Corrections in *Probability Through Problems* by M. Capiński and T. Zastawniak 1st printing, 2001

page iv

Replace email address

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by

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page vi

Insert after the last paragraph in Preface:

It is a pleasure to thank our students and readers for valuable suggestions and comments, which helped us to correct some mistakes present in the 1st printing of this book. In particular, we wish to express our gratitude to Xiaomin Chen, Mark Hunacek and Krzysztof Tokarz.

Readers are invited to visit the Web Page at http://www.hull.ac.uk/php/mastz/ptp/ or write to us at the email addresses below. Your feedback will be greatly appreciated.

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page 18, Problem 2.21

Replace the text of Problem 2.21 by:

From a pack of 52 cards, we draw one-by-one. What is the probability that an Ace will appear for the first time at the fifth turn?

page 59, Problem 5.29

Replace displayed formula by:

$$A_n = \begin{cases} \left(\frac{1}{3} - \frac{1}{n+1}, 1 + \frac{1}{n+1}\right) & \text{if } n = 1, 3, 5, \dots \\ \left(\frac{1}{n+1}, \frac{2}{3} - \frac{1}{n+1}\right) & \text{if } n = 2, 4, 6, \dots . \end{cases}$$

page 66, Solution 5.23

Replace displayed formula by:

$$C = \bigcap_{n=1}^{\infty} C_n,$$

page 99, Hint 7.46

`Replace the text of Hint 7.46 by:`

See Solution 7.44. Is $\mathcal{F}_{\{A_1,\ldots,A_n\}}$ a finite field? If so, what are the atoms in $\mathcal{F}_{\{A_1,\ldots,A_n\}}$? Is each of the atoms independent of B?

page 99, Hint 7.47

`Replace the text of Hint 7.47 by:`

See Solutions 7.44 and 7.46. If E is an atom in $\mathcal{F}_{\{A_1,\ldots,A_n\}}$ and F an atom in $\mathcal{F}_{\{B_1,\ldots,B_m\}}$, are E, F independent?

page 115, Solution 7.44

`Replace the text of Solution 7.44 by:`

The σ-field $\mathcal{F}_{\{B,C\}}$ generated by $\{B, C\}$ is, in fact, a finite field with at most four atoms: $B \cap C$, $B \setminus C$, $C \setminus B$, and $\Omega \setminus (B \cup C)$ (some of these sets may be empty and then should be discarded). For each atom E in $\mathcal{F}_{\{B,C\}}$ the events A, E are independent by Problems 7.43 and 7.34. Each event $D \in \mathcal{F}_{\{B,C\}}$ is a disjoint union of some of these atoms. It follows that A, D are independent.

The last conclusion is based on the following claim: If A, E are independent events, A, F are independent events, and $E \cap F = \emptyset$, then $A, E \cup F$ are independent. The claim can be verified as follows:

$$\begin{aligned} P(A \cap (E \cup F)) &= P((A \cap E) \cup (A \cap F)) = P(A \cap E) + P(A \cap F) \\ &= P(A)P(E) + P(A)P(F) = P(A)(P(E) + P(F)) \\ &= P(A)P(E \cup F). \end{aligned}$$

page 115, Solution 7.46

`Replace the text of Solution 7.46 by:`

The σ-field $\mathcal{F}_{\{A_1,\ldots,A_n\}}$ is, in fact, a finite field with atoms of the form $C_1 \cap \cdots \cap C_n$, where $C_i = A_i$ or $\Omega \setminus A_i$ for $i = 1, \ldots, n$. Since $A_1, \ldots, A_n, B$ are independent events, so are $C_1, \ldots, C_n, B$. It follows that $C_1 \cap \cdots \cap C_n, B$ are independent, that is, E, B are independent for any atom E in $\mathcal{F}_{\{A_1,\ldots,A_n\}}$. Because each $C \in \mathcal{F}_{\{A_1,\ldots,A_n\}}$ is a disjoint union of finitely many atoms, it follows by the claim in Solution 7.44 that C, B are independent.

