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Electron-ion coupling in rapidly varying sources

P. W. Guilbert *Joint Institute for Laboratory Astrophysics, University of Colorado, Boulder, CO 80309, USA*

A. C. Fabian and Susan Stepney *Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*

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Summary. The Thomson depth of sources radiating at the efficiency limit must lie near unity. This allows us to deduce a general condition on the maximum electron temperature attainable in such a source via two-body relaxation; $kT_e \lesssim 22$ keV. The application of this result to the rapid variability observed in NGC 6814 and Cygnus X-1 and to NGC 4151 is discussed.

1 Introduction

The minimum time-scale over which a significant luminosity variation may be observed from a source is simply and generally related to the amplitude of that change through the mass–energy conversion efficiency (Cavallo & Rees 1978; Fabian & Rees 1979). This efficiency limit occurs when the electron-scattering optical depth through the source is unity. Quasars, active galaxies and some galactic X-ray sources can have luminosities and short-time-scale variabilities which indicate that they are close to this limit for an efficiency ~ 10 per cent. For such sources it is necessary to liberate a substantial fraction of the rest mass of the radiating gas in the minimum possible time. As the protons contain most of the energy which is radiated by the electrons, they must be able to pass their energy to the electrons in a time short compared with the gas-cooling time. We show here that in general it is impossible for a source to radiate at its efficiency limit for electron temperatures above 22 keV, if two-body relaxation according to the Spitzer (1956) formula is the main energy-exchange process.

NGC 6814 has been observed to vary on time-scales of 100 s (Tennant *et al.* 1981) and has a luminosity of $\sim 2 \times 10^{36}$ W, so radiates very close to the limit for an efficiency of 5 per cent. The spectrum has only been reported out to 60 keV, which is just compatible with electron-ion coupling, but if the spectrum is seen to extend to above ~ 200 keV then the efficiency must exceed 30 per cent and also a more efficient coupling process must apply. Otherwise geometrical effects and relativistic motions are important. Galactic supernova remnants such as Cas A are already known to require more rapid energy-transfer than that allowed by two-body relaxation (see e.g. McKee & Hollenbach 1980). There may therefore be some macroscopic plasma processes of general astrophysical importance for high-temperature, high-efficiency sources.

2 The efficiency limit and electron–ion coupling

The maximum luminosity that can be observed, when a fraction η of rest-mass energy Mc^2 is radiated, is

$$L_{\max} = E/t_{\min},$$

where $t_{\min}$ is the minimum value of t_e, the time-scale on which most photons can escape from the source. Ignoring electron–ion coupling for the moment,

$$t_e = \frac{R}{c} \times \max[\tau, 1],$$

where R is the characteristic size of the source and τ is the Thomson depth, $n_e \sigma_T R$. Thus,

$$L = 2.8 \times 10^{27} \eta R \times \frac{\tau}{\max[\tau, 1]} \text{ W}.$$

For a given mass $R \propto \tau^{-1/2}$ and thus $L \propto \tau^{1/2}/\max[\tau, 1]$. $L_{\max}$ and the shortest observed time-scale occur when $\tau = 1$, giving the efficiency limit (Fabian & Rees 1979)

$$L_{\max} = 8.4 \times 10^{35} \eta t_{\min} \text{ W}.$$

The time required to exchange energy between protons and electrons is given by Spitzer's (1956) formula for non-relativistic electron and proton energies:

$$t_{ep} = \sqrt{\frac{\pi}{2}} \frac{m_p}{m_e} \frac{n_e}{n_p \ln \Lambda} \frac{(\theta_e + \theta_p)^{3/2}}{n_e \sigma_T c}$$

where $\theta_e = kT_e/m_e c^2$; $\theta_p = kT_p/m_p c^2$; n_e and n_p are the electron and proton densities respectively and $\ln \Lambda$ is the Coulomb logarithm. Since $n_e \sigma_T c = c\tau/R$, we obtain $t_{ep} \simeq 115 (\theta_e + \theta_p)^{3/2} R/c\tau$ s, putting $\ln \Lambda = 20$. This scales with R in exactly the same way as t_e and shows that t_{ep} is greater than t_e for

$$\frac{115}{\tau} (\theta_e + \theta_p)^{3/2} > \max[\tau, 1],$$

i.e.

