

Schedulability Analysis for Fixed Priority Real-Time Systems with Energy- Harvesting

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⁽³⁾ *Real-Time Systems Research Group, University of York*

RTNS'14, 10 October 2014

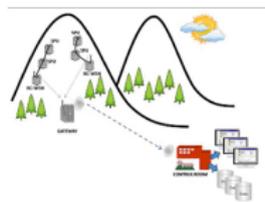
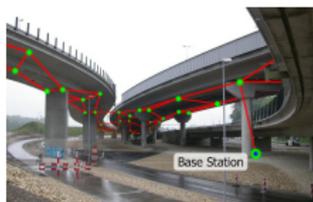
Outline

- 1 Motivation
- 2 Model
- 3 Schedulability Analysis
- 4 Experiments
- 5 Conclusion and Future Work

Energy Harvesting Systems

Energy-Harvesting

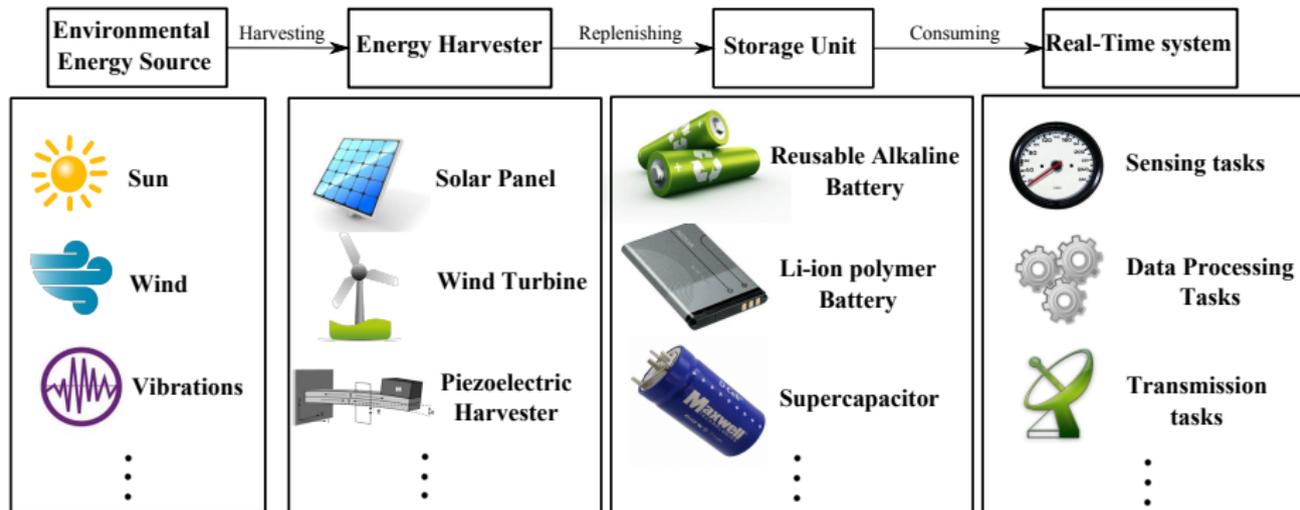
The process by which energy is captured from a system's environment and converted into usable electric power.



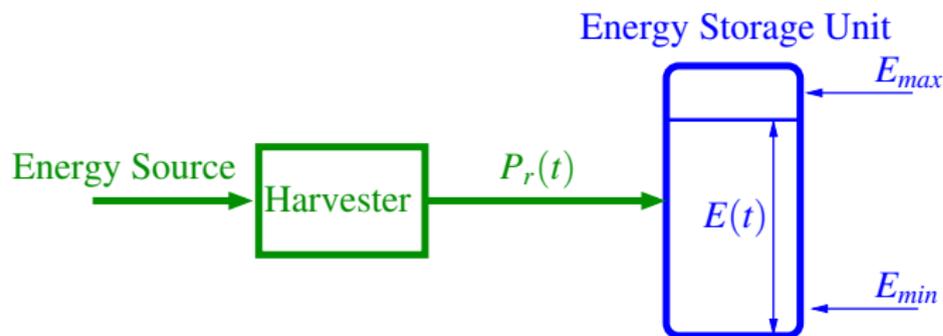
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Energy Model



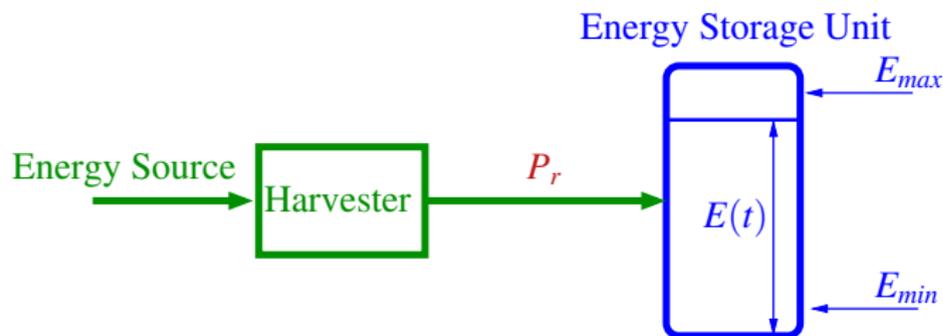
- **Energy Source Model**

- Energy Sources: solar, thermal, mechanical, vibration, ...
- Harvester: transform the environmental energy into electrical power.

- **Energy Storage Unit Model**

- Energy Unit: battery, super-capacitor, ...
- Store the harvested energy: $P_r(t)$ is the energy replenishment function.
 Constant rate of replenishment: $P_r(t) = P_r$
- The energy stored may vary between two levels E_{min} and E_{max} .

Energy Model



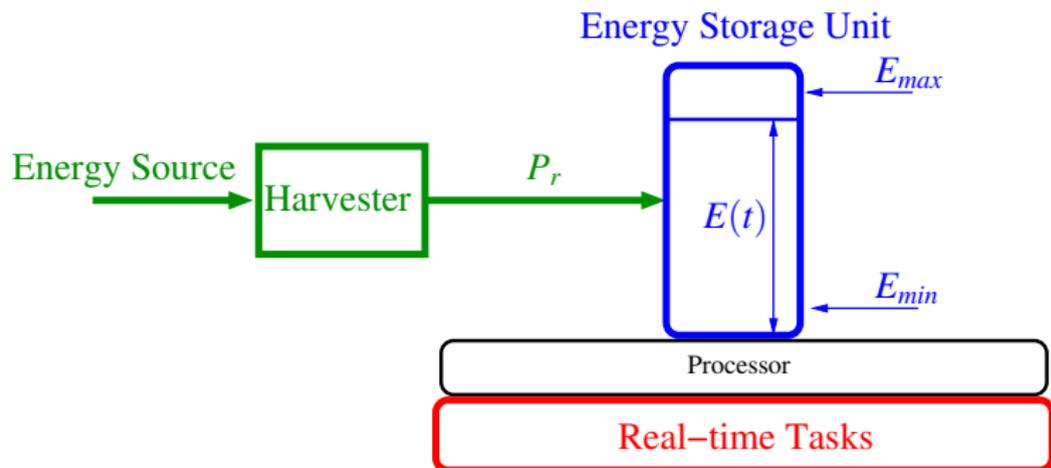
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Task Model



A set of sporadic tasks $\tau_i(C_i, P_i, E_i, T_i, D_i)$

C_i : worst-case execution time,

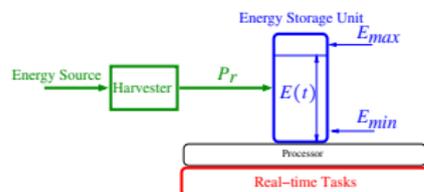
P_i : worst-case power consumption,

$E_i = P_i \times C_i$: worst-case energy consumption,

T_i : minimal inter arrival time,

D_i : relative deadline ($D_i \leq T_i$).

