# OPTIMAL PRIORITY ASSIGNMENT ALGORITHMS FOR PROBABILISTIC REAL-TIME SYSTEMS

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- An alternative approach is to use probabilistic analysis. System reliability is typically expressed in terms of probabilities for hardware failures, memory failures, software faults, etc.
- For example, the reliability requirements placed on the timing behaviour of a system might indicate that the timing failure rate must be less than $10^{-9}$ per hour of operation.

# THE PROBABILISTIC REAL-TIME SYSTEM

- Probabilistic execution times
- Pre-emptive
- Single processor
- Fixed priorities
- Synchronous
- Constrained deadline
- Periodic

The goal: Finding an optimal* priority assignment

**Optimal in the sense that it optimizes some metric related to the probability of deadline failures*

# TASK MODEL

A set of $n$ independent periodic tasks $\Gamma = \{\tau_1, \tau_2, \ldots, \tau_n\}$

Each task $\tau_i$ generates an infinite number of jobs

Jobs are independent of other jobs of the same task and those of other tasks

$\tau_i$ is characterized by:

$$\tau_i = (\mathcal{C}_i, T_i, D_i)$$

$T_i$ being its period;

$D_i$ being its relative deadline;

$\mathcal{C}_i$ being its execution time described by a ***random variable:***

$$\mathcal{C}_i = \begin{pmatrix} c_{i,k} \\ P(C_i = c_{i,k}) \end{pmatrix}$$

# THE PROBABILISTIC EXECUTION TIME

The execution time of task $\tau_i$ is assumed to have a known probability function

$$f_{C_i}(\bullet) = \mathrm{P}(C_i = c)$$

giving the probability that $\tau_i$ has a computation time equal to $c$


$$Example: C_i = \begin{pmatrix} 2 & 3 & 4 \\ 0{,}5 & 0{,}45 & 0{,}05 \end{pmatrix}$$

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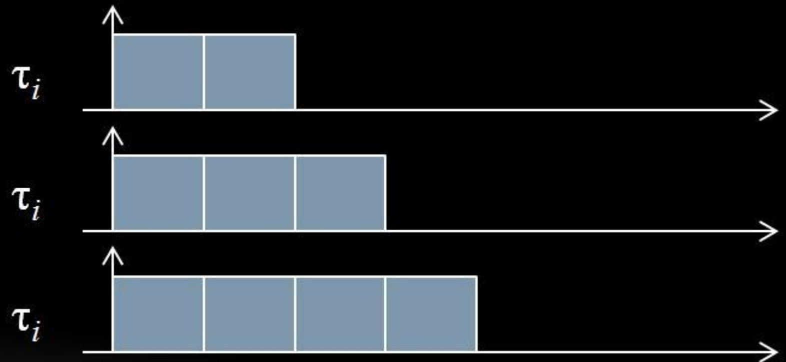

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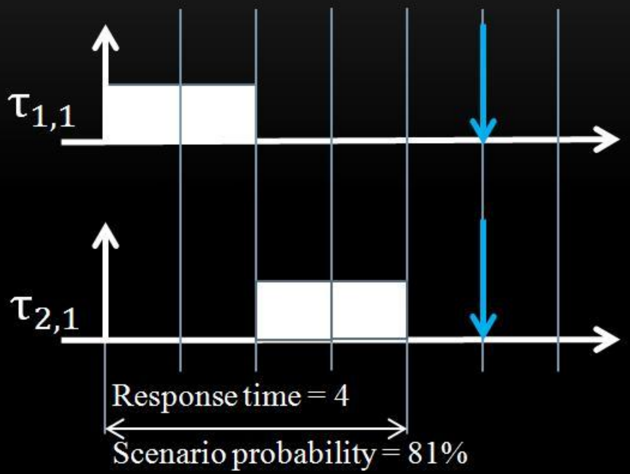

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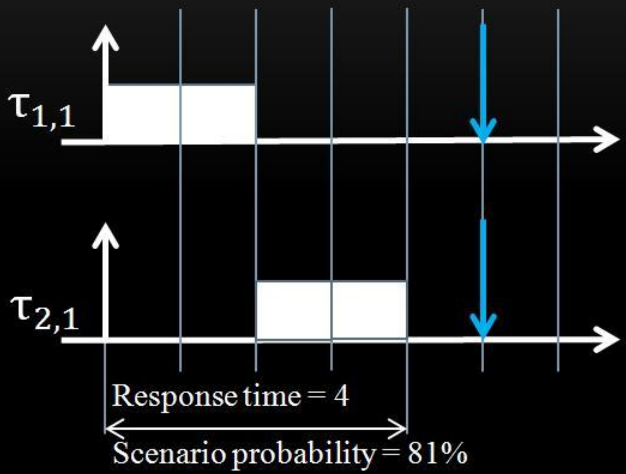


