

Preface

This book of problems has been designed to accompany an undergraduate course in probability. It will also be useful for students with interest in probability who wish to study on their own.

The only prerequisite is basic algebra and calculus. This includes some elementary experience in set theory, sequences and series, functions of one variable, and their derivatives. Familiarity with integrals would be a bonus. A brief survey of terminology and notation in set theory and calculus is provided.

Each chapter is divided into three parts: Problems, Hints, and Solutions. To make the book reasonably self-contained, all problem sections include expository material. Definitions and statements of important results are interlaced with relevant problems. The latter have been selected to motivate abstract definitions by concrete examples and to lead in manageable steps toward general results, as well as to provide exercises based on the issues and techniques introduced in each chapter.

The hint sections are an important part of the book, designed to guide the reader in an informal manner. This makes *Probability Through Problems* particularly useful for self-study and can also be of help in tutorials. Those who seek mathematical precision will find it in the worked solutions provided. However, students are strongly advised to consult the hints prior to looking at the solutions, and, first of all, to try to solve each problem on their own.

Hints are given for all problems, the majority of which also have fully worked solutions. To avoid repetition, we have left out a few solutions that are very similar to the preceding ones. Important items such as definitions,

theorems, and some problems of theoretical nature are marked with ►. Hard problems, which can be omitted on first reading, are designated by *, and those without solution by ◦.

The book begins with a motivating chapter on modeling random experiments. This is followed by Chapters 2 through 6 devoted to the mathematical structures underpinning probability theory: classical probability spaces and related combinatorial problems, fields of sets (also known as algebras), finitely additive probability, sigma fields (sigma algebras), and countably additive probability. Chapter 7 is concerned with the crucial notions of conditional probability and independence. Random variables and their probability distributions are discussed in Chapter 8. The notions of mathematical expectation and variance are studied in Chapter 9, followed by a careful step-by-step exposition of conditional expectation in Chapter 10. Characteristic functions are the subject of Chapter 11. Chapter 12 offers problems designed to illustrate the law of large numbers and the central limit theorem, the emphasis being on understanding the consequences and applications. The Bibliography contains a list of books recommended either as background reading or to consolidate and supplement this course of study.

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