The Model



$$P_r = 3$$

$$E_{max} = 3$$

$$E_{min} = 0$$

$$\tau_1 : C_1 = 2$$

$$P_1 = 6$$

$$E_1 = 12$$

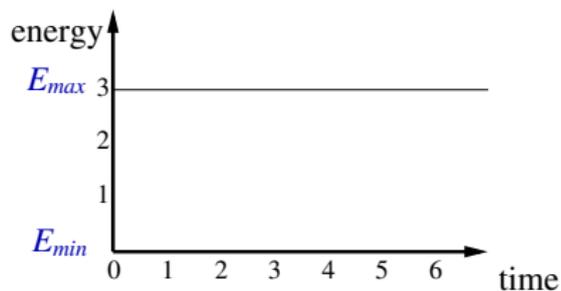
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$$\tau_2 : C_2 = 1$$

$$P_2 = 1$$

$$E_2 = 1$$

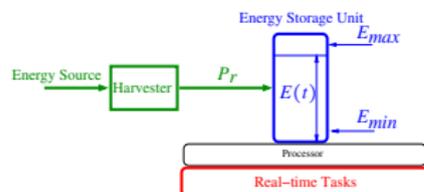
$$T_2 = D_2 = 5$$



τ_1 _____

τ_2 _____

The Model



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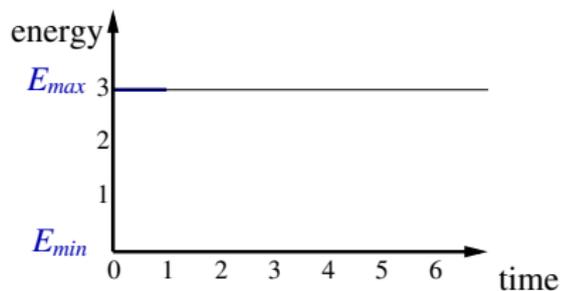
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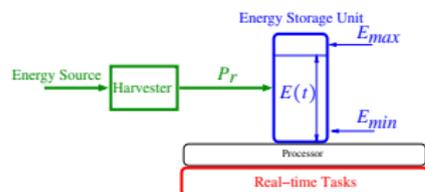
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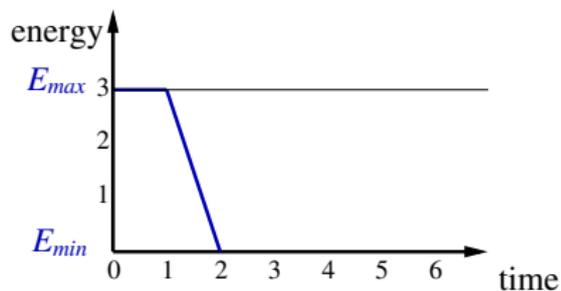
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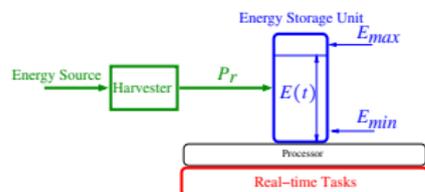
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■ Consuming Task: $P_1 > P_r$

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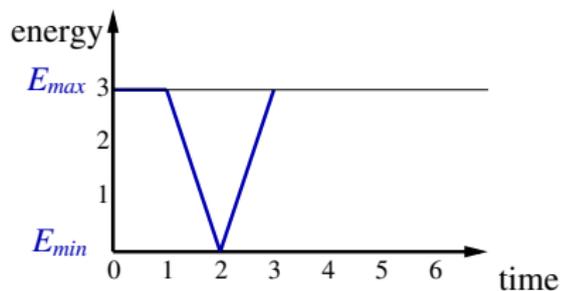
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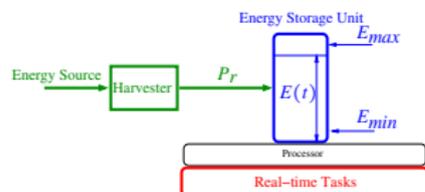
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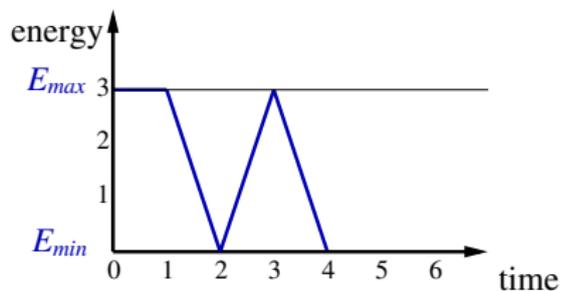
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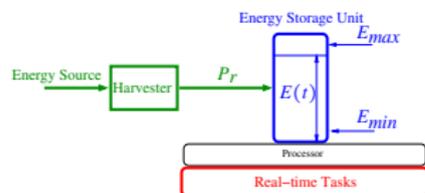
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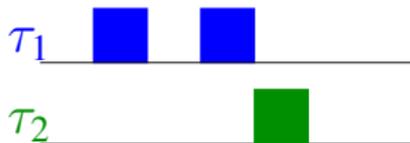
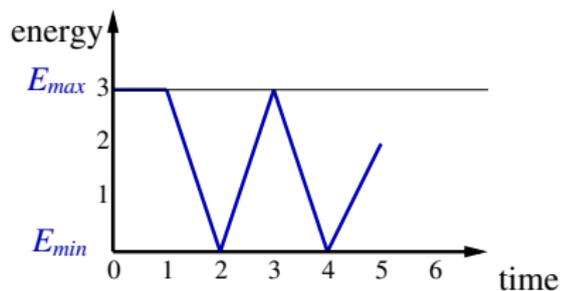
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■ Consuming Task: $P_1 > P_r$

■ Gaining Task: $P_2 \leq P_r$

The Scheduling Problem

The Model

- Storage Unit: Constant rate replenishment P_r and E_{max} , E_{min} the maximal and minimal level of energy.
- A set $\Gamma = \Gamma_c \cup \Gamma_g$ of sporadic tasks $\tau_i = (C_i, P_i, E_i, T_i, D_i)$ in priority order with $D_i \leq T_i$:
 - Consuming Tasks: $\Gamma_c = \{\tau_i \in \Gamma, P_i > P_r\}$
 - Gaining Tasks: $\Gamma_g = \{\tau_i \in \Gamma, 0 \leq P_i \leq P_r\}$

Feasibility

A task set is feasible if all the tasks meet their deadlines: *timing constraints* and $\forall t \geq 0$ the energy level is between E_{min} and E_{max} : *energy constraints*.

Related Work

- An algorithm for Frame-Based Model,



A. Allavena and D. Mossé,

“Scheduling of Frame-based Embedded Systems with Rechargeable Batteries”,
Workshop in conjunction with RTAS, 2001.

- *LSA* Algorithm assumes variable execution time,



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- *PFP_{ASAP}* Algorithm



Y. Abdeddaïm, Y. Chandarli and D. Masson,

“The Optimality of *PFP_{ASAP}* Algorithm for Fixed-Priority Energy-Harvesting Real-Time Systems”,
ECRTS, 2013.

The PFP_{ASAP} Algorithm

- **Execute** tasks whenever there is **enough energy** available in the battery.
- **Replenish** as long as needed **to execute one time unit** of the highest priority active task.

PFP_{ASAP} is an Energy Work-Conserving FPPS Algorithm

The processor is idle only if there is insufficient energy to schedule at least one time unit of the highest priority active task.

