	6	11	3	1	0	1	C
21	7/30/1940	1	9	0	1	0	1	C
22	7/31/1940	7	7	4	2	0	1	A
23	8/1/1940	4	13	3	0	0	1	C
24	8/2/1940	3	7	1	0	0	1	C
25	8/3/1940	0	6	0	0	0	1	C
26	8/4/1940	1	2	2	0	0	1	C
27	8/5/1940	2	8	1	0	1	1	C
28	8/6/1940	6	6	1	0	1	1	R
29	8/7/1940	4	3	0	0	1	1	R
30	8/8/1940	21	24	17	1	3	2	C

Table 1 continued

Day	Date	British Airframe Losses	German Airframe Losses	British Pilots Lost	British Pilots Wounded	British Pilots Slightly Wounded	Phase	Primary Target
31	8/9/1940	3	6	1	0	0	2	R
32	8/10/1940	0	1	0	0	0	2	C
33	8/11/1940	28	38	25	1	3	2	C
34	8/12/1940	18	32	11	6	2	2	C
35	8/13/1940	15	39	4	1	5	2	C
36	8/14/1940	9	20	4	2	1	2	C
37	8/15/1940	35	76	16	8	5	2	C
38	8/16/1940	24	44	9	5	6	2	A
39	8/17/1940	2	5	0	1	0	2	R
40	8/18/1940	33	67	10	11	6	2	A
41	8/19/1940	5	11	2	0	1	0	C
42	8/20/1940	2	8	1	0	0	0	C
43	8/21/1940	4	14	0	0	1	0	C
44	8/22/1940	4	4	2	1	0	0	C
45	8/23/1940	1	8	0	0	0	0	C
46	8/24/1940	20	41	5	7	6	3	R
47	8/25/1940	18	23	11	1	3	3	C
48	8/26/1940	29	42	4	10	9	3	R
49	8/27/1940	7	11	2	0	0	3	A
50	8/28/1940	15	32	6	6	2	3	R
51	8/29/1940	10	24	2	2	4	3	A
52	8/30/1940	25	40	11	4	0	3	A
53	8/31/1940	41	39	9	18	7	3	A
54	9/1/1940	13	16	6	6	0	3	A
55	9/2/1940	14	37	4	10	2	3	A
56	9/3/1940	15	20	6	4	7	3	A
57	9/4/1940	17	28	11	2	4	3	A
58	9/5/1940	20	27	8	7	3	3	A
59	9/6/1940	20	33	8	7	8	3	A
60	9/7/1940	25	41	15	6	7	4	A

Table 1 continued

Day	Date	British Airframe Losses	German Airframe Losses	British Pilots Lost	British Pilots Wounded	British Pilots Slightly Wounded	Phase	Primary Target
61	9/8/1940	5	16	2	2	0	4	A
62	9/9/1940	17	30	6	4	7	4	A
63	9/10/1940	3	13	0	0	0	4	R
64	9/11/1940	29	29	14	8	7	4	L
65	9/12/1940	1	7	1	0	0	4	R
66	9/13/1940	3	7	2	0	1	4	L
67	9/14/1940	13	13	4	7	0	4	L
68	9/15/1940	31	61	16	5	6	4	L
69	9/16/1940	1	10	0	1	0	4	L
70	9/17/1940	6	8	3	1	1	4	L
71	9/18/1940	12	20	4	5	3	4	L
72	9/19/1940	0	10	0	0	0	4	L
73	9/20/1940	8	8	5	0	2	4	L
74	9/21/1940	1	11	1	0	0	4	C
75	9/22/1940	1	6	0	0	0	4	L
76	9/23/1940	11	17	3	4	0	4	C
77	9/24/1940	6	11	2	3	3	4	L
78	9/25/1940	6	16	3	1	0	4	(Filton)
79	9/26/1940	8	9	3	2	1	4	C
80	9/27/1940	28	57	20	4	2	4	L
81	9/28/1940	17	12	10	2	2	4	L
82	9/29/1940	6	9	2	3	0	4	C
83	9/30/1940	21	47	6	5	5	4	L
84	10/1/1940	7	9	4	1	0	4	L
85	10/2/1940	2	18	0	1	1	4	L
86	10/3/1940	1	9	1	1	0	4	C
87	10/4/1940	1	15	1	0	0	4	L
88	10/5/1940	7	14	2	3	1	4	L
89	10/6/1940	2	9	2	0	0	4	L
90	10/7/1940	17	19	9	2	4	4	C

Note:: Pilots Lost, Wounded, or Slightly Wounded are measured by incident. Thus some values represent pilots receiving a slight wound, flying again, and then receiving another wound.