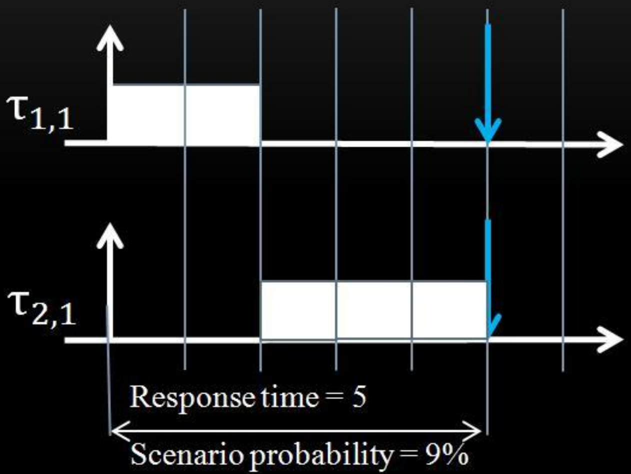

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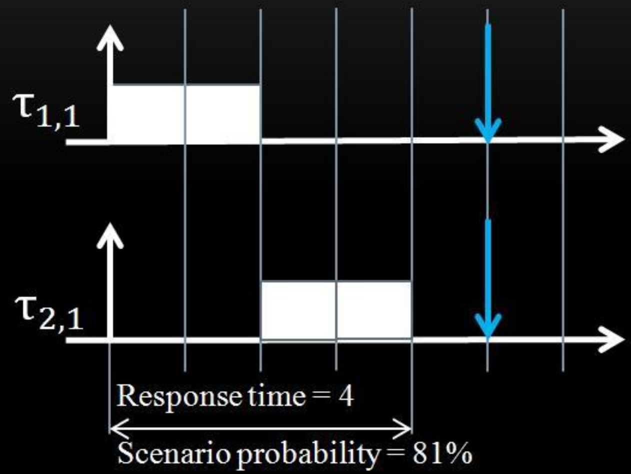


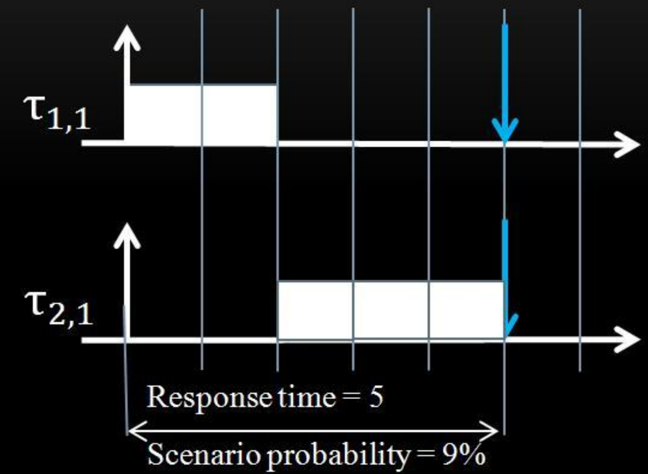


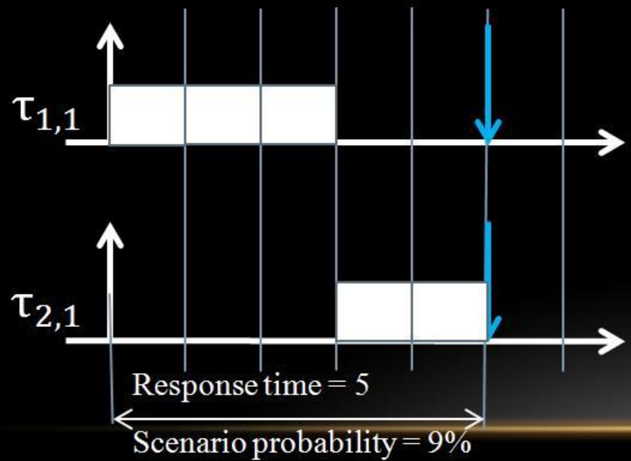

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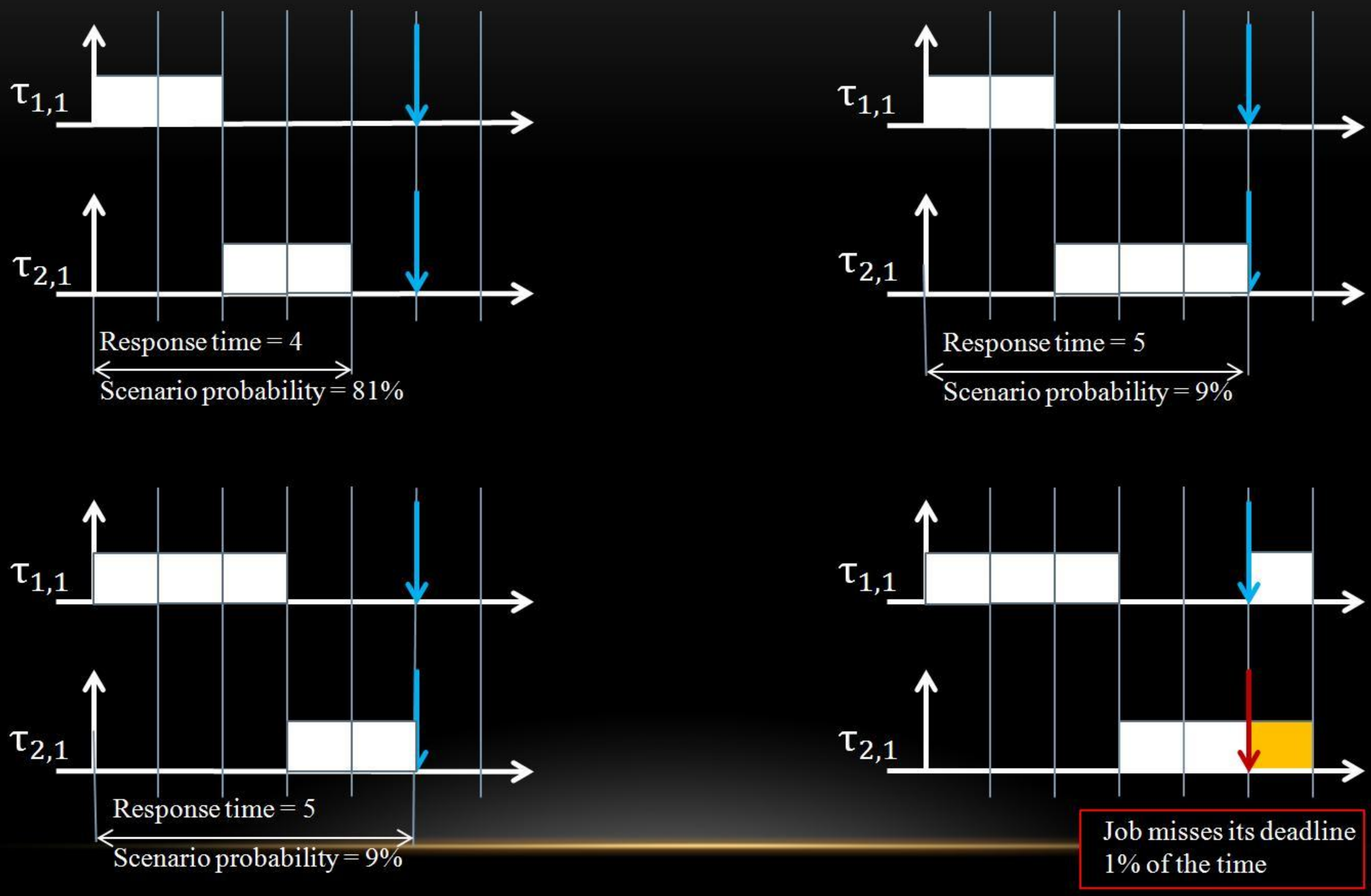

