Susan Stepney

John Horton Conway's Game of Life (GoL) [26, 106] is a simple two-dimensional, two state cellular automaton (CA), remarkable for its complex behaviour [26, 133].

The classic GoL is defined on a regular square lattice. The update rule depends on the state of each cell and its neighbouring eight cells with which it shares a vertex. Each cell has two states, 'dead' and 'alive'. If a cell is alive at time *t*, then it stays alive if and only if it has two or three live neighbours (otherwise it dies of 'loneliness' or 'overcrowding'). If a cell is dead at time *t*, then it becomes alive (is 'born') if and only if it has exactly three live neighbours. This rule gives a famous zoo of GoL patterns, including still lifes, oscillators, and gliders.

Here we show some results of running GoL rules on Penrose tilings. More detail can be found in [127], from which all the figures here are taken. The neighbourhood of a Penrose tile is again all the tiles with which it shares a vertex; now there can be 7–11 of these, depending on details of the tiling. We show some interesting still life patterns and oscillator patterns. For a fuller, but still preliminary, catalogue of Penrose life structures, see [127]. These patterns were discovered by a combination of systematic construction and random search.

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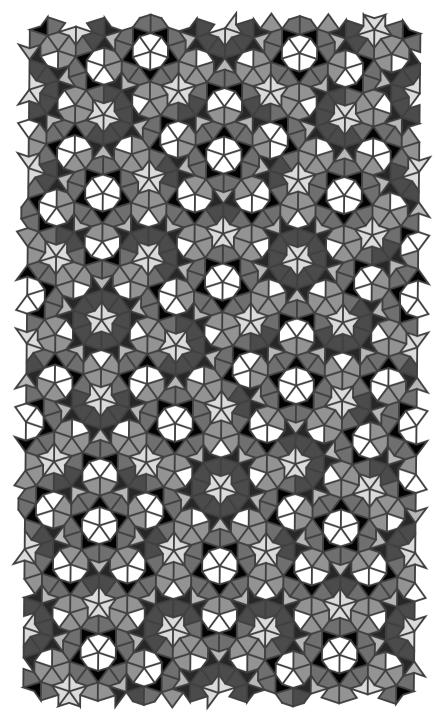
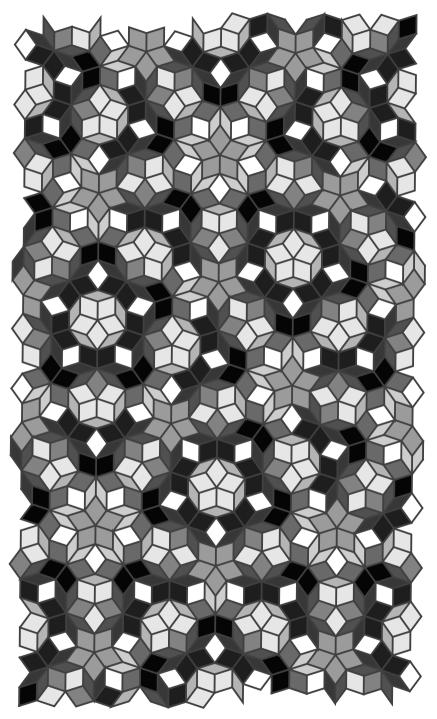
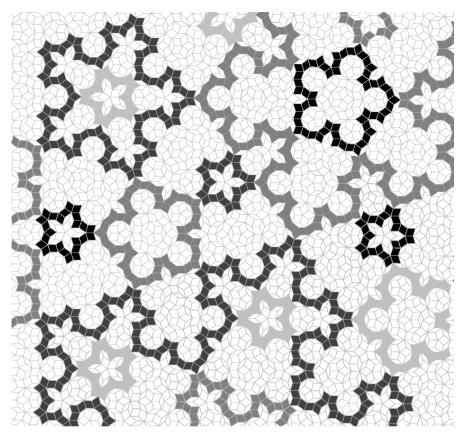


Fig. 1 A kite and dart tiling shaded by the eight distinct neighbourhood types; neighbourhood sizes range from 8-10; darker tiles have more neighbours. Reproduced from [127, fig.18.11].

