THE3 Standard Decision Problems

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Halting Problem (HP)

Input A Turing machine M and a string $w \in \Sigma^*$;
Question Does M halt on w?

Λ-Halting Problem (Λ-HP)

Input A Turing machine M;
Question Does M halt on Λ?

abc-Halting Problem (abc-HP)

Input A Turing machine M;
Question Does M halt on *abc*?

Uniform Halting Problem (UHP)

Input A Turing machine M;
Question Does M halt on all inputs $w \in \Sigma^*$?

Self-Accepting Problem (SAP)

Input A Turing machine M;
Question Is $\mathrm{e}(M) \in \mathrm{L}(M)$?

Membership Problem for Semidecidable Languages (MP)

Input A Turing machine M and a string $w \in \Sigma^*$;
Question Is $w \in \mathrm{L}(M)$?

Accept-Λ Problem (A-Λ)

Input A Turing machine M;
Question Is $\Lambda \in \mathrm{L}(M)$?

Accept-All Problem (AAP)

Input A Turing machine M;
Question Is $\mathrm{L}(M) = \Sigma^*$?

Emptiness Problem (EP)

Input A Turing machine M;
Question Is $\mathrm{L}(M) = \emptyset$?

Equivalence Problem (EQP)

Input Turing machines M_1 and M_2;
Question Is $\mathrm{L}(M_1) = \mathrm{L}(M_2)$?

Inclusion Problem (IP)

Input Turing machines M_1 and M_2;
Question Is $\mathrm{L}(M_1) \subseteq \mathrm{L}(M_2)$?

Property Problem (c.f. Rice's Theorem) (P-P)

Input A Turing machine M;
Question Does M have the property P?

End of Decision Problems

More variations on these problems can be found in the THE3 lecture notes, but the ones listed here are the most important. The author of this document is Oliver Dixon and may be contacted via e-mail at od641@york.ac.uk.