$$(\theta_e + \theta_p) > 0.042 (\tau \max[\tau, 1])^{2/3}.$$

A source therefore cannot radiate at the efficiency limit where $\tau = 1$ if

$$(\theta_e + \theta_p) > 0.042,$$

unless energy is transferred from the protons to the electrons at a faster rate than two-body relaxation. In particular, the efficiency limit (independent of the actual value of η) cannot be attained if $kT_e > 22$ keV or $kT_p > 40$ MeV. When $t_{ep} > t_e$ $(\tau = 1)$ then $t_{\min} = t_{ep}$ and $L_{\max}$ is determined by t_{ep}. The modified efficiency limit is then reached when $t_e = t_{ep}$, i.e.

$$\tau = 10.7 (\theta_e + \theta_p)^{3/4},$$

giving

$$L_{\text{max}}^{\text{ep}} = \frac{L_{\text{max}}}{10.7\,(\theta_e + \theta_p)^{3/4}}, \qquad \theta_e + \theta_p \geqslant 0.042$$

which is lower than the previous value. For sources with $\theta_e + \theta_p \geqslant 0.042$, the minimum variability time is now $t_{\text{min}}^{\text{ep}} \geqslant 1.3\,\eta^{-1} L_{35} (\theta_e + \theta_p)^{3/4}$ s. The assumption that $\theta_p \ll \theta_e$ and $\eta \simeq 0.1$ gives $t_{\text{min}}^{\text{ep}} \geqslant 4 \times 10^{-4}$ s for luminosities and temperatures appropriate to Cyg X-1 ($L \simeq 10^{30}$ W, $\theta_e \simeq 0.2$), and $t_{\text{min}}^{\text{ep}} \geqslant 1000$ s for NGC 4151 ($L \simeq 10^{37}$ W, $\theta_e \simeq 1$).

Although $\theta_e \simeq 1$ implies that the electrons are relativistic, Stepney (in preparation) has shown that Spitzer's (1956) formula for t_{ep} is accurate to within a factor of 2 there, provided that $\theta_p < \theta_e$. The Klein–Nishina reduction in σ requires that an even smaller correction be introduced at these energies, which is ignored here.

NGC 6814 is observed to have a 2–60 keV luminosity of $\simeq 2 \times 10^{36}$ W (Mushotzky *et al.* 1980) and varies on time-scales of ~ 100 s in at least the 2–20 keV band (Tennant *et al.* 1981). This implies that the source lies on the efficiency limit for $\eta = 5$ per cent (with a Hubble constant, H_0, of 50 km s^{-1} Mpc^{-1}, Tennant *et al.* 1981), and is marginally consistent with two-body electron–ion coupling. The spectrum cannot then extend beyond a few tens of keV without requiring that the source be below the efficiency limit. Setting a maximum on η at 30 per cent, we conclude that the spectrum cannot be produced in a general situation by $kT_e \geqslant 180$ keV without the presence of some faster process for electron–ion coupling than two-body relaxation. Relativistic, or geometrical effects (such as a preferred viewing axis), or a much larger value for H_0 must otherwise be invoked.

3 Discussion

Measurements of the hard X-ray spectrum of rapidly-varying X-ray sources such as NGC 6814 and Cyg X-1 provide a model-independent constraint on the electron-heating process. If electron–ion collisions are the dominant process, then the X-ray spectra of those sources that display the fastest variability (at a given luminosity) should cut off at the lowest energies. All models for the X-ray emission, and in particular Compton-scattering models, require τ to be less than a few. (The efficiency limit is, of course, only reached in cases where the cooling time $\leqslant t_e$.) Consequently, sources for which $kT_e \gtrsim 100$ keV cannot radiate at the efficiency limit determined by two-body electron–ion coupling and so must vary on a time-scale much larger than the minimum time-scale. For example, NGC 4151 which appears to radiate at up to ~ 3 MeV (Schoënfelder 1981 and references therein) is unlikely to vary in a time less than ~ 2 hr unless a more efficient process for electron heating is present.

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