Combining the four scenarios we have:

$$\mathcal{R}_{2,1} = \begin{pmatrix} 4 & 5 & 5 & 6 \\ 0{,}81 & 0{,}09 & 0{,}09 & 0{,}1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0{,}9 & 0{,}1 \end{pmatrix} \otimes \begin{pmatrix} 2 & 3 \\ 0{,}9 & 0{,}1 \end{pmatrix} =$$

$$= \begin{pmatrix} 4 & 5 & 6 \\ 0{,}81 & 0{,}18 & 0{,}01 \end{pmatrix}$$

Definition: The ***Response Time*** of a job is the elapsed time between its *release* and its *completion*.

Note: The response time of a job/task is, as the execution time, described by a random variable.

# RESPONSE TIME COMPUTATION

$$R_{i,j} = B_i(\lambda_{i,j}) \otimes I_i(\lambda_{i,j}) \otimes C_i$$

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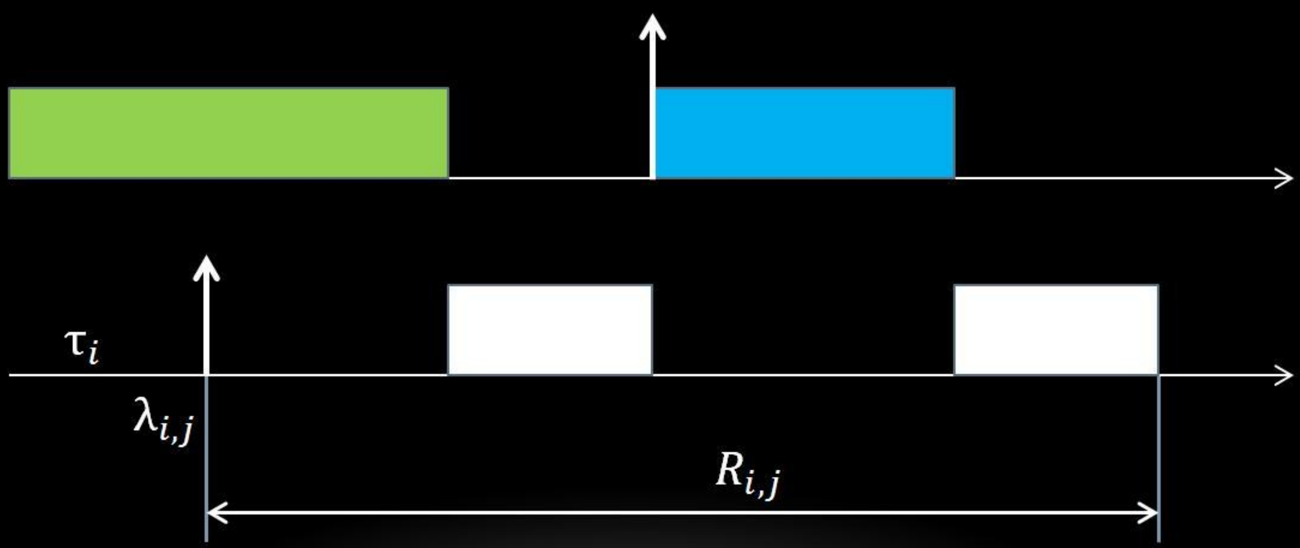

