On the Birman-Corran conjecture

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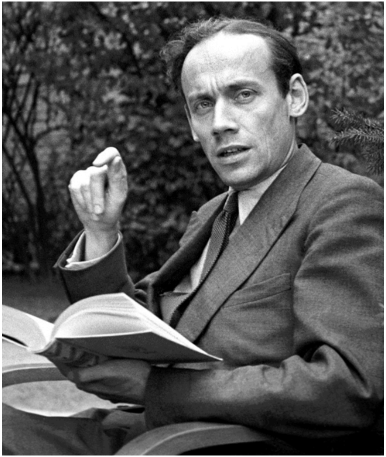

Emil Artin (1898–1962)

Relations

$$\langle \sigma_i \sigma_j \rangle^{m_{ij}} = \langle \sigma_j \sigma_i \rangle^{m_{ij}} \qquad \text{whenever } m_{ij} < \infty; \tag{1}$$

$$\langle \sigma_i \sigma_j \rangle^{m_{ij}-1} \tau_k = \tau_j \langle \sigma_i \sigma_j \rangle^{m_{ij}-1} \qquad \text{for } m_{ij} < \infty,\ k = \begin{cases} i \text{ if } m_{ij} \text{ odd} \\ j \text{ if } m_{ij} \text{ even;} \end{cases} \tag{2}$$

$$\tau_i \tau_j = \tau_j \tau_i \qquad \text{where } m_{ij} = 2; \tag{3}$$

$$\sigma_i \tau_i = \tau_i \sigma_i \qquad \text{for all } i \in I. \tag{4}$$

Some things we need...

$$\begin{aligned} \varphi : \mathcal{S}_M &\to \mathbb{Z}[\mathcal{B}_M] \\ \sigma_i^{\pm 1} &\mapsto \sigma_i^{\pm 1} \\ \tau_i &\mapsto \sigma_i^2 + 1 \qquad \forall i \in I. \end{aligned}$$

$$\begin{aligned} \overline{\varphi} : \mathcal{S}_M^+ &\to \mathbb{Z}\left[\mathcal{B}_M^+\right] \\ \sigma_i &\mapsto \sigma_i \\ \tau_i &\mapsto \sigma_i^2 + 1 \qquad \forall i \in I. \end{aligned}$$

- There exists a fundamental word $\Delta = \mathcal{L}(S)$ in $\mathcal{B}_M^+$ if and only if M is finite type.
- Let $T_1 \subseteq T$, let W be a word over $S \cup S^{-1} \cup T_1$. Then there exists an integer p and a word $\overline{W}$ over $S \cup T_1$ such that

$$W \approx \Delta^p \overline{W}.$$