Table 1 continued

Day	Date	British Airframe Losses	German Airframe Losses	British Pilots Lost	British Pilots Wounded	British Pilots Slightly Wounded	Phase	Primary Target
91	10/8/1940	8	17	7	0	0	4	L
92	10/9/1940	3	9	3	0	0	4	L
93	10/10/1940	8	12	6	1	0	4	L
94	10/11/1940	9	10	4	4	1	4	C
95	10/12/1940	11	13	5	2	3	4	C
96	10/13/1940	4	6	1	3	0	4	L
97	10/14/1940	1	4	1	0	0	4	R
98	10/15/1940	15	16	6	5	2	4	L
99	10/16/1940	3	15	3	0	0	4	(sweeps)
100	10/17/1940	5	16	5	1	0	4	L
101	10/18/1940	6	14	5	0	0	4	R
102	10/19/1940	1	6	2	0	1	4	L
103	10/20/1940	5	11	3	1	0	4	L
104	10/21/1940	2	7	2	0	0	4	L
105	10/22/1940	6	12	4	1	0	4	L
106	10/23/1940	1	4	1	0	0	4	R
107	10/24/1940	3	12	3	0	0	4	L
108	10/25/1940	14	24	6	5	1	4	L
109	10/26/1940	8	10	5	0	0	4	L
110	10/27/1940	14	16	6	1	1	4	L
111	10/28/1940	0	14	0	0	0	4	L
112	10/29/1940	12	28	5	1	2	4	L
113	10/30/1940	9	8	6	1	2	4	L
114	10/31/1940	0	2	0	0	0	4	R

LEGEND

Phases:

- 1 10 Jul - 7 Aug, principally of coastal attacks and armed reconnaissance
- 2 8 Aug - 18 Aug, of heavy attacks on mostly coastal targets
- 0 19 Aug - 23 Aug, of little interaction between the forces
- 3 24 Aug - 6 Sept, of attacks gradually concentrating on aerodromes
- 4 7 Sept - 31 Oct, following the switch to principally bombing London

Primary Targets:

- A aerodromes
- L London, Kent, and Thames estuary
- C docks, shipping, and coastal
- R reconnaissance

Table 2: Sortie and Weather Data

Day	Date	British Sortie	German Sortie	Channel & Coast Weather	London, Kent, and Estuary Weather	Midlands & North Weather
1	7/10/1940	609	NA	R	R	R
2	7/11/1940	452	NA	O	O	R
3	7/12/1940	670	NA	O	R	R
4	7/13/1940	449	NA	O	O	O
5	7/14/1940	593	NA	C	C	C
6	7/15/1940	470	NA	O	O	O
7	7/16/1940	313	NA	O	O	O
8	7/17/1940	253	NA	O	O	O
9	7/18/1940	549	100	O	R	O
10	7/19/1940	701	150	O	O	O
11	7/20/1940	611	100	O	R	R
12	7/21/1940	571	NA	C	C	C
13	7/22/1940	611	100	O	O	R
14	7/23/1940	470	NA	O	O	R
15	7/24/1940	561	NA	O	O	R
16	7/25/1940	641	NA	C	C	O
17	7/26/1940	581	NA	R	R	R
18	7/27/1940	496	NA	C	O	R
19	7/28/1940	794	NA	C	C	C
20	7/29/1940	758	NA	C	C	C
21	7/30/1940	688	NA	R	R	R
22	7/31/1940	395	NA	O	C	C
23	8/1/1940	659	100	O	O	C
24	8/2/1940	477	100	O	R	C
25	8/3/1940	425	50	O	O	O
26	8/4/1940	261	80	C	C	C
27	8/5/1940	402	110	O	C	C
28	8/6/1940	416	60	O	O	O
29	8/7/1940	393	70	C	O	C
30	8/8/1940	621	280	C	R	R

Table 2 continued

Day	Date	British Sortie	German Sortie	Channel & Coast Weather	London, Kent, and Estuary Weather	Midlands & North Weather
31	8/9/1940	409	110	O	O	O
32	8/10/1940	336	80	O	R	R
33	8/11/1940	679	370	O	O	O
34	8/12/1940	732	440	C	C	C
35	8/13/1940	700	450	O	C	C
36	8/14/1940	494	600	C	O	O
37	8/15/1940	974	650	C	C	C
38	8/16/1940	776	800	O	C	C
39	8/17/1940	288	50	C	C	C
40	8/18/1940	755	560	O	O	O
41	8/19/1940	383	400	O	O	O
42	8/20/1940	453	200	O	O	R
43	8/21/1940	589	170	O	O	O
44	8/22/1940	509	220	O	O	O
45	8/23/1940	482	270	O	O	R
46	8/24/1940	936	550	C	C	R
47	8/25/1940	480	325	O	O	O
48	8/26/1940	787	440	C	C	O
49	8/27/1940	288	50	O	O	R
50	8/28/1940	739	400	C	C	C
51	8/29/1940	498	390	O	R	R
52	8/30/1940	1054	600	C	C	C
53	8/31/1940	978	800	O	C	C
54	9/1/1940	661	490	C	C	C
55	9/2/1940	751	750	C	C	C
56	9/3/1940	711	550	O	C	R
57	9/4/1940	678	550	O	C	R
58	9/5/1940	662	460	O	C	C
59	9/6/1940	987	730	C	C	C
60	9/7/1940	817	700	O	O	O