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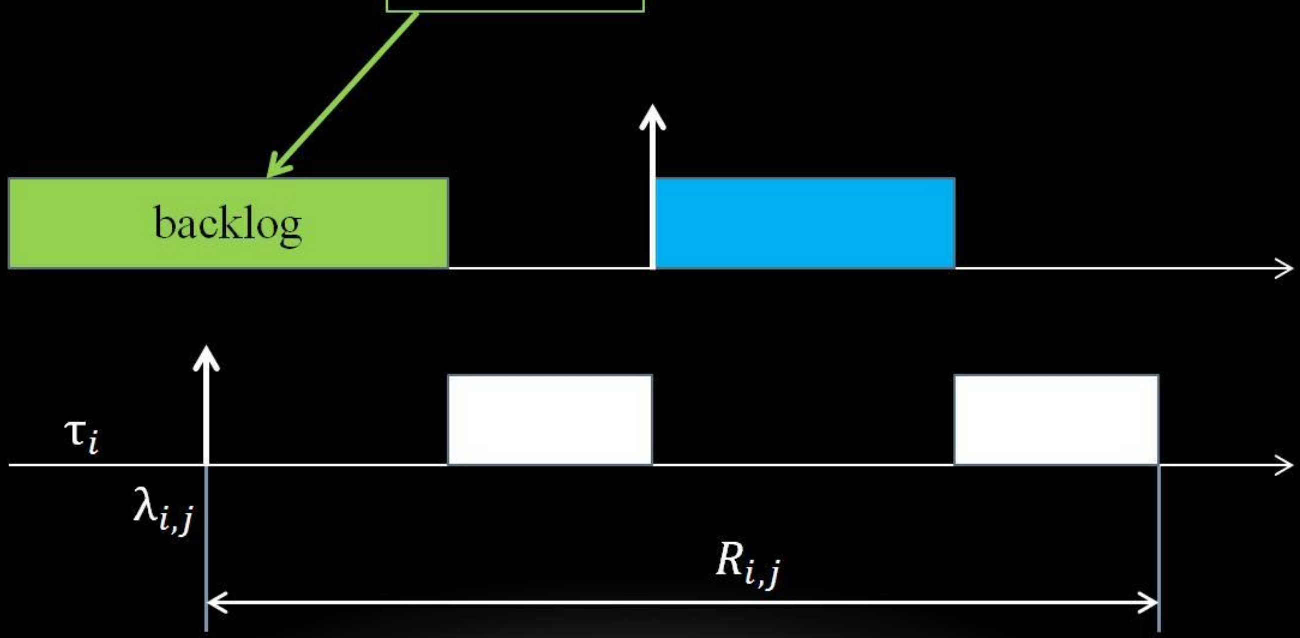

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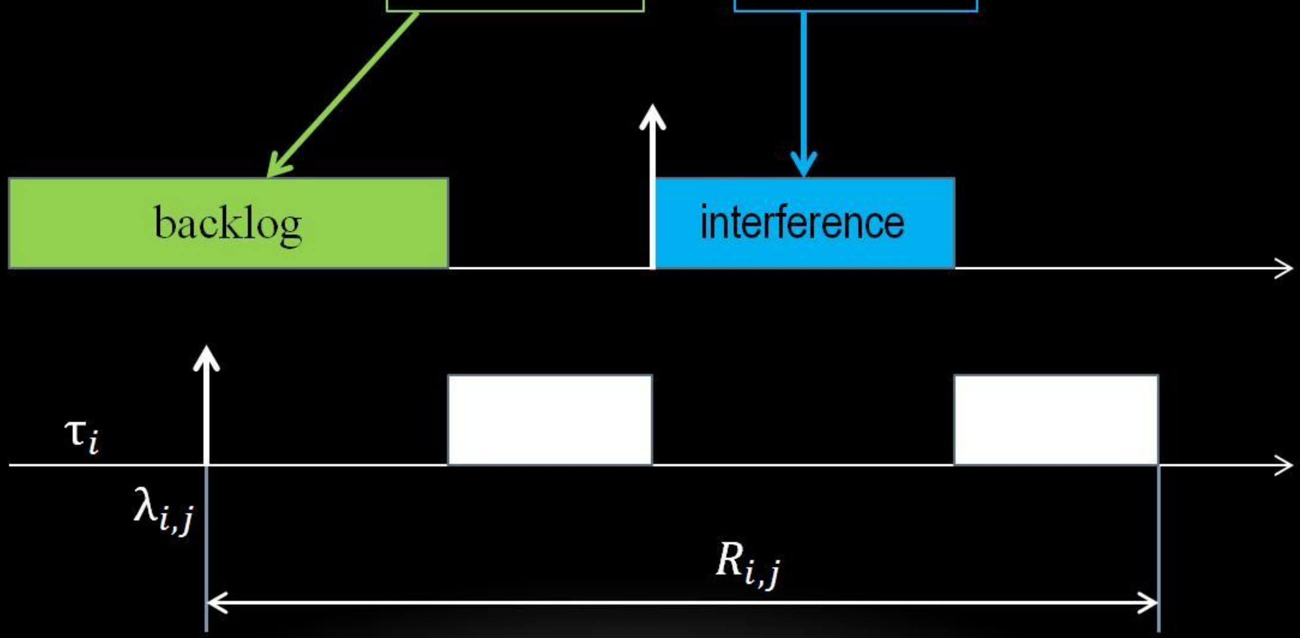

# DEFINITIONS

**Definition** (Job Deadline Miss Probability):

$$\mathrm{DMP}_{i,j} = \mathrm{P}\left(R_{i,j} > \mathrm{D}_i\right)$$

is the probability that $\tau_{i,j}$ misses its deadline.