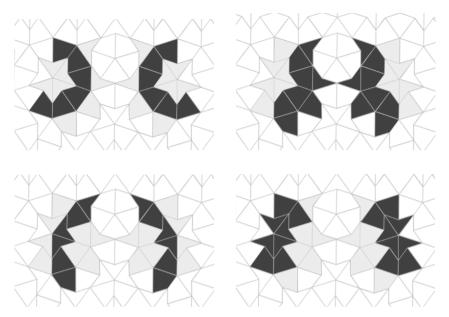


 $\textbf{Fig. 2} \ \, \text{A rhomb tiling shaded by the 11 distinct neighbourhood types; neighbourhood sizes range from 7–11; darker tiles have more neighbours. Reproduced from [127, fig.18.13]. }$ 

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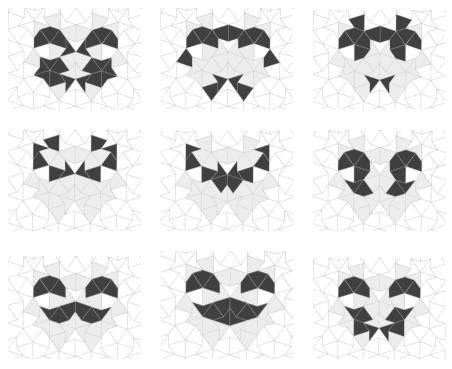


**Fig. 3** Arbitrarily large still lifes can be constructed on the rhomb tiling. Pick a thick rhomb (of a particular type; see [127] for details). Complete the "ribbon" of thick rhombs that is formed from the two thick rhombs adjacent to its edges (all thick rhombs have precisely two such thick rhomb neighbours). The figure shows several such ribbons of thick rhombs; different grey shades show different possible ribbons, although these cannot all exist simultaneously. Reproduced from [127, fig.18.38].

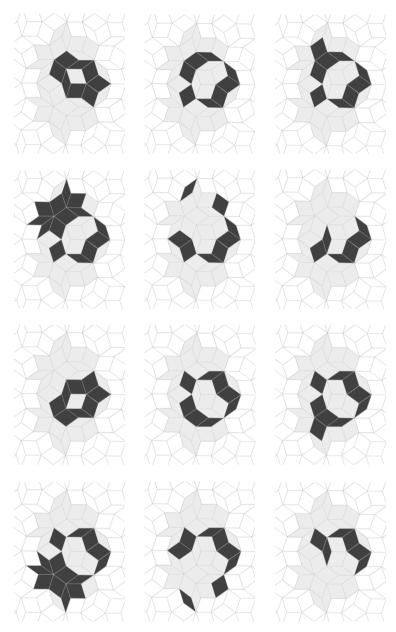


**Fig. 4** The "bat-to-bat" oscillator. Live cells at each step are shown in black; cells that are dead this step but live some other step are shown in grey; cells that are always dead are shown in white. This is an oscillator on the kite-and-dart tiling; it has period 4; it has a total of 28 cells live at some step in the oscillator; it has a minimum of 12 live cells during its oscillation (at steps 1 and 3). It is given the code kd-p4-28-12. Reproduced from [127, fig.18.71d].

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**Fig. 5** The "moustaches" oscillator. This is an oscillator on the kite-and-dart tiling; it has period 9; it has a total of 36 cells live at some step in the oscillator; it has a minimum of 10 live cells during its oscillation (at step 4). It is given the code kd-p9-36-10. Reproduced from [127, fig.18.82].



**Fig. 6** The "fireworks" oscillator. This is an oscillator on the rhomb tiling; it has period 12; it has a total of 33 cells live at some step in the oscillator; it has a minimum of 6 live cells during its oscillation (at steps 6 and 12). It is given the code r-p12-33-6. Note the underlying period 6 behaviour, combined with a reflection. Reproduced from [127, fig.18.96].