Optimality

PFP_{ASAP} is **optimal** in the class of energy work conserving fixed priority pre-emptive scheduling algorithms in the case where **all the task consume energy** ($\Gamma = \Gamma_c$).

Our Goal

Provide a schedulability test for PFP_{ASAP} when the set of tasks is composed of both **consuming** and **gaining** tasks.

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Classical Response Time Analysis

Method

- 1 Find the **worst-case scenario**: scenario where τ_i is subject to the maximum possible delay,
- 2 Compute R_i **the longest response time of task τ_i** : is the response time of τ_i in the worst-case scenario,
- 3 **Exact schedulability test**: If $\forall \tau_i, R_i \leq D_i$ the task set is schedulable.

Work-Conserving FPPS with $D_i \leq T_i$

- 1 Worst-case scenario for task τ_i : **Synchronous release** of all the tasks,
- 2 R_i is given by the smallest $t > 0$ that satisfies $t = F(i, t)$ with:

$$F(i, t) = C_i + \text{Maximum interference}$$
 from higher priority tasks in $[0, t)$

$$F(i, t) = \sum_{h < i} \left\lceil \frac{t}{T_h} \right\rceil \times C_h$$

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Response Time Analysis for PFP_{ASAP}

Response Time of a task τ_i

C_i + Replenishment time + Interference from higher priority tasks.

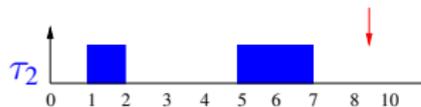
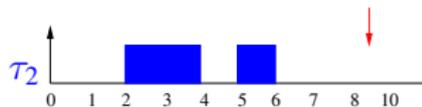
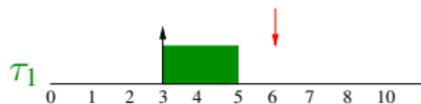
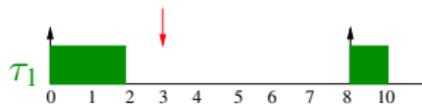
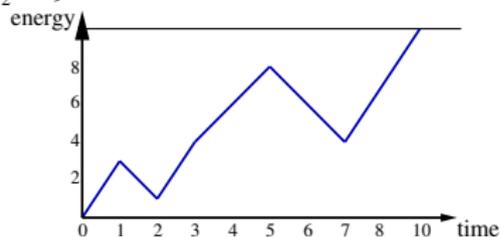
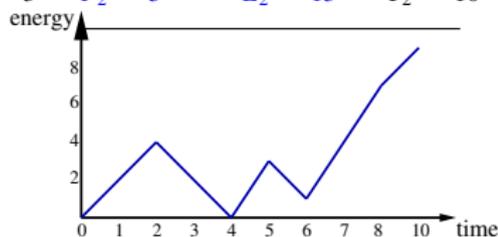
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$\tau_2 : C_2 = 3$	$P_2 = 5$	$E_2 = 15$	$T_2 = 10$	$D_2 = 9$

Response Time Analysis for PFP_{ASAP}

Response Time of a task τ_i

C_i + Replenishment time + Interference from higher priority tasks.

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Response Time $\tau_2 = 6$

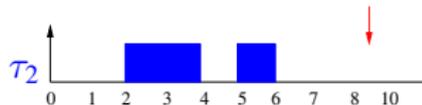
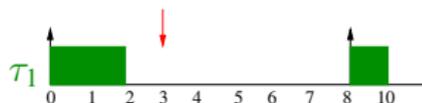
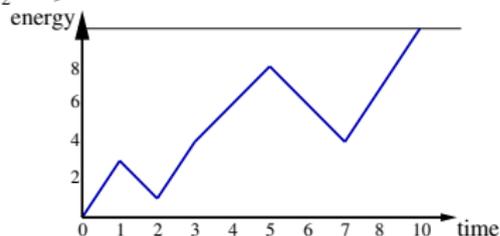
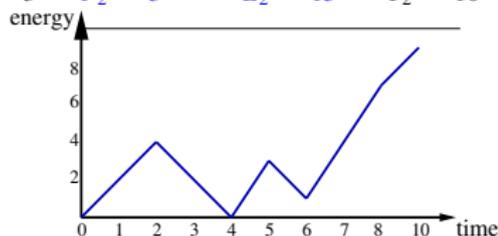
Response Time $\tau_2 = 7$

Response Time Analysis for PFP_{ASAP}

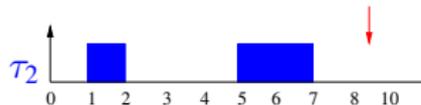
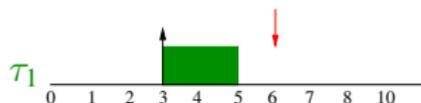
Worst-Case Scenario

The **synchronous release** of all the tasks is **no longer the worst-case scenario**.

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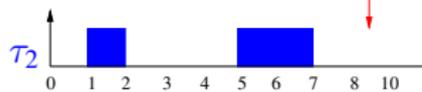
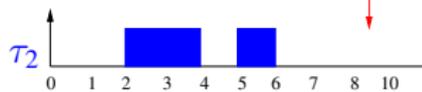
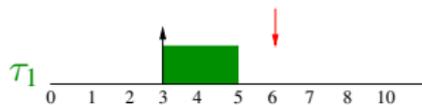
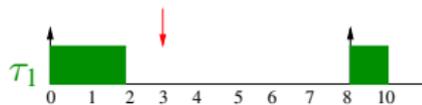
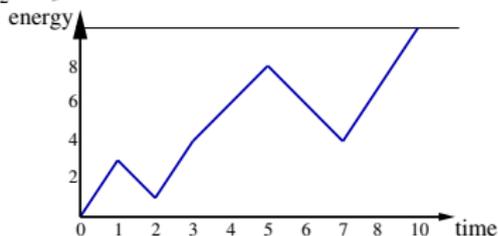
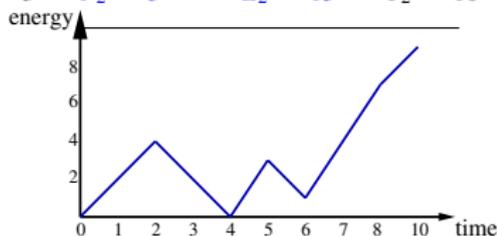
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Response Time Analysis for PFP_{ASAP}

- worst-case scenario is unknown,
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Upper bound R_i to build a sufficient schedulability test.

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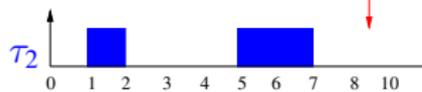
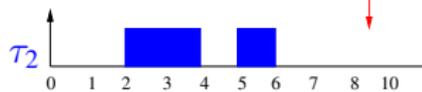
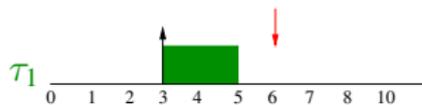
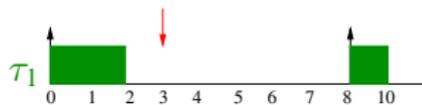
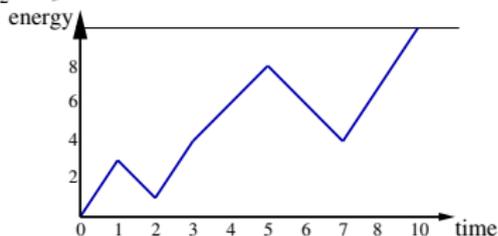
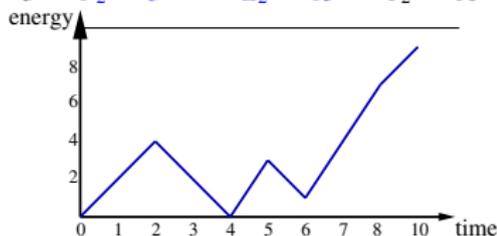
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Response Time $\tau_2 = 7$

Bounding Response Time

- We require a monotonically non-decreasing function $F(i, w)$ that **upper bounds the length of the worst-case response time of task τ_i** within an interval of length w ,
- The upper bound R_i^{UB} of the worst-case response time R_i corresponds to **the smallest $w > 0$ that satisfies $F(i, w) = w$** .
- We define for every task τ_i a **virtual scenario** that:
 - 1 **Maximizes** the amount of **interference** from higher priority tasks,
 - 2 **Maximizes** the amount of **replenishment time** needed.