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Example: $\mathcal{R}_{2,1} = \begin{pmatrix} 4 & 5 & 6 \\ 0{,}81 & 0{,}18 & 0{,}01 \end{pmatrix}$

$\mathrm{D}_2 = 5$ → $\mathrm{DMP}_{2,1} = 0{,}01$

# DEFINITIONS

Definition (Task Deadline Miss Ratio):

$$\mathrm{DMR}_i(a,b) = \frac{\mathrm{P}\left({R_i}^{[a,b]} > \mathrm{D}_i\right)}{n_{[a,b]}} = \frac{1}{n_{[a,b]}} \sum_{j=1}^{n_{[a,b]}} \mathrm{DMP}_{i,j}$$

is the deadline miss ratio of task $\tau_i$ in the interval [a,b] and

$n_{[a,b]} = \left\lceil \frac{b-a}{\mathrm{T}_i} \right\rceil$ is the number of jobs of $\tau_i$ released in [a,b]

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Definition (Task Deadline Miss Ratio): Is the deadline miss ratio of task $\tau_i$ in an interval.

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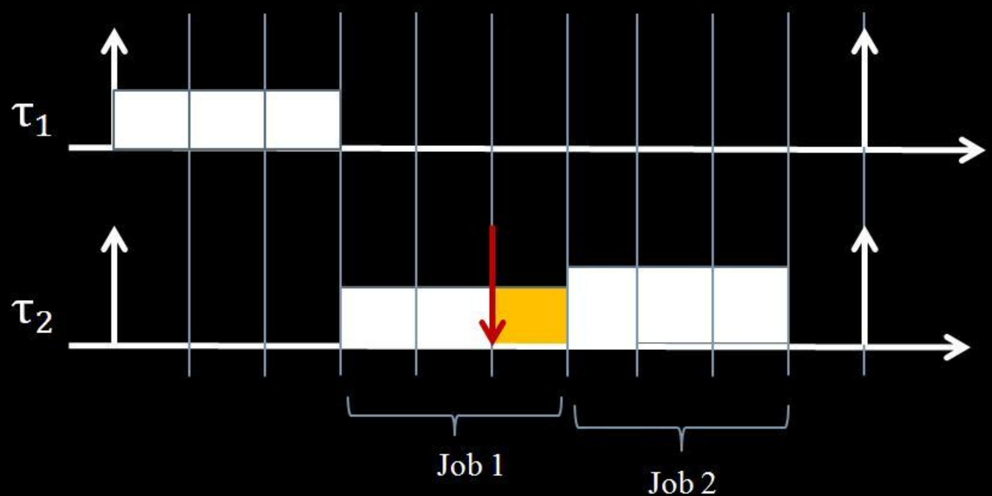

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$$R_{2,1} = \begin{pmatrix} 4 & 5 & 6 \\ 0{,}81 & 0{,}18 & 0{,}01 \end{pmatrix}.$$

$$R_{2,2} = \begin{pmatrix} 2 & 3 & 4 \\ 0{,}991 & 0{,}008 & 0{,}001 \end{pmatrix}$$

$$DMR_2 = \frac{DMR_{2,1} + DMR_{2,2}}{2} = \frac{0.01 + 0}{2} = 0{,}005$$

# PROBLEMS TO SOLVE

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1. Basic Priority Assignment Problem (**BPAP**): Considering that each task has a maximum permitted deadline miss ratio $p_i$, we search for a priority assignment $\Phi$ such that

$$\mathrm{DMR}_i(\Phi) \le p_i$$

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2. Minimization of the Maximum Priority Assignment Problem (**MPAP**): involves finding a priority assignment that minimizes the maximum deadline miss ratio of any task.

$$\max_i\{\mathrm{DMR}_i(a, b, \Phi^*)\} = \min_\Phi\{\max_i \mathrm{DMR}_i(a, b, \Phi)\}$$

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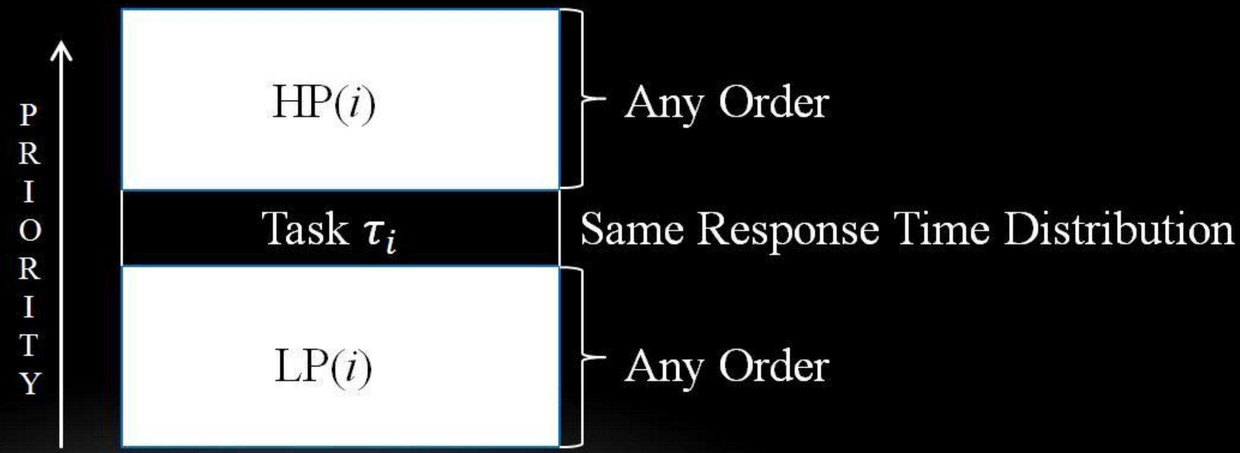
3. Average Priority Assignment Problem (**APAP**): involves finding a priority assignment that minimizes the sum of the deadline miss ratios for all tasks.

$$\Sigma_i\, \mathrm{DMR}_i(a, b, \Phi^*) = \min_\Phi\{\Sigma_i\, \mathrm{DMR}_i(a, b, \Phi)\}.$$

# ORDER OF HIGHER PRIORITY TASKS

**Theorem 1** (Order of higher priority tasks). *Considering a task* $\tau_i$, *if membership of the sets* HP(i) *and* LP(i) *are unchanged, then the response time* $\mathcal{R}_{i,j}$ *of any job of* $\tau_{i,j}$ *is unchanged and the response time* $\mathcal{R}_i^{[a,b]}$ *of task* $\tau_i$ *is unchanged whatever the priority order of tasks within* HP(i) *and within* LP(i).