Table 2 continued

Day	Date	British Sortie	German Sortie	Channel & Coast Weather	London, Kent, and Estuary Weather	Midlands & North Weather
61	9/8/1940	305	200	O	O	O
62	9/9/1940	466	430	C	R	R
63	9/10/1940	224	50	O	O	O
64	9/11/1940	678	500	O	C	C
65	9/12/1940	247	80	R	R	R
66	9/13/1940	209	130	R	R	R
67	9/14/1940	860	400	O	R	R
68	9/15/1940	705	600	C	C	C
69	9/16/1940	428	250	R	R	R
70	9/17/1940	544	350	R	R	R
71	9/18/1940	1165	800	C	C	C
72	9/19/1940	237	75	R	R	R
73	9/20/1940	540	150	O	O	O
74	9/21/1940	563	260	C	C	C
75	9/22/1940	158	140	O	O	O
76	9/23/1940	710	300	C	C	C
77	9/24/1940	880	500	O	O	O
78	9/25/1940	668	290	C	C	C
79	9/26/1940	417	220	C	C	C
80	9/27/1940	939	850	C	R	R
81	9/28/1940	770	300	O	O	C
82	9/29/1940	441	180	C	C	C
83	9/30/1940	1173	650	C	C	C
84	10/1/1940	NA	NA	O	O	O
85	10/2/1940	NA	NA	C	C	C
86	10/3/1940	NA	NA	R	R	R
87	10/4/1940	NA	NA	R	R	R
88	10/5/1940	NA	NA	R	R	R
89	10/6/1940	NA	NA	R	R	R
90	10/7/1940	NA	NA	O	O	O

Table 2 continued

Day	Date	<i>British Sortie</i>	<i>German Sortie</i>	Channel & Coast Weather	London, Kent, and Estuary Weather	Midlands & North Weather
91	10/8/1940	NA	NA	O	O	O
92	10/9/1940	NA	NA	R	O	O
93	10/10/1940	NA	NA	R	R	R
94	10/11/1940	NA	NA	C	R	C
95	10/12/1940	NA	NA	O	O	O
96	10/13/1940	NA	NA	O	O	O
97	10/14/1940	NA	NA	O	R	O
98	10/15/1940	NA	NA	O	C	C
99	10/16/1940	NA	NA	O	O	O
100	10/17/1940	NA	NA	R	R	R
101	10/18/1940	NA	NA	O	O	O
102	10/19/1940	NA	NA	O	O	O
103	10/20/1940	NA	NA	O	O	O
104	10/21/1940	NA	NA	O	O	O
105	10/22/1940	NA	NA	O	O	O
106	10/23/1940	NA	NA	R	R	R
107	10/24/1940	NA	NA	O	C	C
108	10/25/1940	NA	NA	O	O	O
109	10/26/1940	NA	NA	O	O	R
110	10/27/1940	NA	NA	O	O	O
111	10/28/1940	NA	NA	O	O	O
112	10/29/1940	NA	NA	O	O	O
113	10/30/1940	NA	NA	R	R	R
114	10/31/1940	NA	NA	R	O	O

LEGEND

NA	Not Available, no estimate exists
C	Clear
O	Overcast, Cloudy
R	Rainy

Table 3: Summary of Counterfactual (CF) Scenarios

Scenario	Summary	Starting Date	Bootstrap Reweighting (Days of Target or Phase)			
			Airfield P1	Coast P2	London P3	Recon P4
Real	Actual Battle	10 July	16 29	47 11	36 14	13 30
CF1	No switch to London	10 July	29	11	49	0
CF2	Early Start	16 June	32	18.7	23.8	30
CF3	No London and early start	16 June	32	29.7	37.8	0
CF4	Target airfields	10 July	43	33	0	13
CF5	Target airfields and early start	16 June	54	42	0	17

Table 4: Summary of Counterfactual (CF) Scenario Outcomes

Scenario	Summary	Probabilities of British Victory by Threshold		
		1,437.5	1,383	1,328.5
Real	Actual Battle	50.0%	84.0%	97.7%
CF1	No switch to London	8.92%	31.33%	64.17%
CF2	Early Start	0.32%	3.72%	18.65%
CF3	No London and early start	0.11%	1.22%	8.23%
CF4	Target airfields	0.93%	8.81%	33.03%
CF5	Target airfields and early start	0%	0%	0.33%

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