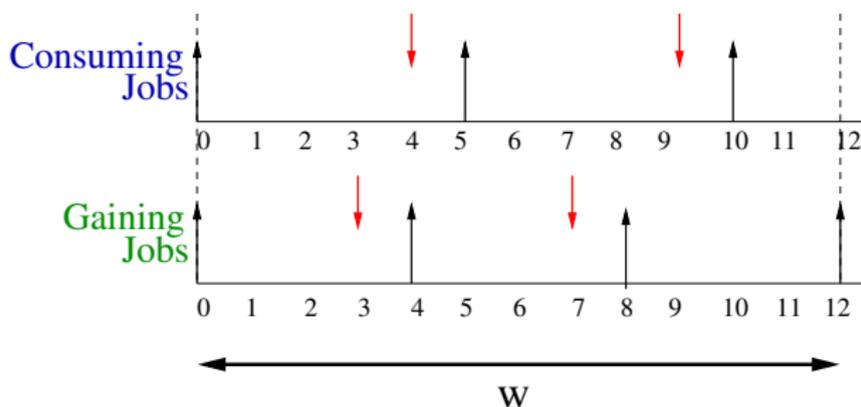
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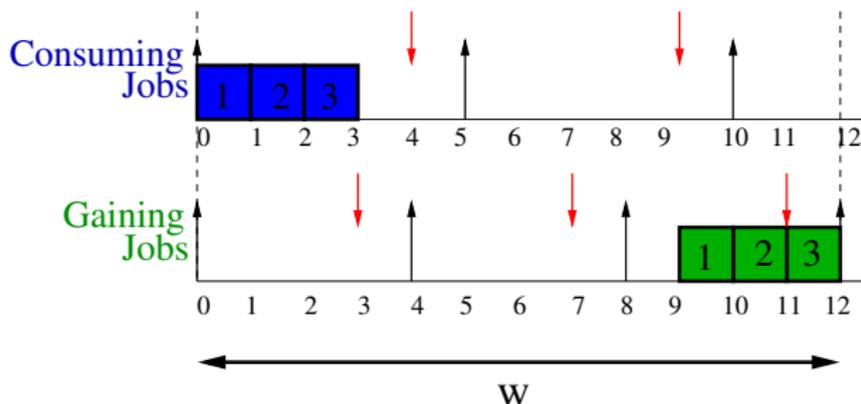
Maximal Interferences

For every task τ_i released in a window of length w , the maximal number of higher priority jobs that are active in this window is

$$\sum_{h < i} \left\lceil \frac{w}{T_h} \right\rceil = \sum_{h < i, \tau_h \in \Gamma_c} \left\lceil \frac{w}{T_h} \right\rceil + \sum_{h < i, \tau_h \in \Gamma_g} \left\lceil \frac{w}{T_h} \right\rceil$$

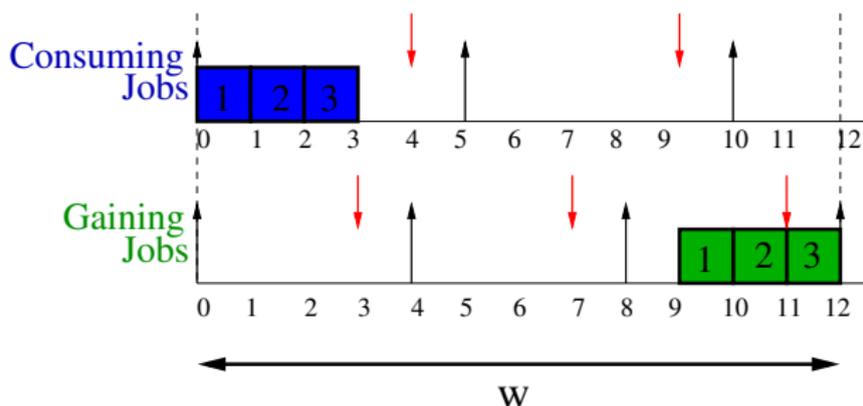


Maximal Replenishment



To upper bound the replenishment in a window of length w we consider a **virtual sequence** where:

- 1 The **battery is empty** at the beginning of the window,
 - to minimize the energy budget of interval w
- 2 All the **consuming jobs** are **before** all the **gaining jobs**.
 - to maximize replenishment periods

Upper Bound R_i^{UB1} 

$$F^{UB1}(i, w) = \underbrace{\sum_{h \leq i, \tau_h \in \Gamma_c} \left\lceil \frac{w}{T_h} \right\rceil \times ((P_h - P_r) \times C_h / P_r)}_{\text{maximum replenishment needed}} + \underbrace{\sum_{h \leq i, \tau_h \in \Gamma_c} \left\lceil \frac{w}{T_h} \right\rceil \times C_h}_{\text{consuming jobs}} + \underbrace{\sum_{h \leq i, \tau_h \in \Gamma_g} \left\lceil \frac{w}{T_h} \right\rceil \times C_h}_{\text{gaining jobs}}$$

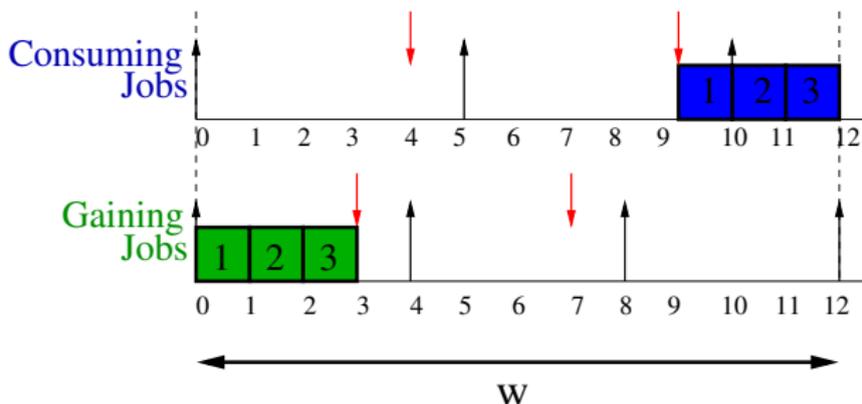
Sufficient Schedulability test $UB1$

- $F^{UB1}(i, w)$ is a **monotonically non-decreasing function** of w and $F^{UB1}(i, w) > C_i$
- R_i^{UB1} the upper bound of the longest response time of task τ_i is given by the smallest $t > 0$ that satisfies $w = F^{UB1}(i, w)$ with:

$$F^{UB1}(i, w) = \left[\frac{\sum_{h \leq i, \tau_h \in \Gamma_c} \left\lceil \frac{w}{T_h} \right\rceil \times E_h}{P_r} \right] + \sum_{h \leq i, \tau_h \in \Gamma_g} \left\lceil \frac{w}{T_h} \right\rceil \times C_h$$