# MONOTONICITY OF THE RESPONSE TIME

**Theorem 2** (Monotonicity of the response time). *Let* $\Phi_1$ *and* $\Phi_2$ *be two priority assignments with the same partial order for all tasks except for* $\tau_i$ *and* $\tau_i$ *is of lower in* $\Phi_1$ *than in* $\Phi_2$*, then the response time of any of its jobs is such that* $R_{i,j}(\Phi_1) \succcurlyeq R_{i,j}(\Phi_2)$. *Consequently, the task response time* $R_i^{[a,b]}(\Phi_1) \succcurlyeq R_i^{[a,b]}(\Phi_2)$.

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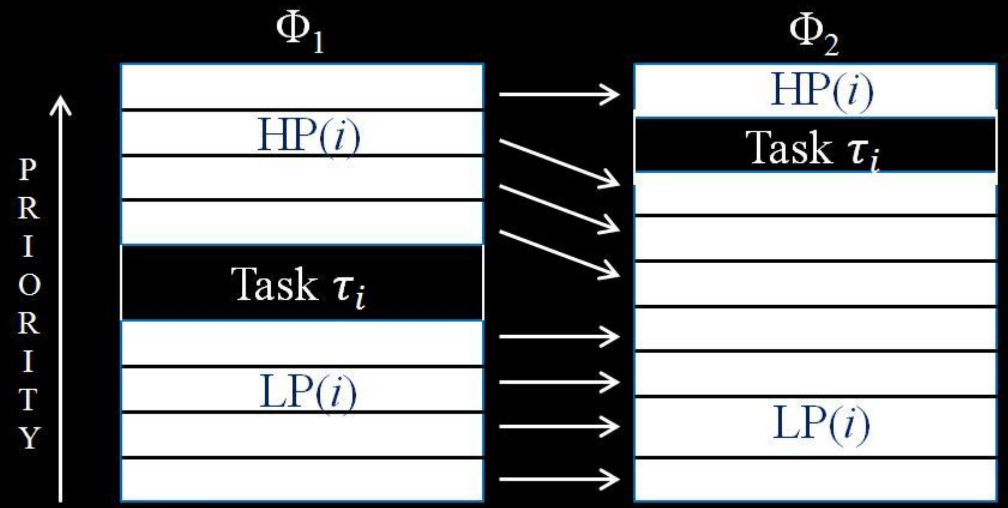

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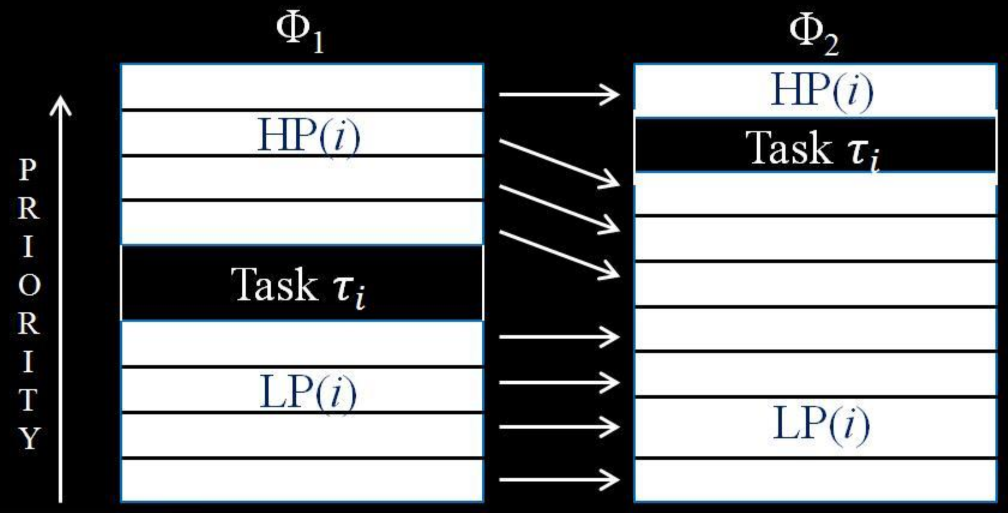
**Corolary**

(Monotonicity of DMP and DMR).

In the same conditions as above:

$DMP_{i,j}(\Phi_1) \succcurlyeq DMP_{i,j}(\Phi_2)$ *and*

$DMR_i^{[a,b]}(\Phi_1) \succcurlyeq DMR_i^{[a,b]}(\Phi_2)$.