page 116, Solution 7.47

`Replace the text of Solution 7.47 by:`

The σ-field $\mathcal{F}_{\{A_1,\ldots,A_n\}}$ is a finite field with atoms of the form $C_1 \cap \cdots \cap C_n$, where $C_i = A_i$ or $\Omega \setminus A_i$ for $i = 1, \ldots, n$. Similarly, $\mathcal{F}_{\{B_1,\ldots,B_m\}}$ is a finite field with atoms of the form $D_1 \cap \cdots \cap D_m$, where $D_j = B_j$ or $\Omega \setminus B_j$ for $j = 1, \ldots, m$. Since $A_1, \ldots, A_n, B_1, \ldots, B_m$ are independent events, so are $C_1, \ldots, C_n, D_1, \ldots, D_m$. As a result, $C_1 \cap \cdots \cap C_n, D_1 \cap \cdots \cap D_m$ are independent, that is, E, F are independent for any atom E in $\mathcal{F}_{\{A_1,\ldots,A_n\}}$ and any atom F in $\mathcal{F}_{\{B_1,\ldots,B_m\}}$. Because each $C \in \mathcal{F}_{\{A_1,\ldots,A_n\}}$ is a disjoint union of finitely many atoms in $\mathcal{F}_{\{A_1,\ldots,A_n\}}$ and each $D \in \mathcal{F}_{\{B_1,\ldots,B_m\}}$ is a disjoint union of finitely many atoms in $\mathcal{F}_{\{B_1,\ldots,B_m\}}$, it follows by the claim in Solution 7.44 that C, D are independent.

page 126, Problem 8.67
Replace displayed formula by:

$$f_{U,V}(u,v) = \tfrac{1}{2} f_{X,Y}\left(\tfrac{u+v}{2}, \tfrac{u-v}{2}\right).$$

page 147, Solution 8.54
Replace displayed formula by:

$$\begin{aligned} P(\{a + bX \in B\}) &= P(\{X \in (B-a)/b\}) \\ &= \int_{(B-a)/b} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \mathrm{d}x \\ &= \int_B \frac{1}{\sqrt{2\pi b^2}} \exp\left(-\frac{(y-a)^2}{2b^2}\right) \mathrm{d}y, \end{aligned}$$

page 185$_{17}$
Replace displayed formula

$$E(X|\{Y = y_1\}), E(X|\{Y = y_1\}), \dots \ .$$

by

$$E(X|\{Y = y_1\}), E(X|\{Y = y_2\}), \dots \ .$$

page 185$_7$
Replace
$E(X|\{Y = y_1\}), E(X|\{Y = y_1\}), \dots \ .$
by
$E(X|\{Y = y_1\}), E(X|\{Y = y_2\}), \dots \ .$

page 190, Problem 10.44
Replace $x^2 + y^2 \leq 1$ by $-2 \leq -y \leq 2x \leq 2$

page 190, Problem 10.46
Replace $f_{X,Y}(x,y) = K\left(x + y^2\right)$ by $f_{X,Y}(x,y) = K\left(x^2 + y^2\right)$

page 194, Hint 10.46
Add sentence:
Alternatively, $E(X|Y)$ can be found without any calculations by considering the symmetry of the problem.

page 202, Solution 10.27
Replace displayed formula

$$\begin{aligned} E(E(X|Y)) &= E(E(X^+|Y)) + E(E(X^-|Y)) \\ &= E(X^+) + E(X^-) = E(X). \end{aligned}$$

by

$$\begin{aligned}E(E(X|Y)) &= E(E(X^+|Y)) - E(E(X^-|Y)) \\ &= E(X^+) - E(X^-) = E(X).\end{aligned}$$

page 209, Solution 10.40

`Replace displayed formula`

$$g(y) = \int_{-\infty}^{+\infty} xh(x|Y)\ \mathrm{d}x$$

by

$$g(Y) = \int_{-\infty}^{+\infty} xh(x|Y)\ \mathrm{d}x.$$

page 209, Solution 10.42

`Replace` $f_Y(x)$ `by` $f_Y(y)$ `in the first displayed formula.`

page 210, Solution 10.43

`Replace` $f_Y(x)$ `by` $f_Y(y)$ `in the first displayed formula.`

page 210, Solution 10.44

`Replace Solution 10.44 by:`

First we compute the density of Y,

$$\begin{aligned}f_Y(y) &= \int_{-\infty}^{\infty} f_{X,Y}(x,y)\mathrm{d}x \\ &= K\int_{-y/2}^{1} (2x+y)\,\mathrm{d}x = K\left(1+y+y^2/4\right)\end{aligned}$$

if $-2 \le y \le 2$, and zero otherwise. Next, we obtain

$$h(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{2x+y}{1+y+y^2/4}$$

if $-2 \le -y \le 2x \le 2$, and zero otherwise. Finally,

$$E(X|Y) = \int_{-\infty}^{\infty} xh(x|Y)\mathrm{d}x = \int_{-Y/2}^{1} \frac{2x^2+xY}{1+Y+Y^2/4}\mathrm{d}x = \frac{2}{3} - \frac{1}{6}Y.$$