- **Sufficient Schedulability test $UB1$:** $\forall \tau_i, R_i^{UB1} \leq D_i$

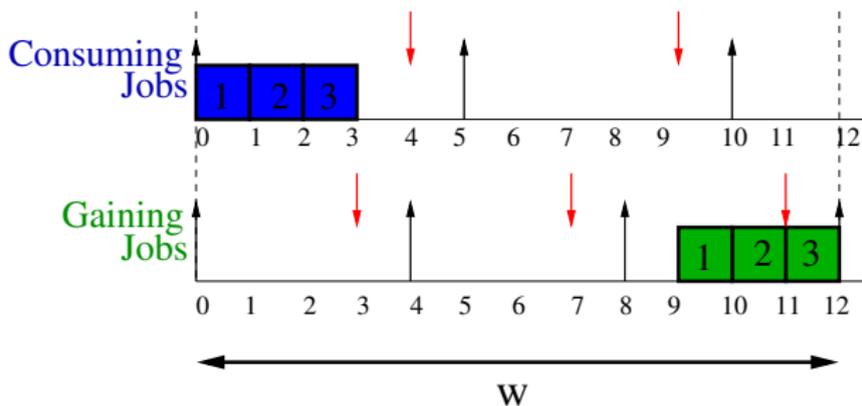
Necessary Schedulability Test *LB1*



To **lower bound** the worst-case response time of a task τ_i released in a window of length w we consider a **virtual sequence** where:

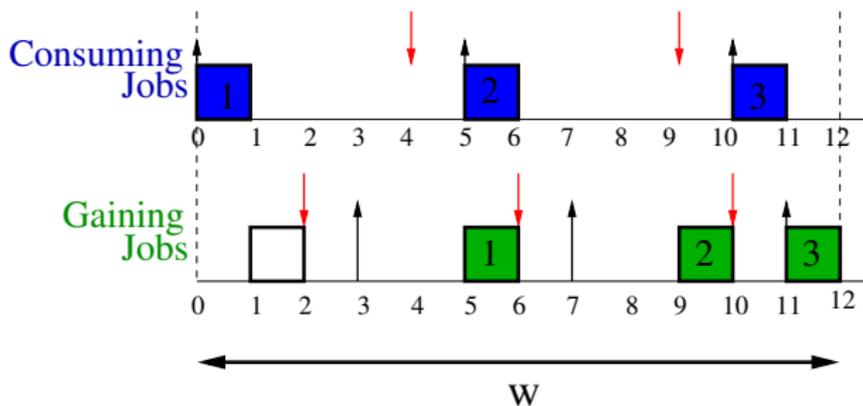
- 1 The **battery is empty** at the beginning of the window,
- 2 All the **gaining jobs** are **before** all the **consuming jobs**.

A Tighter Upper Bound R_i^{UB2}



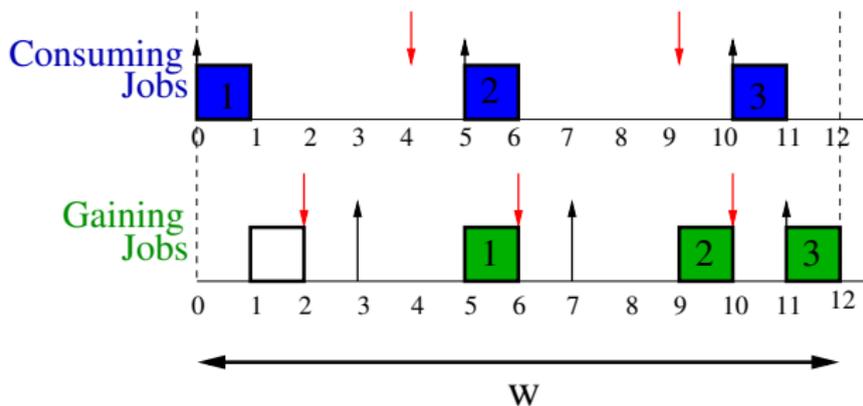
- Consuming jobs as soon as possible,
- Gaining jobs as late as possible

A Tighter Upper Bound R_i^{UB2}



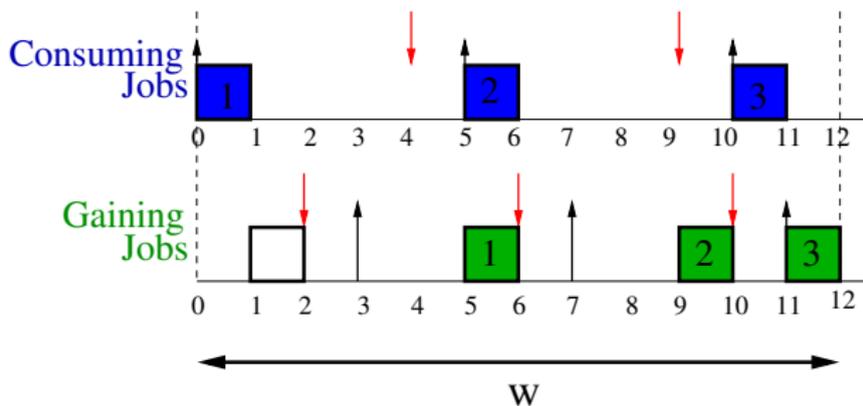
- Consuming jobs as soon as possible,
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Sufficient Schedulability Test $UB2$



- To compute $F^{UB2}(i, w)$, we compute the response time of the **virtual sequence**:

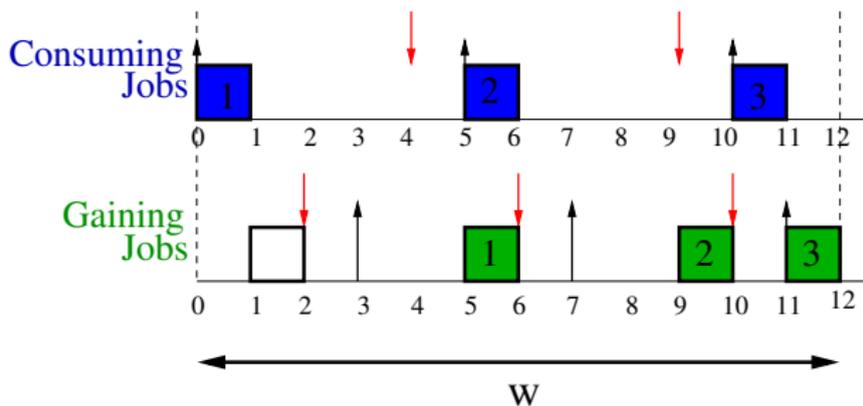
Sufficient Schedulability Test $UB2$



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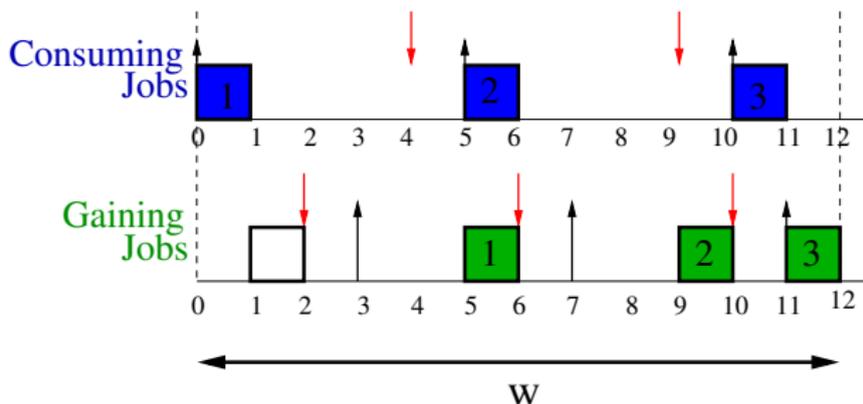
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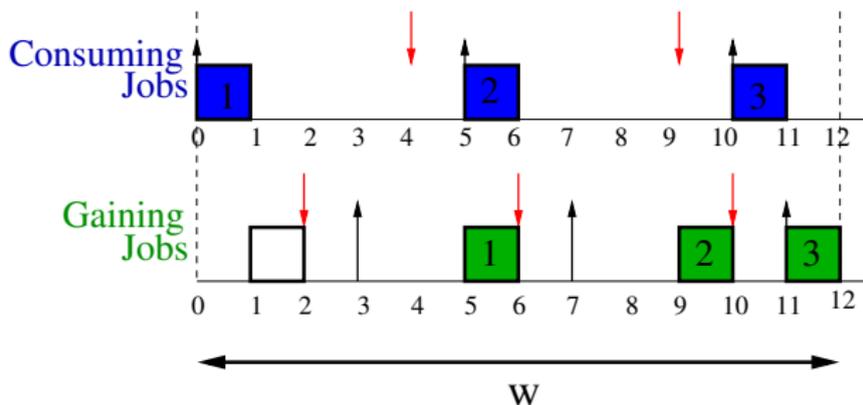
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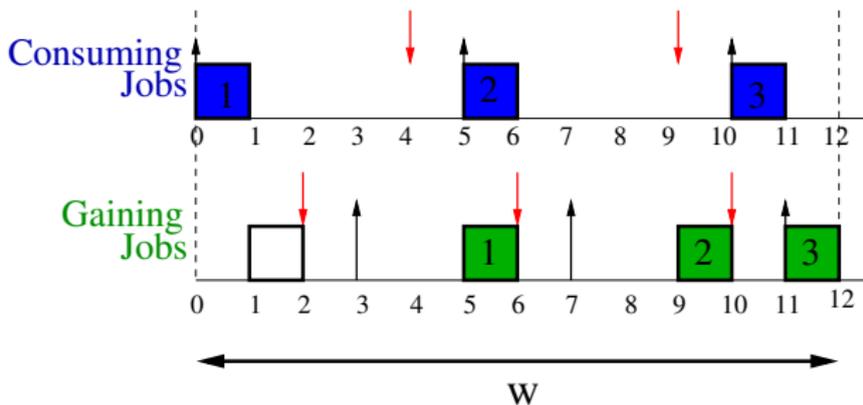
Sufficient Schedulability Test $UB2$



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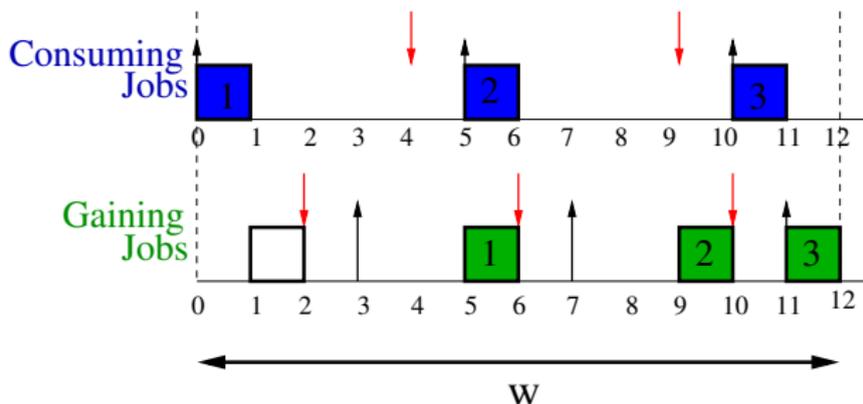
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Performance Comparison

- Competitors

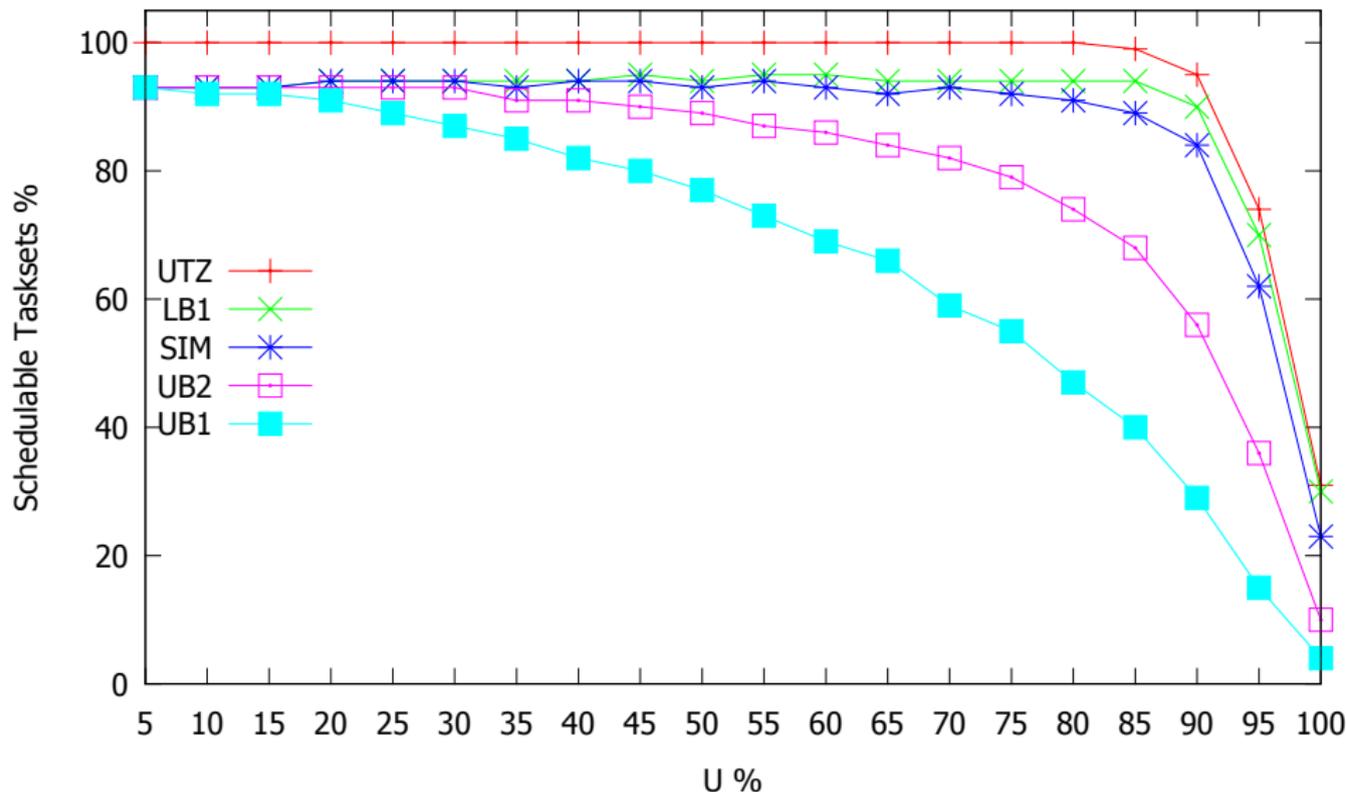
- *UTZ*: the exact test for *FPPS* ignoring energy constraints,
- *SIM*: an empirical necessary test based on simulating the schedule of PFP_{ASAP} over more than twice the hyper-period,
- *UB1*: sufficient schedulability test based on the upper bound R^{UB1} ,
- *UB2*: sufficient schedulability test based on the upper bound R^{UB2} ,
- *LB1*: necessary schedulability test based on the lower bound R^{LB1} .