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***The Rate Monotonic priority assignment policy is not optimal for BPAP.***

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$$\tau_1 = \left(\begin{pmatrix} 2 & 3 \\ 0{,}5 & 0{,}5 \end{pmatrix}, 4, 4, 0.5\right) \qquad \tau_2 = \left(\begin{pmatrix} 2 & 3 \\ 0{,}5 & 0{,}5 \end{pmatrix}, 8, 8, 0.1\right)$$

If $\tau_1$ has the higher priority and $\tau_2$ the lower one, as RM dictates:

$$\mathcal{R}_2 = \begin{pmatrix} 4 & 7 & 8 & {D_2}^{+} \\ 0{,}25 & 0{,}25 & 0{,}375 & 0{,}125 \end{pmatrix}$$

If $\tau_2$ has the higher priority and $\tau_1$ the lower one:

$$\mathcal{R}_1 = \begin{pmatrix} 2 & 3 & 4 & {D_1}^{+} \\ 0{,}0625 & 0{,}1875 & 0{,}3125 & 0{,}4375 \end{pmatrix}$$

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***Arranging the tasks in increasing order of their maximum permitted deadline miss ratio is not a solution for BPAP.***

Example in the paper.

# BPAP

Basic Priority Assignment Problem (**BPAP**): Considering that each task has a maximum permitted deadline miss ratio $p_i$, we search for a priority assignment $\Phi$ such that

$$\text{DM}R_i(\Phi) \leq p_i$$

Main idea: assigning priorities to tasks starting at the lowest priority level and continuing up to the highest priority level.

**Algorithm 1:** Solution to BPAP: the *feasibility* function verifies that for $\forall \tau_i$, $DMR_i < p_i$

**Input**: $\Gamma = \{\tau_i, i \in 1..n\}$ /* source set of tasks */
**Output**: $\Phi$ /* destination sequence of tasks */

```
Φ ← ()
for l ∈ n..1 do
    assignment ← FALSE
    for τ_i ∈ Γ' do
        /* feasibility function such that the computed DMR_i < p_i */
        if feasible(τ_i, Φ) then
            Φ ← Φ.τ_i
            Γ' ← Γ'\{τ_i}
            assignment ← TRUE
            break
    if assignment = FALSE then
        /* could not find a task to put at this priority level */
        break
```

# MPAP

Minimization of the Maximum Priority Assignment Problem (**MPAP**): involves finding a priority assignment that minimizes the maximum deadline miss ratio of any task.

The *Lazy and Greedy algorithm* incrementally builds a solution as a sequence of tasks, starting with the lowest priority first, and adding to $\Phi$ at each iteration an unassigned task.

**Algorithm 2:** Lazy and Greedy Algorithm

**Input:** $\Gamma = \{\tau_i, i \in 1..n\}$ /* source set of tasks */
**Output:** $\Phi$ /* sequence of tasks */, $\text{DMR}_{worst}$ /* worst DMR */

$\Phi \leftarrow ()$
$\text{DMR}_{worst} \leftarrow 0$
/* Loop over the priority levels (from lowest to highest) */
**for** $l \in n..1$ **do**
  /* Search among unassigned tasks */
  $(\tau_{best}, \text{DMR}_{best}) \leftarrow (0, +\infty)$
  **for** $\tau_i \in \Gamma$ **do**
    /* Compute DMR of current task $\tau_i$ */
    $\delta \leftarrow \text{DMR}_i(\Phi)$
    /* If this DMR is better than (or equal to) the current worst DMR in $\Phi$, be lazy: pick this task and stop the search. */
    **if** $\delta \leq \text{DMR}_{worst}$ **then**
      $(\tau_{best}, \text{DMR}_{best}) \leftarrow (\tau_i, \delta)$
      **break**
    /* If this DMR improves on other unassigned tasks, remember this task. */
    **if** $\delta < \text{DMR}_{best}$ **then**
      $(\tau_{best}, \text{DMR}_{best}) \leftarrow (\tau_i, \delta)$
  /* The search is done. The task in $\tau_{best}$ can be assigned at the current priority level. */
  $\Gamma \leftarrow \Gamma \backslash \{\tau_{best}\}$
  $\Phi \leftarrow \Phi.\tau_{best}$
  /* Update the value of the worst DMR in $\Phi$. */
  **if** $\text{DMR}_{worst} < \text{DMR}_{best}$ **then**
    $\text{DMR}_{worst} \leftarrow \text{DMR}_{best}$
**return** $(\Phi, \text{DMR}_{worst})$

# MPAP

Minimization of the Maximum Priority Assignment Problem (MPAP): involves finding a priority assignment that minimizes the maximum deadline miss ratio of any task.

*Greedy*: At each iteration, it performs a for loop over the unassigned tasks to search for the one that has the best DMR at the current priority level.

*Lazy*: whenever a task is found with a deadline miss ratio better than or equal to the current worst DMR, the search is cancelled and this task is assigned.

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# APAP

Average Priority Assignment Problem (**APAP**): involves finding a priority assignment that minimizes the sum of the deadline miss ratios for all tasks.

The Lazy and Greedy Algorithm is not optimal for this problem

Counter example in the paper.

**Algorithm 3:** Depth-First Search

```
f_best = +∞ /* best value so far (glob. var) */
Γ = {τ_i, i ∈ 1..n} /* source set of tasks */
(Φ, g) ← RECUR(Γ, (), n, 0)
return (Φ, g)

/* Function recursively completing the current solution Φ. */
RECUR(Γ, Φ, l, g) /* Note: g = g(Φ) */
/* If priority level 0 is attained, we have a complete solution. */
if l = 0 then
    /* Is this solution the new best solution? */
    if g < g^best then
        g^best ← g
    return (Φ, g)
/* Otherwise, if the current partial solution is worse than the best solution
so far, then backtrack. */
if g ≥ g^best then
    return (Φ, g)
/* Try each unassigned task τ_i at the current priority level. */
(Φ^min, g^min) ← ((), +∞)
for τ_i ∈ Γ do
    δ ← DMR_i(Φ)
    /* Get the best solution completing Φ.τ_i. */
    (Φ', g') ← RECUR(Γ\{τ_i}, Φ.τ_i, l − 1, g + δ)
    /* Memorize the best completed solution. */
    if g' < g^min then
        (Φ^min, g^min) ← (Φ', g')
    /* If task τ_i has a null DMR, then backtrack. */
    if δ = 0 then
        break
/* Return the best completed solution. */
return (Φ^min, g^min)
```

# APAP

Average Priority Assignment Problem (**APAP**): involves finding a priority assignment that minimizes the sum of the deadline miss ratios for all tasks.