- Input Data:

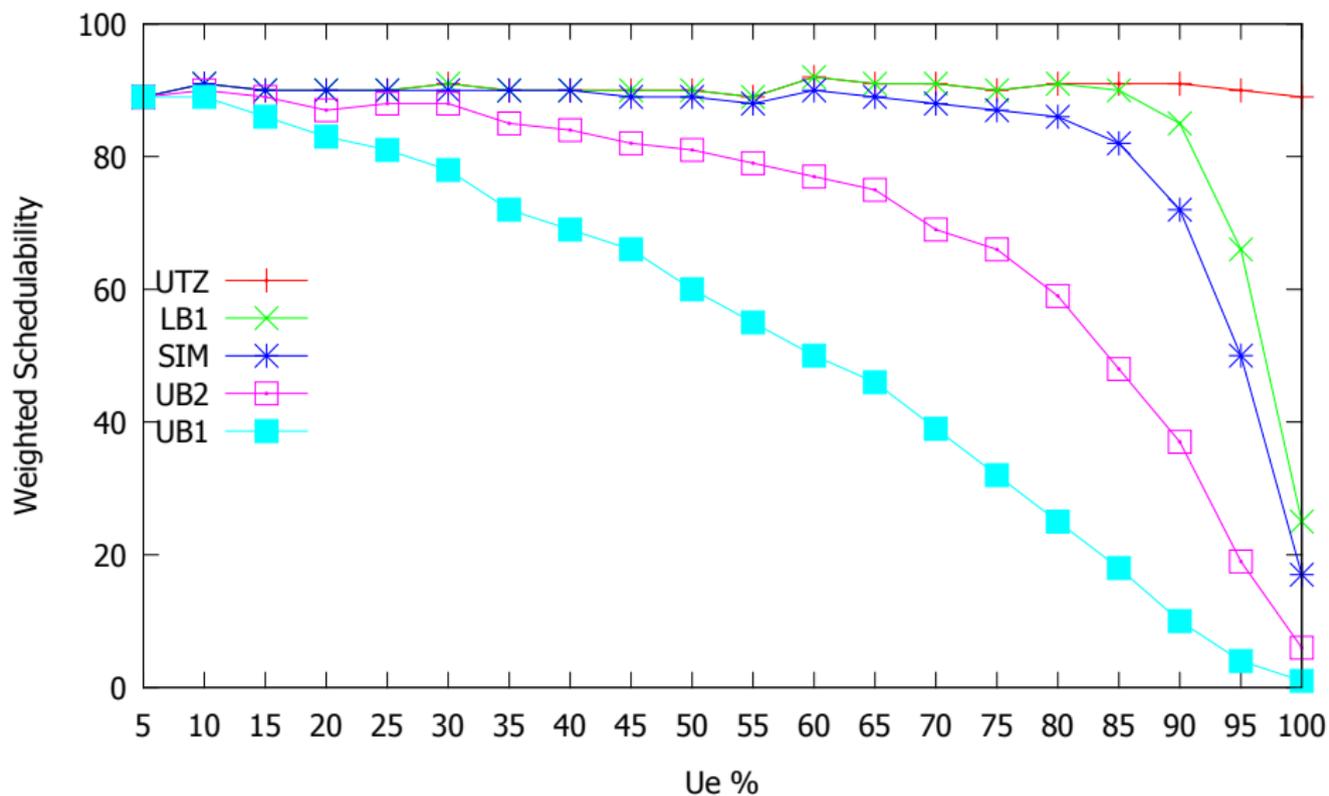
- 40000 task sets randomly generated using UUniFast-Discard algorithm coupled with a technique of hyper-period limitation,
- Processor utilization varied from 0.05 to 1,
- Energy utilization varied from 0.05 to 1,
- Percentage of gaining tasks varied from 0% to 100%.

- Simulation tool: YARTISS Real-time systems simulator

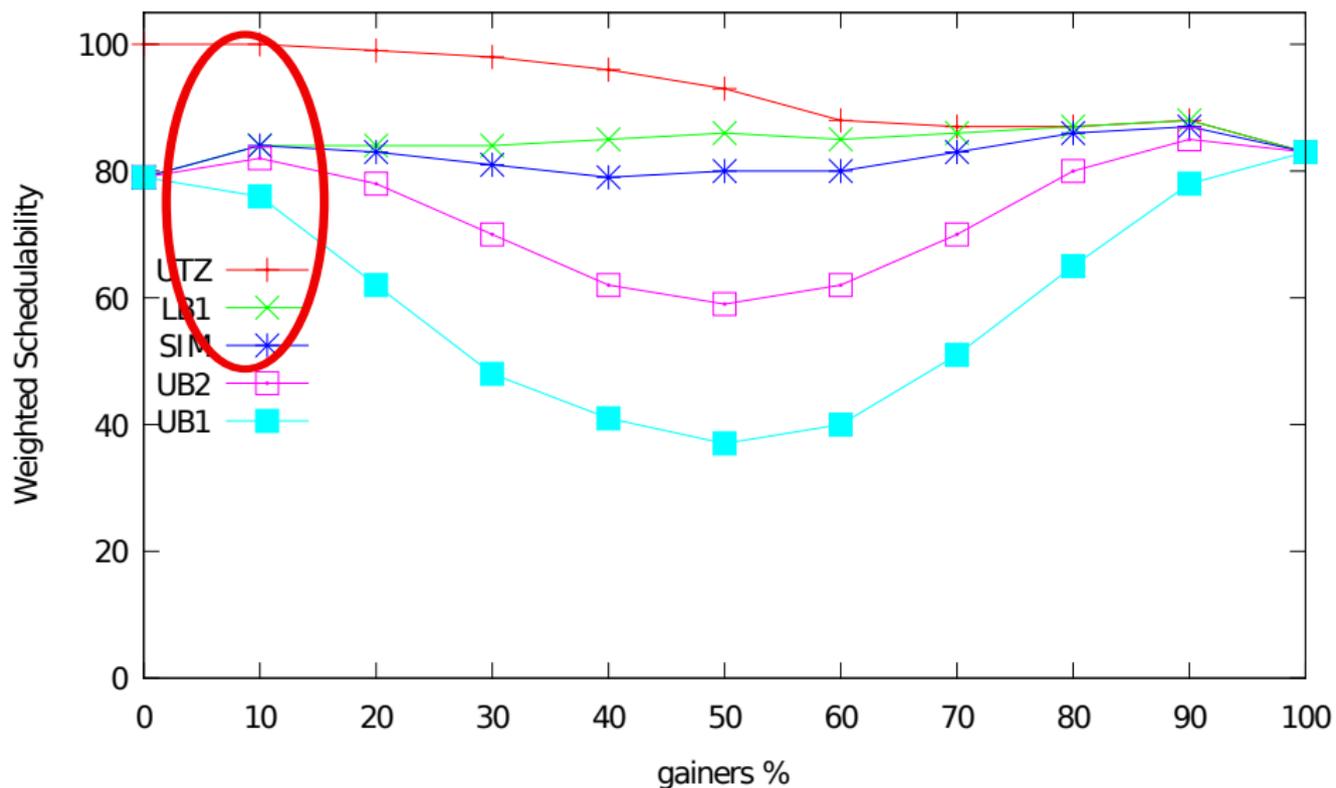
Varying the Processor Utilization



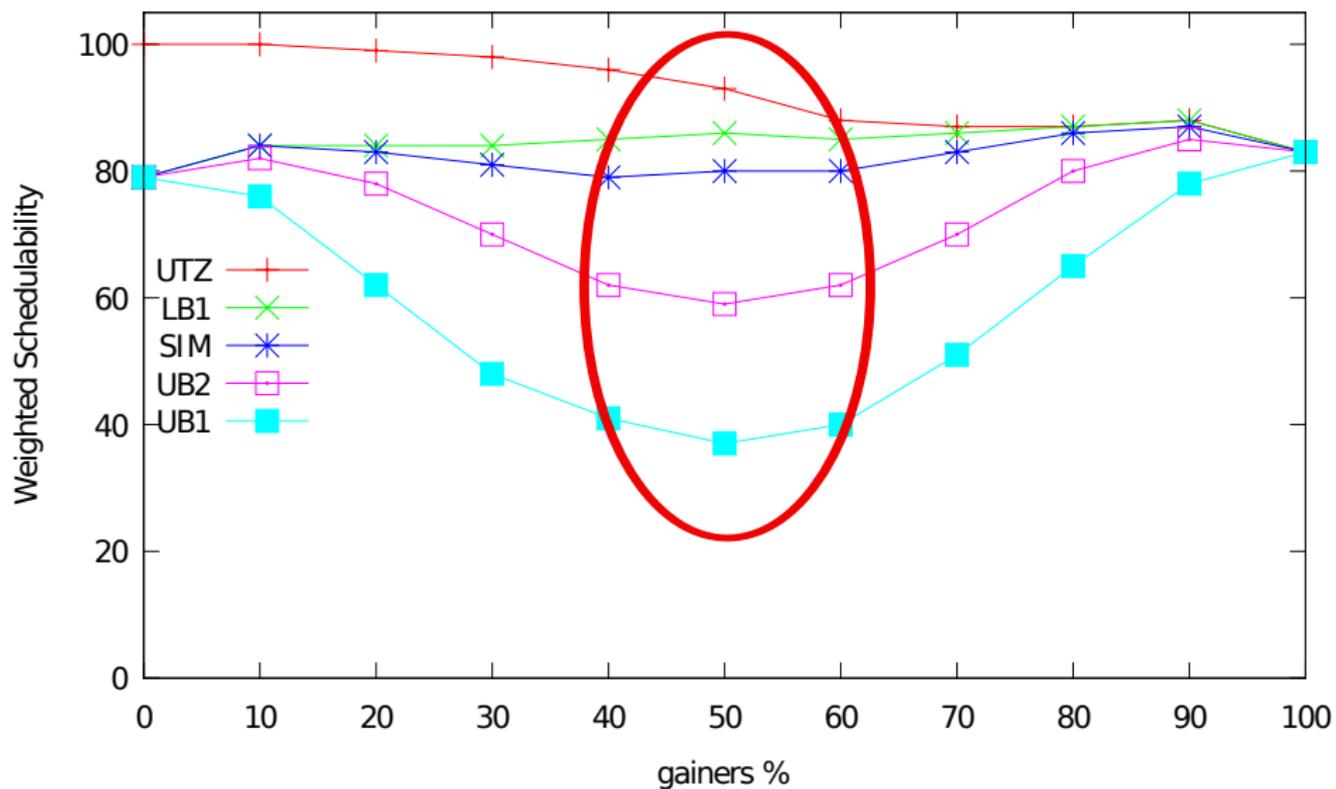
Varying the Energy Utilization



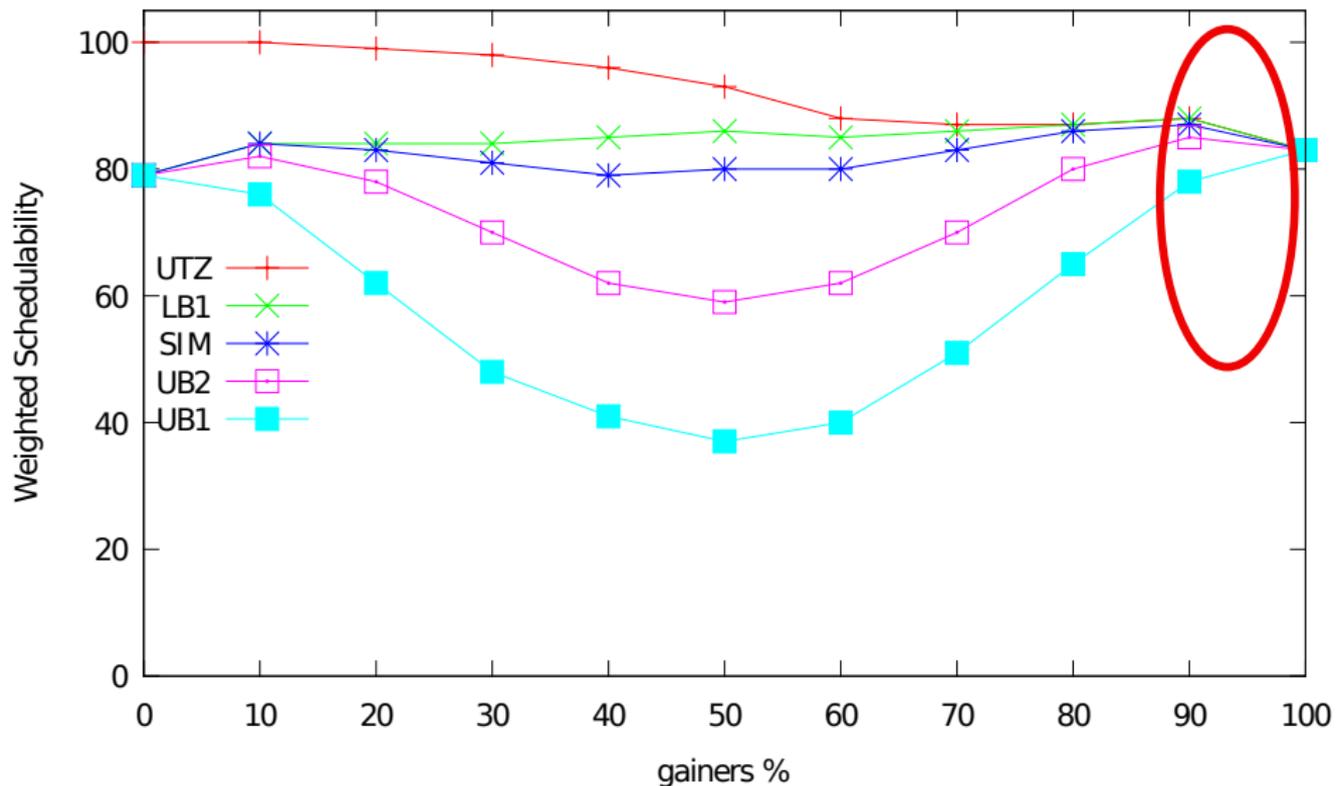
Varying the Gaining Tasks Ratio



Varying the Gaining Tasks Ratio



Varying the Gaining Tasks Ratio



Conclusion and Future Work

- 1 In this paper we
 - Showed that under PFP_{ASAP} , the **worst-case scenario** for task sets with both **consuming** and **gaining** tasks is not necessarily synchronous release with all other tasks,
 - Derived two **sufficient schedulability tests** based on two upper bounds on task response times,
 - Derived a **necessary schedulability test** based on a lower bound on task response time,
 - **Evaluated the performance** of the sufficient tests in comparison with a number of necessary tests.
- 2 As future work we plan to:
 - Investigate the problem of **optimal priority assignment**,
 - Investigate analysis for **more complex replenishment functions** and **additional costs** of entering and exiting low power modes needed for energy replenishment.

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Questions ?

Battery Capacity

For the upper bounds to be valid, the battery capacity must be sufficient to store the maximum amount of energy needed in the virtual sequences.

- 1 For $UB1$: E_{max} must be sufficient to store the energy needed to execute one time unit of the most consuming task:

$$E_{max}^{UB1} \geq \max(\max_{\forall i}(E_i/C_i) - P_r, P_r)$$

- 2 For $UB2$: E_{max} must be sufficient to store the energy needed to execute consuming jobs in any possible energy busy period:

$$E_{max}^{UB2} \geq \max \left(\sum_{\forall i} \left[\frac{\max_{\forall j}(D_j)}{T_i} \right] \times \max(E_i - C_i \times P_r, 0), P_r \right)$$