To optimize g(Φ) = ∑ $_i$ DMR$_i$(Φ), a simple approach is to use a tree search algorithm enumerating all solutions.

Among various possible tree search algorithms, we choose here Depth-First Search (DFS), which explores each branch as far as possible before backtracking.

**Algorithm 3:** Depth-First Search

$f_{best} = +\infty$ /* best value so far (glob. var) */
$\Gamma = \{\tau_i, i \in 1..n\}$ /* source set of tasks */
$(\Phi, g) \leftarrow$ RECUR$(\Gamma, (), n, 0)$
**return** $(\Phi, g)$

/* Function recursively completing the current solution Φ. */
RECUR$(\Gamma, \Phi, l, g)$ /* Note: $g = g(\Phi)$ */
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        $g^{best} \leftarrow g$
    **return** $(\Phi, g)$
/* Otherwise, if the current partial solution is worse than the best solution so far, then backtrack. */
**if** $g \geq g^{best}$ **then**
    **return** $(\Phi, g)$
/* Try each unassigned task $\tau_i$ at the current priority level. */
$(\Phi^{min}, g^{min}) \leftarrow ((), +\infty)$
**for** $\tau_i \in \Gamma$ **do**
    $\delta \leftarrow$ DMR$_i(\Phi)$
    /* Get the best solution completing $\Phi.\tau_i$. */
    $(\Phi', g') \leftarrow$ RECUR$(\Gamma \backslash \{\tau_i\}, \Phi.\tau_i, l-1, g+\delta)$
    /* Memorize the best completed solution. */
    **if** $g' < g^{min}$ **then**
        $(\Phi^{min}, g^{min}) \leftarrow (\Phi', g')$
    /* If task $\tau_i$ has a null DMR, then backtrack. */
    **if** $\delta = 0$ **then**
        **break**
/* Return the best completed solution. */
**return** $(\Phi^{min}, g^{min})$

# APAP

Average Priority Assignment Problem (**APAP**): involves finding a priority assignment that minimizes the sum of the deadline miss ratios for all tasks.

As in previous algorithms, we start with the lowest priority, extending the partial priority ordering $\Phi$ progressively as we go down the tree.

Because of the different criteria optimized in APAP, one cannot be as lazy as in MPAP. Nevertheless, if a task is encountered with a DMR equal to zero, then the search loop can also be interrupted early.

**Algorithm 3:** Depth-First Search

$f_{best} = +\infty$ /* best value so far (glob. var) */
$\Gamma = \{\tau_i, i \in 1..n\}$ /* source set of tasks */
$(\Phi, g) \leftarrow \text{RECUR}(\Gamma, (), n, 0)$
**return** $(\Phi, g)$

/* Function recursively completing the current solution $\Phi$. */
$\text{RECUR}(\Gamma, \Phi, l, g)$ /* Note: $g = g(\Phi)$ */
/* If priority level 0 is attained, we have a complete solution. */
**if** $l = 0$ **then**
 /* Is this solution the new best solution? */
 **if** $g < g^{best}$ **then**
  $g^{best} \leftarrow g$
 **return** $(\Phi, g)$
/* Otherwise, if the current partial solution is worse than the best solution so far, then backtrack. */
**if** $g \geq g^{best}$ **then**
 **return** $(\Phi, g)$
/* Try each unassigned task $\tau_i$ at the current priority level. */
$(\Phi^{min}, g^{min}) \leftarrow ((), +\infty)$
**for** $\tau_i \in \Gamma$ **do**
 $\delta \leftarrow \text{DMR}_i(\Phi)$
 /* Get the best solution completing $\Phi.\tau_i$. */
 $(\Phi', g') \leftarrow \text{RECUR}(\Gamma \setminus \{\tau_i\}, \Phi.\tau_i, l-1, g+\delta)$
 /* Memorize the best completed solution. */
 **if** $g' < g^{min}$ **then**
  $(\Phi^{min}, g^{min}) \leftarrow (\Phi', g')$
 /* If task $\tau_i$ has a null DMR, then backtrack. */
 **if** $\delta = 0$ **then**
  **break**
/* Return the best completed solution. */
**return** $(\Phi^{min}, g^{min})$

# CONCLUSIONS

Probabilistic analysis as a way to reduce the overprovisioning of real-time systems

Three analysis frameworks (problems):

- **BPAP** – Greedy (Audsley) algorithm
- **MPAP** – Greedy and Lazy algorithm
- **APAP** – Tree Search Algorithm

# CONCLUSIONS

Probabilistic analysis as a way to reduce the overprovisioning of real-time systems

Three analysis frameworks (problems):

- **BPAP** – Greedy (Audsley) algorithm
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# FUTURE WORK

Probabilistic arrivals (periodic model)

More than one probabilistic parameter (for example probabilistic execution and probabilistic arrival)

# THANK YOU FOR YOUR ATTENTION